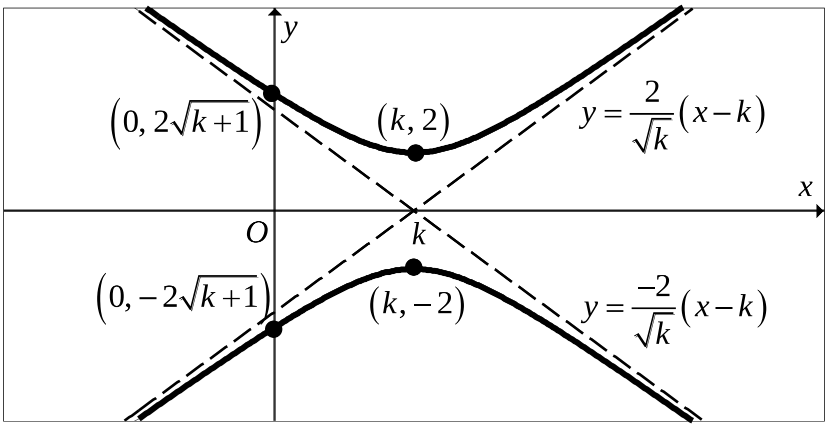
2014 H2 Mathematics Promotional Examination

Qn	Solution
1	$1521x+103y+35.6z=459.81 \quad ----(1)$ $1806x+68y+41.1z=533.16 \quad ----(2)$ $1089x+97y+33.0z=343.11 \quad ----(3)$ From G.C., $x=0.26,\ y=0.21,\ z=1.2$ Let k be the gas usage on Denise's bill. $1616(0.26)+k(0.21)+38.2(1.2)=481.83 \Rightarrow k=75.381=75.4$ (to 3 s.f.) Denise's household used 75.4 kWh of gas in that month.
2(i)	As $n\to\infty,\ u_n\to L$ and $u_{n+1}\to L$ Thus $L=\frac{1}{L}+3$ $\Rightarrow L^2-3L-1=0$ $\Rightarrow L=\frac{3\pm\sqrt{9+4}}{2}=\frac{3+\sqrt{13}}{2}$ (reject $\frac{3-\sqrt{13}}{2}$)
2(ii)	$u_{n+1}-u_n=\frac{1}{u_n}+3-u_n=\frac{1+3u_n-u_n{}^2}{u_n}$ $=\frac{-\left(u_n-\frac{3+\sqrt{13}}{2}\right)\left(u_n-\frac{3-\sqrt{13}}{2}\right)}{u_n}$ $=\frac{-(u_n-L)(u_n-(-0.3028))}{u_n}$ $=\frac{(L-u_n)(u_n+0.3028)}{u_n}$ Since $u_n<L$ and u_n is positive $\Rightarrow \frac{(L-u_n)(u_n+0.3028)}{u_n}>0$. Thus $u_{n+1}-u_n>0 \Rightarrow u_{n+1}>u_n$.
