2015 Promo Exam Solution

<table>
<tr><th>Qn</th><th>Solution</th><th>Comments</th></tr>
<tr><td>1</td><td>Let k be the price of kang kong per kg,
c be the price of chye sim per kg and
n be the price of nai bai per kg.

$1.5k + c + 2n = 12.70$ (1)
$\left(2k + 3c + 0.5n\right) - 1 = 12.60$ (2)
$0.97\left(0.5k + 2c + 1.1n\right) = 9.70$ (3)

Therefore,
$1.5k + c + 2n = 12.70$ (1)
$2k + 3c + 0.5n = 13.60$ (2)
$0.5k + 2c + 1.1n = 10$ (3)

Using GC to solve: $k = 2.10$, $c = 2.55$, $n = 3.50$</td><td>Equations must always be written for SoE qns.

Students must take note that $\frac{100\%}{97\%} \neq 103\%$.</td></tr>
<tr><td>2</td><td>$4x^2 + y^2 + 8mx - 4y + 4 = 0$
$4(x^2 + 2mx) + y^2 - 4y + 4 = 0$
$4(x+m)^2 - 4m^2 + (y-2)^2 = 0$
$4(x+m)^2 + (y-2)^2 = 4m^2$
$\left(\frac{x+m}{m}\right)^2 + \left(\frac{y-2}{2m}\right)^2 = 1$
Hence $a = m$, $b = m$, $c = -2$, $d = 2m$

<u>Method 1</u>
$x^2 + (y - \frac{1}{m})^2 = 1 \xrightarrow[\text{with } \frac{x}{m}]{\text{Replace } x} \left(\frac{x}{m}\right)^2 + (y - \frac{1}{m})^2 = 1$
$\downarrow$ Replace x with $x+m$
$\left(\frac{x+m}{m}\right)^2 + (y - \frac{1}{m})^2 = 1$
$\downarrow$ Replace y with $\frac{y}{2m}$
$\left(\frac{x+m}{m}\right)^2 + \left(\frac{y}{2m} - \frac{1}{m}\right)^2 = 1$
$\left(\frac{x+m}{m}\right)^2 + \left(\frac{y-2}{2m}\right)^2 = 1$
(1) Scaling with scale factor m parallel to the x-axis, followed by
(2) Translation of m units in the negative x-direction, followed by
(3) Scaling with scale factor $2m$ parallel to the y-axis.

<u>Method 2</u></td><td>Some students made mistakes when completing the square. Some students wrote b = m^2, d = $4m^2$.

For the 2 methods given, Method 1 is more straightforward and is preferred.

Common mistakes:
Some students mixed up the order of transformation (scaling and translation). The order is $x \to \frac{x}{m} \to \frac{x+m}{m}$.

