



MATHEMATICS

9740

7 October 2013

3 hours

Additional Materials: Answer Paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your name and CT group on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **7** printed pages.

- 1 Sophia has a total saving of \$90 million in three accounts A , B and C with \$ x million, \$ y million and \$ z million respectively. She transfers funds among the accounts based on the table below.

Percentage of Fund transferred from initial amount in	To Account A	To Account B	To Account C
Account A	—	37.5%	12.5%
Account B	5%	—	5%
Account C	10%	20%	—

For instance, 37.5% and 12.5% of the initial amount in Account A are transferred to Account B and Account C respectively.

As a result of the funds transfer, the amount in Account A decreases by \$16 million and the amount in Account B increases by \$19 million.

- (i) By considering the amount in Account A , show that

$$0.5x - 0.05y - 0.1z = 16. \quad [1]$$

- (ii) By forming a system of linear equations, find the values of x , y and z . [3]

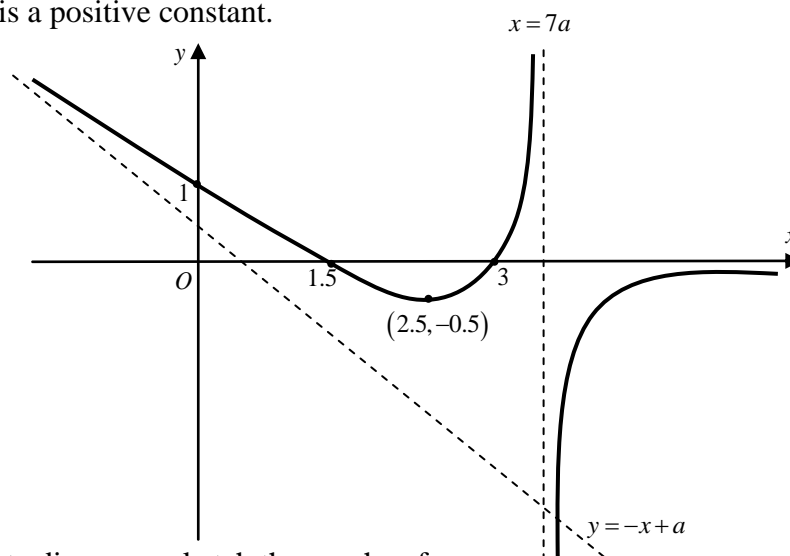
- 2 It is given that the expansion of $(2 + px)^{-q}$ in ascending powers of x , up to and including the term in x , is $\frac{1}{4} - x$. Find the values of p and q .

Find, in terms of n , the coefficient of x^n in the above expansion. [4]

- 3 A water tank contains 8000 litres of water initially. At the beginning of each day, 500 litres of water is added to the tank. At the end of each day, 10% of the amount of water in the tank will be used.

- (i) Show that the amount of water in the tank after 3 days is 7051.5 litres. [1]
 (ii) Find the least number of days it will take for the water in the tank to be less than 5000 litres. [3]
 (iii) Will the tank ever dry up? Justify your answer. [1]

- 4 The diagram below shows the graph of $y = f(x)$. It cuts the axes at the points $(0, 1)$, $(1.5, 0)$ and $(3, 0)$. It has a minimum point at $(2.5, -0.5)$. The horizontal, vertical and oblique asymptotes are $y = 0$, $x = 7a$ and $y = -x + a$ respectively, where a is a positive constant.



On separate diagrams, sketch the graphs of

(i) $y = \frac{1}{f(x)}$, [3]

(ii) $y = f'(x)$, [3]

showing clearly the axial intercepts, the stationary points and the equations of the asymptotes where applicable.

- 5 A sequence of real numbers $\{u_n\}$, for $n \in \mathbb{Z}^+$, satisfies the recurrence relation

$$\frac{u_{n+1} + a}{u_n + b} = \frac{a}{b}, \text{ with } u_1 = a, \text{ where } a \text{ and } b \text{ are fixed non-zero real constants and } a \neq b.$$

(i) Given that the limit l of the sequence $\{u_n\}$ exists, find the value of l . [2]

(ii) By expressing u_{n+1} in terms of u_n , find an expression for u_n , leaving your answer in terms of a , b and n . [2]

(iii) Given that the sum to infinity S for the sequence $\{u_n\}$ exists, state an inequality satisfied by a and b . Find S in terms of a and b . [2]

