HWA CHONG INSTITUTION
JC1 PROMOTIONAL EXAMINATION 2014

MATHEMATICS
Higher 2

9740/01

Paper 1

Wednesday **1 October 2014** **3 hours**

Additional materials: Answer paper
List of Formula (MF15)

READ THESE INSTRUCTIONS FIRST

Write your name and CT class on all the work you hand in, including the Cover Page which is found on Page 2.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Do not write anything on the List of Formula (MF15).

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question. At the end of the examination, place the completed cover page on top of your answer scripts and fasten all your work securely together with the string provided.

HWA CHONG INSTITUTION
2014 JC1 PROMOTIONAL EXAMINATION
Higher 2

MATHEMATICS

9740
1 October 2014
3 hours

Name:

CT: 1 4

COVER PAGE

1. Write your name, CT group and calculator model(s) in the spaces provided.
2. Arrange your answers in numerical order.
3. Detach this cover page and place it on top of your answer paper and fasten them securely together with the string provided.

For Examiner's Use			
Question No.	**Marks Obtained**	**Total Marks**	**Remarks**
1		4	
2		5	
3		6	
4		6	
5		6	
6		7	
7		9	
8		9	
9		11	
10		11	
11		13	
12		13	
TOTAL		**100**	

Graphing Calculator Model:

Scientific Calculator Model:

1 The table below shows the household utilities usage and bills of four neighbours Anne, Beth, Cathy and Denise for a particular month. The unit charge for electricity, gas and water are $\$x$, $\$y$ and $\$z$ respectively. Unfortunately, the record for gas usage on Denise's bill was smudged and could not be read.

	Anne	Beth	Cathy	Denise
Electricity (kWh)	1521	1806	1089	1616
Gas (kWh)	103	68	97	
Water (m^3)	35.6	41.1	33.0	38.2
Total amount ($)	459.81	533.16	343.11	481.83

Find the amount of gas used by Denise's household in that month. [4]

2 A sequence $\{u_n\}$ of positive terms has recurrence relation $u_{n+1} = \frac{1}{u_n} + 3,\ u_n \geq 1$.

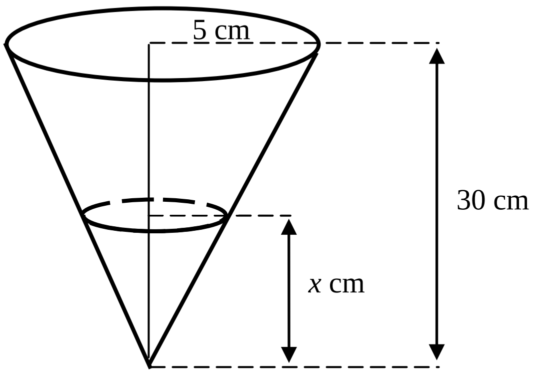
(i) Given that $u_n \to L$ as $n \to \infty$, find the exact value of L. [2]

(ii) By considering $u_{n+1} - u_n$, show algebraically that $u_{n+1} > u_n$ if $u_n < L$. [3]

3 Prove by mathematical induction that $\sum_{r=2}^{n} \ln\left(1 + \frac{2}{r-1}\right) = \ln\left[\frac{n(n+1)}{2}\right],\ n \geq 2$. [4]

Hence find $\sum_{r=1}^{n} \ln\left(1 + \frac{2}{r}\right)$. [2]

4 A large paper drinking cup, in the shape of a cone of height $30\,\text{cm}$ and radius $5\,\text{cm}$ as shown in the diagram, is initially empty. Water is poured into the cup at a constant rate of $3\ \text{cm}^3\text{s}^{-1}$. It is given that the depth of the water in the cup is x cm at time t seconds.



(i) Show that the volume of the water in the cup at time t seconds is given by $\frac{\pi x^3}{108}\ \text{cm}^3$. [2]

(ii) Find the rate of change of x at $t = 2$ seconds. [4]

[The volume of a cone of radius r and height h is $\frac{1}{3}\pi r^2 h$.]

[Turn over

5 A curve C has equation $4(x-k)^2 = k\left(y^2-4\right)$, where k is a positive constant.

Sketch C, indicating clearly the equations of asymptotes, coordinates of turning points and axial intercepts. [4]

Describe a sequence of transformations that maps the graph of $y^2 - x^2 = 4$ onto C. [2]

6 A grain storage facility holds 10 000 bushels of grain at the end of December 2013. An insurance policy provides a pay-out to the facility based on the amount of grain held by the facility. This pay-out is then used to purchase more grain for the facility, adding 2% of grain before the end of every month.