3.	Let $\mathrm{P}(n)$ be the proposition $\sum_{r=2}^{n}\ln\left(1+\frac{2}{r-1}\right)=\ln\left[\frac{n(n+1)}{2}\right],\ n\ge 2$. Show $\mathrm{P}(2)$ is true: LHS = $\sum_{r=2}^{2}\ln\left(1+\frac{2}{r-1}\right)=\ln\left(1+\frac{2}{1}\right)=\ln 3$ and RHS = $\ln\left[\frac{2(3)}{2}\right]=\ln 3$ $\therefore$ $\mathrm{P}(2)$ is true. Assume $\mathrm{P}(k)$ is true for some $k\in\mathbb{Z},\ k\ge 2$. Show $\mathrm{P}(k+1)$ is true: LHS $=\sum_{r=2}^{k+1}\ln\left(1+\frac{2}{r-1}\right)$

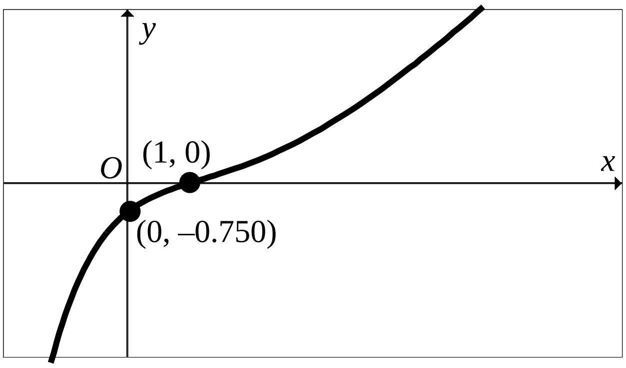
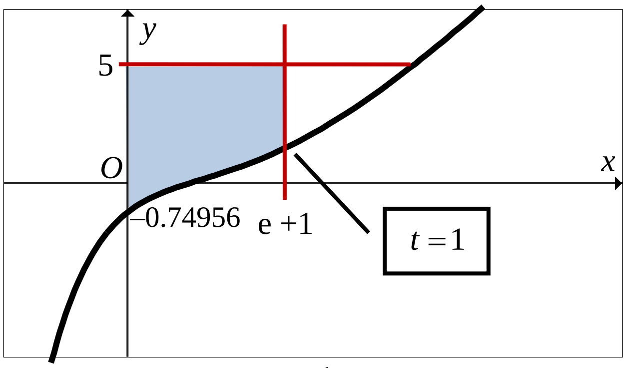
	$= \sum_{r=2}^{k} \ln\left(1+\frac{2}{r-1}\right) + \ln\left(1+\frac{2}{k}\right)$ $= \ln\left[\frac{k(k+1)}{2}\right] + \ln\left(\frac{k+2}{k}\right)$ $= \ln\left[\frac{k(k+1)}{2} \cdot \frac{k+2}{k}\right] = \ln\left[\frac{(k+1)(k+2)}{2}\right]$ $\therefore$ $\mathrm{P}(k+1)$ is true. Since $\mathrm{P}(2)$ is true, and $\mathrm{P}(k)$ is true $\Rightarrow \mathrm{P}(k+1)$ is true, by mathematical induction, $\mathrm{P}(n)$ is true for all $n \in \mathbb{Z},\ n \geq 2$.
	Method 1 (Starting from previous result) From $\sum_{r=2}^{n} \ln\left(1+\frac{2}{r-1}\right) = \ln\left[\frac{n(n+1)}{2}\right]$ Replace r by $r+1$: $\Rightarrow \sum_{r+1=2}^{r+1=n} \ln\left(1+\frac{2}{r}\right) = \ln\left[\frac{n(n+1)}{2}\right]$ $\Rightarrow \sum_{r=1}^{n-1} \ln\left(1+\frac{2}{r}\right) = \ln\left[\frac{n(n+1)}{2}\right]$ Finally, replace n by $n+1$: $\Rightarrow \sum_{r=1}^{n} \ln\left(1+\frac{2}{r}\right) = \ln\left[\frac{(n+1)(n+2)}{2}\right]$ Method 2 (Starting from current question) From $\sum_{r=1}^{n} \ln\left(1+\frac{2}{r}\right)$ Replace r by $r-1$: $\Rightarrow \sum_{r-1=1}^{r-1=n} \ln\left(1+\frac{2}{r-1}\right)$ $\Rightarrow \sum_{r=2}^{r=n+1} \ln\left(1+\frac{2}{r-1}\right)$ Since $\sum_{r=2}^{n} \ln\left(1+\frac{2}{r-1}\right) = \ln\left[\frac{n(n+1)}{2}\right]$, therefore by replacing n by $n+1$ in this result, $\sum_{r=2}^{n+1} \ln\left(1+\frac{2}{r-1}\right) = \ln\left[\frac{(n+1)(n+2)}{2}\right]$ That is, $\sum_{r=1}^{n} \ln\left(1+\frac{2}{r}\right) = \ln\left[\frac{(n+1)(n+2)}{2}\right]$
