HWA CHONG INSTITUTION
JC1 PROMOTIONAL EXAMINATION 2015

MATHEMATICS
Higher 2

9740/01

Paper 1

Wednesday **30 September 2015** **3 hours**

Additional materials: Answer paper
List of Formula (MF15)

READ THESE INSTRUCTIONS FIRST

Write your name and class on the Cover Page (Page 2) and all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Do not write anything on the List of Formula (MF15).

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question. At the end of the examination, place the completed cover page on top of your answer scripts and fasten all your work securely together with the string provided.

HWA CHONG INSTITUTION
2015 JC1 PROMOTIONAL EXAMINATION
Higher 2

MATHEMATICS

9740
30 September 2015
3 hours

Name:

CT: 1 5

COVER PAGE

1. Write your name, CT group and calculator model(s) in the spaces provided.
2. Arrange your answers in numerical order.
3. Detach this cover page and place it on top of your answer paper and fasten them securely together with the string provided.

For Examiner's Use			
Question No.	**Marks Obtained**	**Total Marks**	**Remarks**
1		4	
2		6	
3		6	
4		7	
5		8	
6		8	
7		9	
8		9	
9		9	
10		10	
11		11	
12		13	
TOTAL		**100**	

Graphing Calculator Model:

Scientific Calculator Model:

1 Three neighbours, Mrs Toh, Mrs Ng and Mrs Wee buy three different types of vegetables from a supermarket. To promote a healthy lifestyle, a \$1 rebate is given to a customer who buys more than 2.5 kilograms of any one type of vegetables from the supermarket. Mrs Wee belongs to the Pioneer Generation and she holds a Pioneer Card which entitles her to a 3% discount of her purchases. The three neighbours are unable to recall the individual prices per kilogram for each type of the vegetables but they know the total amounts that they have to pay. The masses of the vegetables in kilogram and the total amount the three neighbours have to pay are shown in the following table.

	Mrs Toh	Mrs Ng	Mrs Wee
Kang Kong (kg)	1.5	2.0	0.5
Chye Sim (kg)	1.0	3.0	2.0
Nai Bai (kg)	2.0	0.5	1.1
Total amount paid (\$)	12.70	12.60	9.70

Assuming that the price per kilogram for each type of vegetable to be paid by each of the neighbours is the same, calculate the price per kilogram for each type of vegetables. [4]

2 The curve C has equation

$$4x^2+y^2+8mx-4y+4=0, \text{ where } m\in\mathbb{R},\ m>0.$$

Show that C can be expressed in the form $\left(\frac{x+a}{b}\right)^2+\left(\frac{y+c}{d}\right)^2=1$, where a, b, c and d are constants to be determined in terms of m.

Hence describe a sequence of transformations, in terms of m, which transforms the graph of $x^2+\left(y-\frac{1}{m}\right)^2=1$ to C. [6]

3 Prove that $\frac{3r^2-2r-3}{(r+1)!}=\frac{3}{(r-1)!}-\frac{5}{r!}+\frac{2}{(r+1)!}$, showing your workings clearly. [1]

Hence evaluate $\sum_{r=1}^{n}\frac{3r^2-2r-3}{(r+1)!}$, express your answer in the form $1-\mathrm{f}(n)$ where $\mathrm{f}(n)$ is a single fraction in terms of n. [3]

Deduce that $\sum_{r=1}^{n}\frac{r^2-r-1}{(r+1)!}<\frac{1}{3}$. [2]

[Turn over

4 The point A has coordinates $\left(-2,\ \frac{9}{2},\ 3\right)$, the line l_1 has equation $\frac{x+3}{2}=\frac{7-y}{5},\ z=3$ and the plane π has equation $\mathbf{r}.\begin{pmatrix}1\\-2\\1\end{pmatrix}=10$.

(i) Verify that A lies on l_1. [1]

(ii) Find the exact perpendicular distance from A to π. [2]

(iii) A line l_2 passes through A, is parallel to π and is perpendicular to l_1.

(a) Write down the distance from l_2 to π. [1]

(b) Find a vector equation of the line l_2. [3]

5 The function f and g are defined by

$$\mathrm{f}: x \mapsto 2\cos x,\ x\in[-2\pi, 2\pi],$$

$$\mathrm{g}: x \mapsto \frac{2x-1}{x-1},\ x\in\mathbb{R},\ x<1.$$

(i) Give a reason why f does not have an inverse. [1]

(ii) The function f has an inverse if its domain is restricted to $\frac{\pi}{2}\le x\le b$.