(a) Calculate the amount of grain in the facility at the end of December 2014. [1]

(b) Starting in January 2014, the grain storage facility has a one-year partnership with an agricultural research laboratory, supplying x bushels of grain in each month to the laboratory for research. The grain is supplied to the laboratory before the addition of 2% of grain to the facility due to the insurance pay-out.

(i) If $x = 100$, show that the amount of grain in the facility at the end of March 2014 is $10\,299.9$ bushels, correct to 1 decimal place. [2]

(ii) Find the maximum amount of grain that can be supplied for research in each month. [4]

7 The curve G has equation $\mathrm{e}^{x+y} = x - y$. It is given that G has only one turning point.

(i) Show that $1 + \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2}{1+\mathrm{e}^{x+y}}$. [4]

(ii) Hence show that $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -\dfrac{1}{2}\left(1 - \dfrac{\mathrm{d}y}{\mathrm{d}x}\right)\left(1 + \dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2$. [3]

(iii) Use the result in part **(ii)** to explain whether the turning point of G is a maximum or a minimum. [2]

8 A point $P(1,\ 4, -3)$ lies on the line L with equation $1 - x = \dfrac{z+3}{2}$, $y = 4$. The plane p_1 has equation $2x - y - 2z = 16$.

(i) Find Q, the point of intersection between L and p_1. [3]

(ii) Find the exact value of the sine of the acute angle between L and p_1. Hence find the length of projection of $\overrightarrow{PQ}$ onto p_1. [4]

(iii) Find, in the form $\mathbf{r} \cdot \mathbf{n} = d$, the equation of the plane p_2 that contains L and is perpendicular to p_1. [2]

9 A curve H has parametric equations $x = \mathrm{e}^t + t$, $y = t^3 + t$.

(i) Find an equation of the tangent to H at the point where it is parallel to the line $y = \frac{1}{2}x + 2$. [4]

(ii) Sketch H, showing the axial intercepts. [3]

(iii) Find the area of the region bounded by H, the lines $x = \mathrm{e} + 1$, $y = 5$ and the vertical axis. [4]

10. Relative to the origin O, the points A, B and C have position vectors given by $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{OC} = \mathbf{c}$ and $4\mathbf{a} = 5\mathbf{b} - \mathbf{c}$.

(i) Show that A, B and C are collinear. [2]

Given that OAB is an equilateral triangle.

(ii) Show that $\mathbf{a} \cdot \mathbf{b} = \frac{1}{2}|\mathbf{b}|^2$. [2]

(iii) The point D lies on OB produced such that $OD = \lambda OB$ and OD is perpendicular to CD. Find the value of λ. [4]

(iv) Hence by using the vector product, find the area of ΔOCD, leaving your answer in the form of $k|\mathbf{b}|^2$, where the exact value of k is to be determined.[3]

11 **(a)** Find $\int x \ln\left(x^2 - 4\right) \mathrm{d}x$. [4]

(b) Use the substitution $x = a\cos u$ to find $\int_0^{\frac{a}{2}} \sqrt{a^2 - x^2}\, \mathrm{d}x$ where a is a positive constant. Leave your answer in exact form. [5]

(c) The region R is bounded by the curve $y = \frac{\sqrt{4x^4 - 1}}{x^2}$, the lines $y = 0$ and $y = 1$.

Find the volume of the solid obtained when R is rotated through 2 right angles about the y-axis, leaving your answer in exact form. [4]

12 The function f is defined by $\mathrm{f} : x \mapsto \cos\left(\frac{\pi x}{2}\right), \quad x \in \mathbb{R}, -2 < x \leq 0$.

(i) Sketch the graph of $y = \mathrm{f}(x)$, indicating clearly the coordinates of all axial intercepts and end points. [2]

(ii) Show that f^{-1} exists, and find its rule and domain. [4]

The function g is defined by $\mathrm{g} : x \mapsto (2x+1)^{\frac{2}{3}}, \quad x \in \mathbb{R}, -2 < x \leq 2$.

(iii) Find the set of values of x such that $\mathrm{g}(x) \geq \mathrm{f}(x)$. [4]

(iv) Explain clearly why gf exists. Hence, find the range of gf. [3]