4(i)	Using proportion of triangles, $\frac{x}{r} = \frac{30}{5} \Rightarrow r = \frac{1}{6}x$ Thus $V = \frac{1}{3}\pi r^2 x = \frac{1}{3}\pi\left(\frac{1}{6}x\right)^2 x = \frac{\pi x^3}{108}$ $\therefore$ Volume of water in the cup $= \frac{\pi x^3}{108}$ cm^3

4(ii)	$\frac{dV}{dx}=\frac{\pi x^2}{36}$ $\frac{dx}{dt}=\frac{dx}{dV}\times\frac{dV}{dt}=\frac{108}{\pi x^2}$ At $t = 2$ s, volume of water in the container is 6 cm^3. Thus $6=\frac{\pi x^3}{108}\Rightarrow x=\sqrt[3]{\frac{648}{\pi}}$ cm (≈ 5.90847) $\therefore \frac{dx}{dt}=\frac{108}{\pi}\left(\frac{\pi}{648}\right)^{\frac{2}{3}}=0.985\text{ cms}^{-1}$ (to 3.s.f)
5	From $4(x-k)^2=k(y^2-4)$ $\Rightarrow ky^2-4(x-k)^2=4k$ $\Rightarrow \frac{y^2}{2^2}-\frac{(x-k)^2}{(\sqrt{k})^2}=1\Rightarrow$ the curve is a NS-hyperbola with centre $(k, 0)$. Thus the curve is a NS-hyperbola with centre $(k, 0)$. Thus the turning points are $(k, 2)$ and $(k, -2)$. Asymptotes are $\frac{y}{2}=\pm\frac{x-k}{\sqrt{k}}$ $\Rightarrow y=\frac{2}{\sqrt{k}}x-2\sqrt{k}$ or $y=-\frac{2}{\sqrt{k}}x+2\sqrt{k}$ Substitute $x=0$ into equation of curve $\Rightarrow 4(0-k)^2=k(y^2-4)$ $\Rightarrow y=\sqrt{4k+4}=2\sqrt{k+1}$ Thus axial intercepts are $(0, 2\sqrt{k+1})$ and $(0, -2\sqrt{k+1})$. 

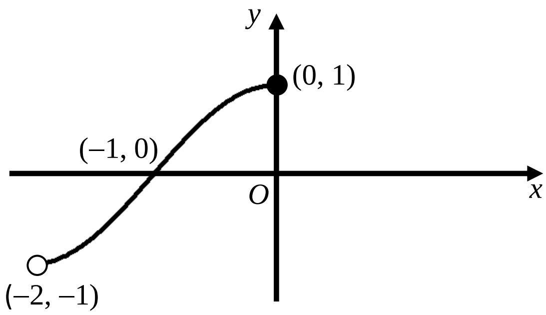
5	<u>Method 1 (Scale then Translate)</u> Start from $y^2 - x^2 = 4$: $\boxed{1}$ Stretch by factor $\frac{\sqrt{k}}{2}$ parallel to the x-axis to obtain $y^2 - \left(\frac{2}{\sqrt{k}}x\right)^2 = 4 \Rightarrow y^2 - \frac{4x^2}{k} = 4$. $\boxed{2}$ Translate k units in the positive x-direction to obtain $y^2 - \frac{4(x-k)^2}{k} = 4 \Rightarrow 4(x-k)^2 = k(y^2-4)$ <u>Method 2 (Translate then Scale)</u> Start from $y^2 - x^2 = 4$: $\boxed{1}$ Translate $2\sqrt{k}$ units in the positive x-direction to obtain $y^2 - (x-2\sqrt{k})^2 = 4$ $\boxed{2}$ Stretch parallel to the x-axis by a factor of $\frac{\sqrt{k}}{2}$ to obtain $y^2 - \left(\frac{2}{\sqrt{k}}x - 2\sqrt{k}\right)^2 = 4 \Rightarrow y^2 - \frac{4}{k}(x-k)^2 = 4$ $\Rightarrow 4(x-k)^2 = k(y^2-4)$
6(a)	Amount of grain = $10000(1.02)^{12}$ = 12700 bushels (to 3 s.f.)