Some students got the direction of translation wrong. To obtain $x + m$ from x, we need to translate by m units in the negative x-direction, not positive x-direction.</td></tr>
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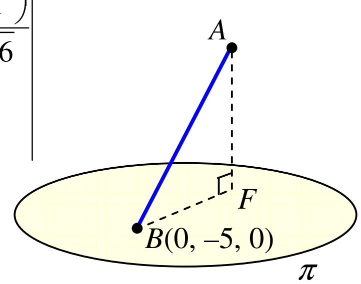
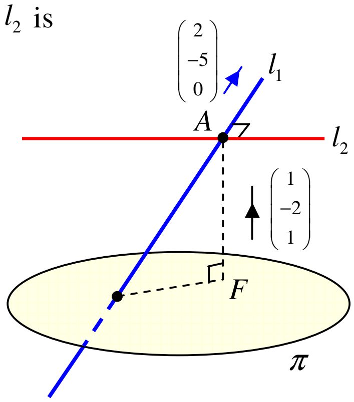
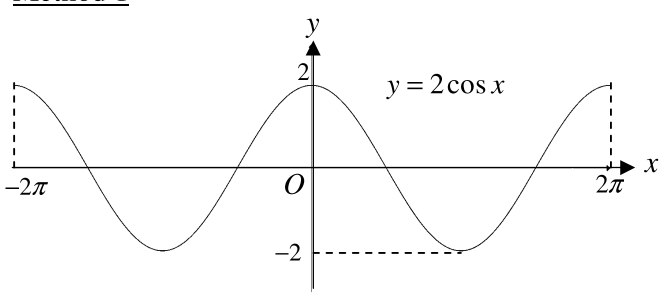
	$x^2+(y-\frac{1}{m})^2=1 \xrightarrow[\text{with } x+1]{\text{Replace } x} (x+1)^2+(y-\frac{1}{m})^2=1$ $\downarrow$ Replace x with $\frac{x}{m}$ $\left(\frac{x}{m}+1\right)^2+(y-\frac{1}{m})^2=1$ $\downarrow$ Replace y with $\frac{y}{2m}$ $\left(\frac{x}{m}+1\right)^2+\left(\frac{y}{2m}-\frac{1}{m}\right)^2=1$ $\left(\frac{x+m}{m}\right)^2+\left(\frac{y-2}{2m}\right)^2=1$ (1) Translation of 1 unit in the negative x-direction, followed by (2) Scaling with scale factor m parallel to the x-axis, followed by (3) Scaling with scale factor $2m$ parallel to the y-axis.	Students must describe the transformations used, i.e. translation and/or scaling. Some students just say replace x or y by this and that. No marks will be awarded.
3	$\frac{3}{(r-1)!}-\frac{5}{r!}+\frac{2}{(r+1)!}$ $=\frac{3(r+1)(r)}{(r+1)(r)(r-1)!}-\frac{5(r+1)}{(r+1)r!}+\frac{2}{(r+1)!}$ $=\frac{3r^2-2r-3}{(r+1)!}$ $\sum_{r=1}^{n}\frac{3r^2-2r-3}{(r+1)!}=\sum_{r=1}^{n}\left(\frac{3}{(r-1)!}-\frac{5}{r!}+\frac{2}{(r+1)!}\right)$ $=\left(\begin{array}{l}\frac{3}{0!}-\frac{5}{1!}+\frac{2}{2!}\\+\frac{3}{1!}-\frac{5}{2!}+\frac{2}{3!}\\+\frac{3}{2!}-\frac{5}{3!}+\frac{2}{4!}\\ \quad\vdots\\+\frac{3}{(n-2)!}-\frac{5}{(n-1)!}+\frac{2}{n!}\\+\frac{3}{(n-1)!}-\frac{5}{n!}+\frac{2}{(n+1)!}\end{array}\right)$ $=1-\frac{3}{n!}+\frac{2}{(n+1)!}$	**Common mistakes:** • 0! = 0 • A handful of students did not show cancellation of terms for at least 3 rows. • Fail to simplify $1-\frac{3}{n!}+\frac{2}{(n+1)!}$ to give answer $1-\left[\frac{3n+1}{(n+1)!}\right]$. • Showing $\sum_{r=1}^{n}\frac{r^2-r-1}{(r+1)!}<\frac{1}{3}$ is not done properly.

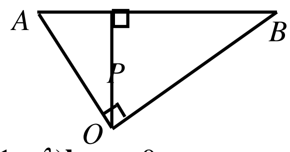
	$=1-\left(\dfrac{3(n+1)}{n!(n+1)}-\dfrac{2}{(n+1)!}\right)$ $=1-\left[\dfrac{3n+1}{(n+1)!}\right]$ $\displaystyle\sum_{r=1}^{n}\frac{r^2-r-1}{(r+1)!}=\frac{1}{3}\sum_{r=1}^{n}\frac{3r^2-3r-3}{(r+1)!}$ $\displaystyle<\frac{1}{3}\sum_{r=1}^{n}\frac{3r^2-2r-3}{(r+1)!}$ $=\dfrac{1}{3}\left\{1-\left[\dfrac{3n+1}{(n+1)!}\right]\right\}$ $<\dfrac{1}{3}\qquad\left(\because \dfrac{3n+1}{(n+1)!}>0\right)$	
4(i)	At $A(-2,\ \frac{9}{2},\ 3)$, $\quad \dfrac{x+3}{2}=\dfrac{-2+3}{2}=\dfrac{1}{2}$; $\dfrac{7-y}{5}=\dfrac{7-\frac{9}{2}}{5}=\dfrac{1}{2}$; $z=3$ Since coordinates of A satisfy equation of l_1, point A lies on l_1.	
(ii)	By inspection, a point B on π is $(0,-5,0)$. The perpendicular distance from A to π is $AF=\left\lvert\overrightarrow{AB}\cdot\hat{\mathbf{n}}\right\rvert$ $=\left\lvert\left[\begin{pmatrix}0\\-5\\0\end{pmatrix}-\begin{pmatrix}-2\\ \frac{9}{2}\\3\end{pmatrix}\right]\cdot\dfrac{\begin{pmatrix}1\\-2\\1\end{pmatrix}}{\sqrt{6}}\right\rvert$ $=\dfrac{18}{\sqrt{6}}$ $=3\sqrt{6}$  	**Common mistakes:** • Did not use length of projection.
(iii) (a)	$3\sqrt{6}$	
(iii) (b)	A vector equation of l_1 is $\mathbf{r}=\begin{pmatrix}-3\\7\\3\end{pmatrix}+\lambda\begin{pmatrix}2\\-5\\0\end{pmatrix},\ \lambda\in\mathbb{R}$.	• Fail to use $\begin{pmatrix}1\\-2\\1\end{pmatrix}\times\begin{pmatrix}2\\-5\\0\end{pmatrix}=\begin{pmatrix}5\\2\\-1\end{pmatrix}$ to find a