- 6 (a) By using the substitution $u = 9 + 4x^2$, find $\int x^3 \sqrt{9 + 4x^2} \, dx$. [4]
- (b) Evaluate $\int_0^1 x^2 \tan^{-1} x \, dx$, giving your answer in exact form. [4]
- 7 The coordinates of 3 points A , B and C are $(2, 0, -1)$, $(-3, 1, 2)$ and $(1, -2, -4)$ respectively.
- (a) Find the point D on the x -axis such that there exists a point P on line AB where C , D and P are collinear. [4]
- (b) Find two possible points E on the x - y plane, such that \overrightarrow{OE} is a unit vector and $\angle AOE = 150^\circ$. [4]
- 8 (i) Express $\frac{2}{r(r+1)(r+3)}$ in partial fractions. [2]
- (ii) Hence find $\sum_{r=1}^n \frac{1}{2r(r+1)(r+3)}$. [3]
- (iii) Using the result in part (ii), determine the value of $\sum_{r=5}^{\infty} \frac{1}{2r(r-2)(r-3)}$. [3]
- 9 Prove by mathematical induction that for all $n \in \mathbb{Z}^+$,
- $$1 + (1+2) + (1+2+3) + (1+2+3+4) + \dots + (1+2+3+\dots+n) = \frac{1}{6}n(n+1)(n+2).$$
- [5]
- Hence find, in terms of n ,
- (i) $3 + (3+6) + (3+6+9) + (3+6+9+12) + \dots + (3+6+9+\dots+(6n-3))$, [2]
- (ii) $3 \times (3 \times 9) \times (3 \times 9 \times 27) \times \dots \times (3 \times 9 \times 27 \times 81 \times \dots \times 3^n)$. [2]

10 The functions f and g are defined as follows.

$$f(x) = \sqrt{|2-x|} + 1, \quad x \in \mathbb{R},$$

$$g(x) = \begin{cases} -\frac{1}{3}x + \frac{2}{3}, & 0 \leq x < 2, \\ 1 - (x-3)^2, & x \geq 2. \end{cases}$$

- (i) Show that f^{-1} does not exist. [1]
- (ii) If the domain of f is restricted to $[k, \infty)$ such that f^{-1} exists, state the least value of k and define f^{-1} in a similar form. [3]

Use the new domain of f found in part (ii) for the following parts.

- (iii) Show algebraically that there is no value of x for which $f^{-1}(x) = f(x)$. [2]
- (iv) Find the range of the composite function gf . [2]
- (v) Find the value of x such that $gf(x) = 1$. [1]

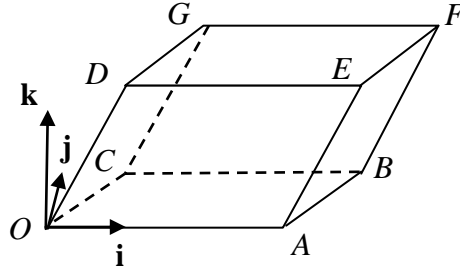
11 Sketch the graph of $y = \frac{2x^2 - 3}{x - 2}$, showing clearly the axial intercepts, the stationary points and the equations of the asymptotes where applicable. [3]

- (a) Solve the inequality $\frac{2x^2 - 3}{x - 2} \geq 1$. [2]

Deduce the solution of the inequality $\frac{2\sin^2 x - 3}{\sin x - 2} \geq 1$, where $0 \leq x \leq 2\pi$. [2]

- (b) Describe fully a sequence of transformations which would transform the graph of $y = 2x + \frac{5}{x}$ to the graph of $y = \frac{2x^2 - 3}{x - 2}$. [3]

- 12** An art structure, which is a parallelepiped (made of 6 faces of parallelograms) has a horizontal base $OABC$, with OA , OC and OD as its three sides and remaining vertices are B , E , F , and G as shown in the diagram below.



It is given that $\overrightarrow{OA} = 5\mathbf{i}$ and $\overrightarrow{OC} = \mathbf{i} + 7\mathbf{j}$. The lines l_1 and l_2 have equations given by $l_1: \mathbf{r} = (5 + \lambda)\mathbf{i} + (7\lambda - 14)\mathbf{j} + 6\mathbf{k}$, where λ is a real parameter and $l_2: 3x = z + 15, y = 0$. E and F are on line l_1 , and A and D are on line l_2 .

- (i) Find the position vector of E . [2]
- (ii) Find the equation, in scalar product form, of the plane $ABFE$. [3]
- (iii) Find the projection vector of \overrightarrow{AE} onto the base $OABC$. Hence, or otherwise, find the area of the projection of the plane $ABFE$ onto the base. [2]
- (iv) Find the equation of the line l_3 , which is the reflection of line AE about the base $OABC$. [2]
- (v) An architect wants to add a shelter which has the plane equation $x + ay + bz = c$, where a , b and c are unknown constants. He wants the shelter to meet the plane $ABFE$ at EF . What can be said about the values of a , b and c ? [2]

- 13 (a)** Using differentiation, find the equation of the tangent at the point $(-2, 1)$ on the curve $x^3 - y^3 = 3(x - y)$. [3]

- (b)** A spherical balloon is inflated such that 0.1 m^3 of air is pumped into the balloon every second. Find the rate of change of its surface area when the diameter is 1 m . [4]

[Volume of sphere = $\frac{4}{3}\pi r^3$ and surface area of sphere = $4\pi r^2$.]

- (c)** When designing the floor plan of his new house, Mr Lim wants to build a triangular garage with 2 adjacent walls of fixed lengths a and b meters and making an angle of θ radians. On the third side of his triangular garage, he intends to build 4 square-shaped rooms of equal size (see diagram). Find the value of θ when the total area covered by the garage and the 4 rooms is a maximum. [5]