6(b)(i)	End Jan 2014: $10000(1.02) - 100(1.02)$ End Feb 2014: $10000(1.02)^2 - 100(1.02)^2 - 100(1.02)$ End Mar 2014: $10000(1.02)^3 - 100\left((1.02)^3 + (1.02)^2 + (1.02)\right) = 10299.9$ (to 1 d.p.)
6(b)(ii)	End Dec 2014: $10000(1.02)^{12} - x\left((1.02)^{12} + (1.02)^{11} + \ldots + (1.02)\right)$ $= 10000(1.02)^{12} - x(1.02)\frac{1.02^{12}-1}{1.02-1}$ Let $10000(1.02)^{12} - x(1.02)\frac{1.02^{12}-1}{1.02-1} \geq 0$ $\Rightarrow x \leq 927.0549$ $\therefore$ max = 927 bushels (to 3 s.f.)
7(i)	From $e^{x+y} = x - y$, differentiate w.r.t x: $\Rightarrow e^{x+y}\left(1 + \frac{dy}{dx}\right) = 1 - \frac{dy}{dx}$ $\Rightarrow e^{x+y} + e^{x+y}\frac{dy}{dx} + \frac{dy}{dx} = 1$ $\Rightarrow \frac{dy}{dx}\left(1 + e^{x+y}\right) = 1 - e^{x+y}$ $\Rightarrow \frac{dy}{dx} = \frac{1-e^{x+y}}{1+e^{x+y}}$ $\Rightarrow 1 + \frac{dy}{dx} = 1 + \frac{1-e^{x+y}}{1+e^{x+y}} = \frac{1+e^{x+y}+1-e^{x+y}}{1+e^{x+y}}$ $\Rightarrow 1 + \frac{dy}{dx} = \frac{2}{1+e^{x+y}}$

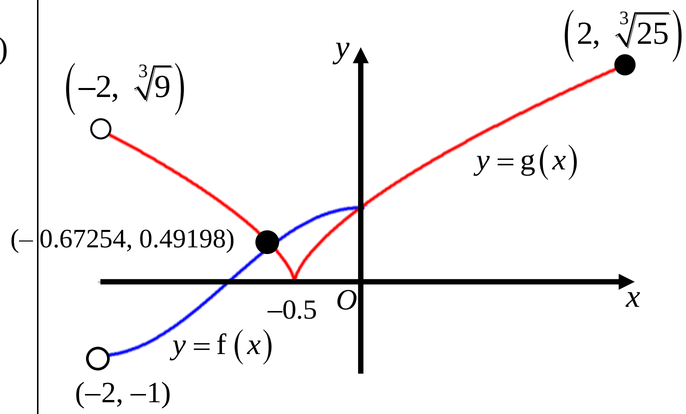
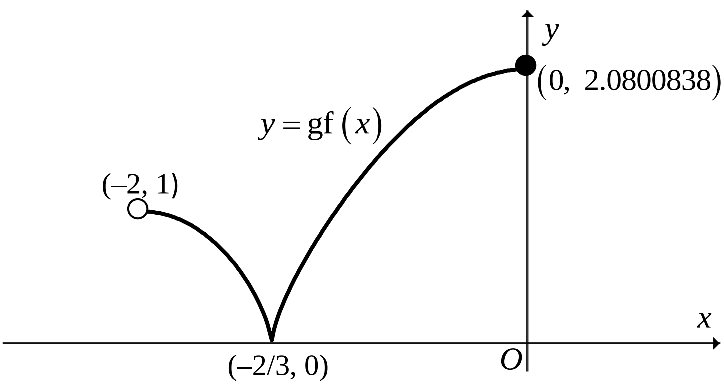
7(ii)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-2\left(1+e^{x+y}\right)^{-2}\left(e^{x+y}\left(1+\frac{\mathrm{d}y}{\mathrm{d}x}\right)\right)$ $=\frac{-2e^{x+y}\left(1+\frac{\mathrm{d}y}{\mathrm{d}x}\right)}{\left(1+e^{x+y}\right)^2}=\frac{-2\left(\frac{1-\frac{\mathrm{d}y}{\mathrm{d}x}}{1+\frac{\mathrm{d}y}{\mathrm{d}x}}\right)\left(1+\frac{\mathrm{d}y}{\mathrm{d}x}\right)}{\left(\frac{2}{1+\frac{\mathrm{d}y}{\mathrm{d}x}}\right)^2}=-\frac{1}{2}\left(1-\frac{\mathrm{d}y}{\mathrm{d}x}\right)\left(1+\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$
7(iii)	When $\frac{\mathrm{d}y}{\mathrm{d}x}=0$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{1}{2}$ Since $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}<0$, hence the turning point is a maximum point.