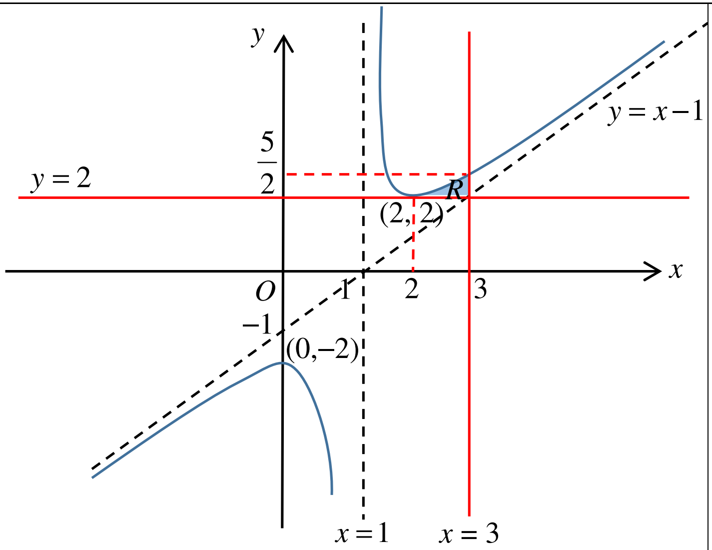
	Hence a direction vector of l_2 is $\begin{pmatrix}1\\-2\\1\end{pmatrix}\times\begin{pmatrix}2\\-5\\0\end{pmatrix}=\begin{pmatrix}5\\2\\-1\end{pmatrix}$ Hence, an equation of l_2 is $\mathbf{r}=\begin{pmatrix}-2\\ \frac{9}{2}\\3\end{pmatrix}+\mu\begin{pmatrix}5\\2\\-1\end{pmatrix},\ \mu\in\mathbb{R}.$ 	direction vector of l_2.
5(i)	<u>Method 1</u>  The horizontal line $y=k,\ -2\le k\le 2$ cuts the graph at more than one point, so f is not 1-1 and f^{-1} does not exist. <u>Method 2</u> Since $\mathrm{f}\left(\frac{\pi}{2}\right)=\mathrm{f}\left(-\frac{\pi}{2}\right)=0$, f is not 1-1 and f^{-1} does not exist.	Errors for Method 1 • Many students did not sketch the curve out. • Stated the horizontal line as $y = k$ but didn't state the k value or wrote $k\in\mathbb{R}$ (which is wrong). • Didn't write f is not 1-1. • Some merely state f is not 1-1 without justification. • Some state it does not pass through the horizontal line test but didn't state what the test is. • Wrong statement: <u>Every</u> horizontal line cuts the graph > once. You should just need 1 counter-example.
(ii)	Max $b=\pi$ $y=2\cos x\ \Rightarrow\ x=\cos^{-1}\frac{y}{2}$ $\mathrm{f}^{-1}:x\mapsto\cos^{-1}\frac{x}{2},\ -2\le x\le 0$	• Note that it is max value of b, not value of b. • Need to give answer in the <u>similar format</u> as f(x)! • Students forgot about the domain for answer. • [0,-2] is wrong!! Should start from a smaller number to bigger number!
(iii)	$\mathrm{R}_{(\mathrm{gf})^{-1}}=\mathrm{D}_{\mathrm{gf}}=\mathrm{D}_{\mathrm{f}}=\left[\frac{\pi}{2},\pi\right]$ Let $a=(\mathrm{gf})^{-1}\left(\frac{3}{2}\right)$ $\therefore\ \mathrm{gf}(a)=\frac{3}{2}$	• $(\mathrm{gf})^{-1}\ne g^{-1}\mathrm{f}^{-1}$ • $(\mathrm{gf})^{-1}\ne g\circ\mathrm{f}^{-1}$ • Note presentation of function term. Students should write $(\mathrm{gf})^{-1}$ instead of gf^{-1} which means $g\circ\mathrm{f}^{-1}$.