Find the greatest value of b. Define f^{-1} in similar form. [3]

(iii) Under the restricted domain of f in part **(ii)**, state the range of the composite function $(\mathrm{gf})^{-1}$. Hence find $(\mathrm{gf})^{-1}\left(\frac{3}{2}\right)$. [4]

6 **(a)** Find $\int x\sec^2 x\,\mathrm{d}x$. [3]

(b) By using the substitution $x=\frac{1}{2}(1+\sin\theta)$, find the exact value of

$$\int_{\frac{1}{2}}^{1}\sqrt{x-x^2}\,\mathrm{d}x.$$ [5]

7 Use the method of mathematical induction to prove that

$$\sum_{r=2}^{n}\ln\left[\frac{r^2-1}{r^2}\right]=\ln\left[\frac{n+1}{2n}\right].$$ [5]

Hence express $\sum_{r=5}^{15}\ln\left[\frac{2(r^2-1)}{r^2}\right]$ in the form $A\ln 2+B\ln 3+C\ln 5$, where A, B and C are integers to be found. [4]

8 William joined a company with a starting monthly pay of \$2000 in January 2004. In January 2005, he received an increment of 50% of his previous monthly pay. In January for each subsequent year, he received an increment of 50% of his previous increment. In other words, in January 2005, his increment was \$1000; in January 2006, his increment was \$500 and so on.

Let U_n denote the pay William received in the n^{th} year (where 2004 was the 1^{st} year, 2005 was the 2^{nd} year, and so on).

(i) Find U_1, U_2, U_3 and show that $U_n = 48000\left(1-0.5^n\right)$. Hence by considering $\sum_{r=1}^{n} U_r$, find the total pay William received in the first n years. [6]

(ii) William decided to quit if his increment fell below \$10. In which year would he quit the company? [3]

9 Relative to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. The point P has position vector $\mathbf{p} = \lambda\mathbf{a} + (1-\lambda)\mathbf{b}$, for $0 < \lambda < 1$. Show that A, B and P are collinear. [2]

Given that OA is perpendicular to OB and $\angle APO = 90°$. By using a scalar product, show that $(1-\lambda)|\mathbf{b}|^2 = \lambda|\mathbf{a}|^2$. [3]

Hence by using a vector product, find the values of λ if the area of the triangle OBP is $\frac{\sqrt{2}}{6}|\mathbf{b}|^2$. [4]

10 The curve G has equation $y = \dfrac{x^2 - 2x + 2}{x - 1}$.

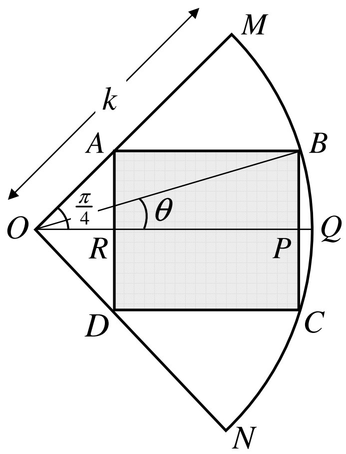
Sketch G, giving the equations of any asymptotes and the coordinates of any stationary points. [4]

The finite region bounded by G and the lines $y = 2$ and $x = 3$ is denoted by R.

(a) Find the exact area of R. [3]

(b) Find the volume of revolution formed when R is rotated through 2π radians about the y-axis, giving your answer correct to 3 decimal places. [3]

11



The diagram shows a plot of land $OMQN$ in the shape of a quadrant, where $\measuredangle MON$ is a right angle. It is given that the length of OM is k metres, where k is a constant and $\measuredangle MOQ$ is $\frac{\pi}{4}$ radians. A rectangular grass patch $ABCD$ is to be laid within $OMQN$ such that the vertex A is on OM, the vertex D is on ON, and the vertices B and C are on the arc MQN. The points R and P are on OQ such that the straight line $ORPQ$ is perpendicular to AD, and $\measuredangle BOQ$ is θ radians.

(i) By considering the triangle OBP, find the length of BP in terms of k and θ. [1]

(ii) Show that the length of PR is $k(\cos\theta - \sin\theta)$ metres. [2]

(iii) Let S be the area of $ABCD$. By using differentiation, find the maximum value of S in terms of k. [6]

(iv) Sketch the graph of S against θ. State briefly how the graph relates to the result found in part **(iii)**. [2]

12 A curve has parametric equations

$$x = p(\cos 2\theta - 2\cos\theta), \quad y = p\sin\frac{3\theta}{2},$$

where p is a positive constant and $0 < \theta < \pi$.

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{3}{8}\operatorname{cosec}\frac{\theta}{2}$. [3]

(ii) Find, in terms of p, the exact equation of the normal to the curve at the point when $x = -p$. [4]

(iii) Find the exact value of p such that the normal found in part **(ii)** intersects another curve with equation $y = -\frac{\sqrt{2}p}{x}$ at exactly one point. [3]

(iv) Given that y decreases at a rate of 1.5 units per second, find the rate of change of $x + y$ when $\theta = 1$ radian. Give your answer correct to 4 decimal places. [3]