8(i)	<u>Method 1 (Using equation of L in vector form)</u> Write $L:\mathbf{r}=\begin{pmatrix}1\\4\\-3\end{pmatrix}+\lambda\begin{pmatrix}-1\\0\\2\end{pmatrix},\quad \lambda\in\mathbb{R}$ From $p_1: 2x-y-2z=16 \Rightarrow \mathbf{r}\cdot\begin{pmatrix}2\\-1\\-2\end{pmatrix}=16$ Substitute equation of line into plane: $\begin{pmatrix}1-\lambda\\4\\-3+2\lambda\end{pmatrix}\bullet\begin{pmatrix}2\\-1\\-2\end{pmatrix}=16$ $2(1-\lambda)-4-2(-3+2\lambda)=16$ $-6\lambda+4=16$ $\lambda=-2$ $\therefore \overrightarrow{OQ}=\begin{pmatrix}3\\4\\-7\end{pmatrix}\Rightarrow Q=(3,4,-7)$ <u>Method 2 (Direct use of equation of L)</u> From equation of $L: x=1-\frac{z+3}{2}$ Substitute x and y into $2x-y-2z=16$: $2\left(1-\frac{z+3}{2}\right)-(4)-2z=16$ $2-z-3-4-2z=16$ $z=-7$ $\therefore\ x=1-\frac{(-7)+3}{2}=3\ \therefore \overrightarrow{OQ}=\begin{pmatrix}3\\4\\-7\end{pmatrix}\Rightarrow Q=(3,4,-7)$

8(ii)	Let $\mathbf{d}=\begin{pmatrix}-1\\0\\2\end{pmatrix}$ and $\mathbf{n}_1=\begin{pmatrix}2\\-1\\-2\end{pmatrix}$ and let acute angle between L and p_1 be θ. $\sin\theta=\left\lvert\frac{\mathbf{d}\cdot\mathbf{n}_1}{\lvert\mathbf{d}\rvert\lvert\mathbf{n}_1\rvert}\right\rvert=\left\lvert\frac{-2+0-4}{\left(\sqrt{5}\right)(3)}\right\rvert\Rightarrow\sin\theta=\frac{2}{\sqrt{5}}$ $\overrightarrow{PQ}=\overrightarrow{OQ}-\overrightarrow{OP}=\begin{pmatrix}-1\\4\\1\end{pmatrix}-\begin{pmatrix}1\\4\\-3\end{pmatrix}=\begin{pmatrix}-2\\0\\4\end{pmatrix}$ <u>Method 1 (Using cosine of angle)</u> Length of projection of $\overrightarrow{PQ}$ onto Π_1 $=\left\lvert\overrightarrow{PQ}\right\rvert\cos\theta=\sqrt{(-2)^2+0^2+4^2}\left(\frac{1}{\sqrt{5}}\right)$ = 2 units <u>Method 2 (Direct use of angle)</u> Find $\theta=63.43494882°$ Length of projection of $\overrightarrow{PQ}$ onto $\Pi_1=\left\lvert\overrightarrow{PQ}\right\rvert\cos\theta$ $=\sqrt{(-2)^2+0^2+4^2}\ \cos\left(63.43494882°\right)$ = 2.002439516 = 2.00 units (to 3 s.f.)
8(iii)	$\mathbf{d}\times\mathbf{n}_1=\begin{pmatrix}-1\\0\\2\end{pmatrix}\times\begin{pmatrix}2\\-1\\-2\end{pmatrix}=\begin{pmatrix}2\\2\\1\end{pmatrix}$ Let normal vector of p_2 be $\mathbf{n}_2=\begin{pmatrix}2\\2\\1\end{pmatrix}$. Thus p_2 : $\mathbf{r}\cdot\begin{pmatrix}2\\2\\1\end{pmatrix}=\begin{pmatrix}1\\4\\-3\end{pmatrix}\cdot\begin{pmatrix}2\\2\\1\end{pmatrix}\Rightarrow p_2$: $\mathbf{r}\cdot\begin{pmatrix}2\\2\\1\end{pmatrix}=7$ $\boxed{\text{Or use } \begin{pmatrix}-1\\4\\1\end{pmatrix}\cdot\begin{pmatrix}2\\2\\1\end{pmatrix}}$
9(i)	$\frac{\mathrm{d}x}{\mathrm{d}t}=\mathrm{e}^t+1$ $\qquad$ $\frac{\mathrm{d}y}{\mathrm{d}t}=3t^2+1$ $\frac{\mathrm{d}x}{\mathrm{d}t}\times\frac{\mathrm{d}t}{\mathrm{d}y}=\frac{\mathrm{d}y}{\mathrm{d}x}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3t^2+1}{\mathrm{e}^t+1}$ Since tangent is parallel to the line, the gradients are equal $\Rightarrow\frac{3t^2+1}{\mathrm{e}^t+1}=\frac{1}{2}\Rightarrow 6t^2-\mathrm{e}^t+1=0$ From GC, $t=0,\ 0.18287975$ or 5.02862. Thus equation of tangent is $y-\left(t^3+t\right)=\frac{1}{2}\left(x-\left(\mathrm{e}^t+t\right)\right)$ $\Rightarrow y=\frac{1}{2}(x-1)$ or $y=\frac{1}{2}x-0.503$ or $y=\frac{1}{2}x+53.3$

9 (ii)	 When $y = t^3 + t = 0 \Rightarrow t = 0$ Thus $x = \mathrm{e}^t + t = 1 \Rightarrow (1, 0)$ When $x = \mathrm{e}^t + t = 0$, by GC, $t = -0.56714$ Thus $y = t^3 + t = -0.74956 = -0.750$ (to 3 s.f.)
9 (iii)	 Area of the region $= \int_0^{\mathrm{e}+1} 5 - y \, \mathrm{d}x$ $= \int_{-0.56714}^{1} \left(5 - t^3 - t\right)\left(\mathrm{e}^t + 1\right) \mathrm{d}t$ = 16.592 = 16.6 unit^2 (to 3 s.f.)
10(i)	<u>Method 1</u> $\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = \mathbf{b} - \frac{1}{4}(5\mathbf{b} - \mathbf{c}) = \frac{1}{4}(\mathbf{c} - \mathbf{b}) \Rightarrow \overrightarrow{AB} = \frac{1}{4}\overrightarrow{BC}$ Since $\overrightarrow{AB} \,//\, \overrightarrow{BC}$ and B is a common point, A, B and C are collinear. (Shown) <u>Method 2</u> $4\overrightarrow{OA} = 5\overrightarrow{OB} - \overrightarrow{OC}$ $= 4\overrightarrow{OB} + \overrightarrow{OB} - \overrightarrow{OC}$ $4\left(\overrightarrow{OA} - \overrightarrow{OB}\right) = \overrightarrow{OB} - \overrightarrow{OC}$ $4\overrightarrow{BA} = \overrightarrow{CB}$ Since $\overrightarrow{BA} \,//\, \overrightarrow{CB}$ and B is a common point, A, B and C are collinear. (Shown)

10 (ii)	Given ΔOAB is equilateral, $\lvert\mathbf{a}\rvert = \lvert\mathbf{b}\rvert$ and $\angle AOB = \dfrac{\pi}{3}$. $\mathbf{a}\cdot\mathbf{b} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos(\angle AOB)$ $= \lvert\mathbf{b}\rvert^2 \cos\left(\dfrac{\pi}{3}\right) = \dfrac{1}{2}\lvert\mathbf{b}\rvert^2$ (shown)
10 (iii)	Since D lies on OB produced, let $\overrightarrow{OD} = \lambda\mathbf{b}$. Then $4\overrightarrow{OA} = 5\overrightarrow{OB} - \overrightarrow{OC} \Rightarrow \overrightarrow{OC} = 5\mathbf{b} - 4\mathbf{a}$. $\Rightarrow \overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC} = \lambda\mathbf{b} - (5\mathbf{b} - 4\mathbf{a}) = (\lambda - 5)\mathbf{b} + 4\mathbf{a}$ Since $OD \perp CD$ $\Rightarrow \overrightarrow{OD}\cdot\overrightarrow{OC} = 0$ $\Rightarrow \lambda\mathbf{b}\cdot\left[(\lambda - 5)\mathbf{b} + 4\mathbf{a}\right] = 0$ $\Rightarrow \lambda(\lambda - 5)\mathbf{b}\cdot\mathbf{b} + 4\lambda\mathbf{b}\cdot\mathbf{a} = 0$ But $\mathbf{b}\cdot\mathbf{b} = \lvert\mathbf{b}\rvert^2$ and $\mathbf{b}\cdot\mathbf{a} = \dfrac{1}{2}\lvert\mathbf{b}\rvert^2 \Rightarrow \lambda(\lambda - 5)\lvert\mathbf{b}\rvert^2 + 2\lambda\lvert\mathbf{b}\rvert^2 = 0$ $\Rightarrow \lambda^2 - 3\lambda = 0 \Rightarrow \lambda = 3$ (note that $\lambda > 1$)
10 (iv)	Using $\overrightarrow{OD} = 3\mathbf{b}$ and $\overrightarrow{OC} = 5\mathbf{b} - 4\mathbf{a}$ <u>Method 1</u> Area of $\Delta OCD = \dfrac{1}{2}\left\lvert\overrightarrow{OC}\times\overrightarrow{OD}\right\rvert = \dfrac{1}{2}\lvert 3\mathbf{b}\times(5\mathbf{b} - 4\mathbf{a})\rvert$ $= \dfrac{1}{2}\lvert 15\mathbf{b}\times\mathbf{b} - 12\mathbf{b}\times\mathbf{a}\rvert$ $= \dfrac{1}{2}\lvert 0 - 12\mathbf{b}\times\mathbf{a}\rvert$ $= 6\lvert\mathbf{b}\times\mathbf{a}\rvert$ $= 6\left\lvert \lvert\mathbf{b}\rvert\lvert\mathbf{a}\rvert\sin\left(\dfrac{\pi}{3}\right)\right\rvert$ $= 3\sqrt{3}\lvert\mathbf{b}\rvert^2$ square units. Thus $k = 3\sqrt{3}$. <u>Method 2</u> Use $\overrightarrow{CD} = (\lambda - 5)\mathbf{b} + 4\mathbf{a} = 4\mathbf{a} - 2\mathbf{b}$ $\Delta OCD = \dfrac{1}{2}\left\lvert\overrightarrow{OD}\times\overrightarrow{CD}\right\rvert = \dfrac{1}{2}\lvert 3\mathbf{b}\times(4\mathbf{a} - 2\mathbf{b})\rvert$ $= \dfrac{1}{2}\lvert 12\mathbf{b}\times\mathbf{a} - 6\mathbf{b}\times\mathbf{b}\rvert$ $= \dfrac{1}{2}\lvert 12\mathbf{b}\times\mathbf{a} - 0\rvert$ $= 6\lvert\mathbf{b}\times\mathbf{a}\rvert$ $= 6\left\lvert \lvert\mathbf{b}\rvert\lvert\mathbf{a}\rvert\sin\left(\dfrac{\pi}{3}\right)\right\rvert$ $= 3\sqrt{3}\lvert\mathbf{b}\rvert^2$ square units. Thus $k = 3\sqrt{3}$.

11 (a)	Using integration by parts: $\int x\ln\left(x^2-4\right)\mathrm{d}x$ $=\frac{x^2}{2}\ln\left(x^2-4\right)-\int\frac{2x^3}{2\left(x^2-4\right)}\,\mathrm{d}x$ $=\frac{x^2}{2}\ln\left(x^2-4\right)-\int x+\frac{4x}{x^2-4}\,\mathrm{d}x$ Use long division $=\frac{x^2}{2}\ln\left(x^2-4\right)-\frac{1}{2}x^2-2\ln\left(x^2-4\right)+C$
11 (b)	From $x=a\cos u\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}u}=-a\sin u$ Thus $\int_0^{\frac{a}{2}}\sqrt{a^2-x^2}\,\mathrm{d}x$ $=\int_{\frac{\pi}{2}}^{\frac{\pi}{3}}\sqrt{a^2-\left(a\cos u\right)^2}\left(-a\sin u\right)\mathrm{d}u$ $=-a^2\int_{\frac{\pi}{2}}^{\frac{\pi}{3}}\sin^2u\,\mathrm{d}u=-a^2\int_{\frac{\pi}{2}}^{\frac{\pi}{3}}\frac{1-\cos 2u}{2}\mathrm{d}u$ $=\frac{a^2}{2}\left[\frac{\sin 2u}{2}-u\right]_{\frac{\pi}{2}}^{\frac{\pi}{3}}$ $=\frac{a^2}{2}\left[\left(\frac{\sin\frac{2\pi}{3}}{2}-\frac{\pi}{3}\right)-\left(\frac{\sin\pi}{2}-\frac{\pi}{2}\right)\right]$ $=\frac{a^2}{2}\left(\frac{\sqrt{3}}{4}-\frac{\pi}{3}+\frac{\pi}{2}\right)=\frac{a^2}{2}\left(\frac{\sqrt{3}}{4}+\frac{\pi}{6}\right)=a^2\left(\frac{\sqrt{3}}{8}+\frac{\pi}{12}\right)$
11 (c)	Make x^2 the subject: $x^4=\frac{4x^4-1}{y^2}\Rightarrow y^2x^4-4x^4=-1$ $\Rightarrow x^4=\frac{1}{4-y^2}\Rightarrow x^2=\frac{1}{\sqrt{4-y^2}}$ Therefore required volume $=\int_0^1\pi x^2\,\mathrm{d}y=\pi\int_0^1\frac{1}{\sqrt{4-y^2}}\mathrm{d}y=\pi\left[\sin^{-1}\left(\frac{y}{2}\right)\right]_0^1=\frac{\pi^2}{6}$ units^3
12(i)	

12 (ii)	Since any horizontal line cuts the graph of $y=\mathrm{f}(x)$ at no more than one point, thus f is one-to-one, and thus its inverse exists. Let $y=\cos\left(\frac{\pi x}{2}\right)\Rightarrow x=\frac{2}{\pi}\cos^{-1}(y)$ Thus the rule is $\mathrm{f}^{-1}(x)=\frac{2}{\pi}\cos^{-1}(x)$ The domain of f^{-1} is the range of f $=(-1,1]$.
12 (iii)	 From GC, the intersection between the graphs is at $x=-0.67254$. Thus, from the sketch, the solution set is $\{x\in\mathbb{R}: -2<x\le -0.673 \text{ or } x=0\}$.
12 (iv)	$R_{\mathrm{f}}=(-1,1]$ and $D_{\mathrm{g}}=(-2,2]$ Since $R_{\mathrm{f}}\subseteq D_{\mathrm{g}}$, thus gf exists. <u>Method 1 (Find by separate functions)</u> $D_{\mathrm{f}}=(-2,0]\xrightarrow{\mathrm{f}}R_{\mathrm{f}}=(-1,1]\xrightarrow{\mathrm{g}}\left[0,\sqrt[3]{9}\right]$. <u>Method 2 (Direct from composite function)</u> $\mathrm{gf}(x)=\left[2\cos\left(\frac{\pi x}{2}\right)+1\right]^{\frac{2}{3}},\ -2<x\le 0$  From graph, $R_{\mathrm{gf}}=[0, 2.0800838]=[0, 2.08]$ (to 3 s.f.)