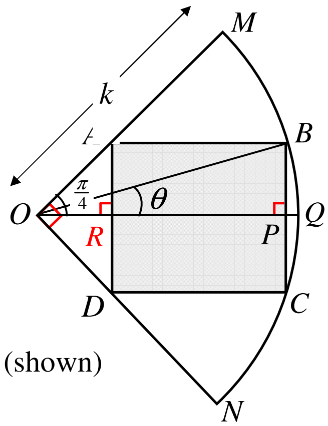
	$\Rightarrow \dfrac{4\cos(a)-1}{2\cos(a)-1}=\dfrac{3}{2}$ $\Rightarrow \cos(a)=-\dfrac{1}{2}$ $\Rightarrow a=\dfrac{2\pi}{3}$	• Inverse function $(\mathrm{gf})^{-1}(x)$ is NOT reciprocal $\dfrac{1}{\mathrm{gf}(x)}$. • Students should realise the answer is a special angle, leave answer in exact.
6(a)	$\int x\sec^2 x\,\mathrm{d}x = x\tan x-\int \tan x\,\mathrm{d}x$ $= x\tan x-\ln\lvert\sec x\rvert+C$ or $= x\tan x+\ln\lvert\cos x\rvert+C$	The answer to $\int \tan x\ \mathrm{d}x$ can be obtained from MF15. 1 mark is deducted if you forget to write '+ C'.
(b)	$\int_{\frac{1}{2}}^{1}\sqrt{x-x^2}\,\mathrm{d}x$ $=\int_0^{\frac{\pi}{2}}\sqrt{\frac{1}{2}(1+\sin\theta)-\frac{1}{4}(1+\sin\theta)^2}\left(\frac{1}{2}\cos\theta\,\mathrm{d}\theta\right)$ $=\int_0^{\frac{\pi}{2}}\sqrt{\frac{1}{4}(1+\sin\theta)(2-1-\sin\theta)}\left(\frac{1}{2}\cos\theta\,\mathrm{d}\theta\right)$ $=\int_0^{\frac{\pi}{2}}\frac{1}{4}\sqrt{1-\sin^2\theta}\cos\theta\,\mathrm{d}\theta$ $=\int_0^{\frac{\pi}{2}}\frac{1}{4}\cos^2\theta\,\mathrm{d}\theta$ $=\frac{1}{4}\int_0^{\frac{\pi}{2}}\frac{\cos 2\theta+1}{2}\,\mathrm{d}\theta$ $=\frac{1}{8}\left[\frac{\sin 2\theta}{2}+\theta\right]_0^{\frac{\pi}{2}}$ $=\frac{1}{8}\times\frac{\pi}{2}=\frac{\pi}{16}$ $x=\frac{1}{2}(1+\sin\theta)$ $\Rightarrow \frac{\mathrm{d}x}{\mathrm{d}\theta}=\frac{1}{2}\cos\theta$ When $x=\frac{1}{2}$, $\sin\theta=0$ $\Rightarrow \theta=0$ When $x=1$, $\sin\theta=1$ $\Rightarrow \theta=\frac{\pi}{2}$	**Common mistakes:** • did not substitute dx • did not substitute the limits • not using principal values • using double angle formula wrongly • integrated cos 2θ wrongly • using degrees instead of radians • did not give exact answer
7	(*)Let $P(n)$ be the statement $\sum_{r=2}^{n}\ln(\frac{r^2-1}{r^2})=\ln(\frac{n+1}{2n})$ for $n\in\mathbb{Z}^+$, $n\geq 2$. When $n=2$, $\text{LHS}=\sum_{r=2}^{2}\ln(\frac{r^2-1}{r^2})=\ln\left(\frac{2^2-1}{2^2}\right)=\ln\left(\frac{3}{4}\right)$ $\text{RHS}=\ln(\frac{2+1}{2(2)})=\ln\left(\frac{3}{4}\right)$ Since LHS = RHS, therefore $P(2)$ is true. (*)Assume that $P(k)$ is true for some $k\in\mathbb{Z}^+$, $k\geq 2$.	Many students missed out the set of values of n to be proved in the $P(n)$ statement, assumption and conclusion. Some students committed the mistake on writing $n\in\mathbb{R}$ instead of $n\in\mathbb{Z}^+$. Some students start with base case n = 1, which is wrong.

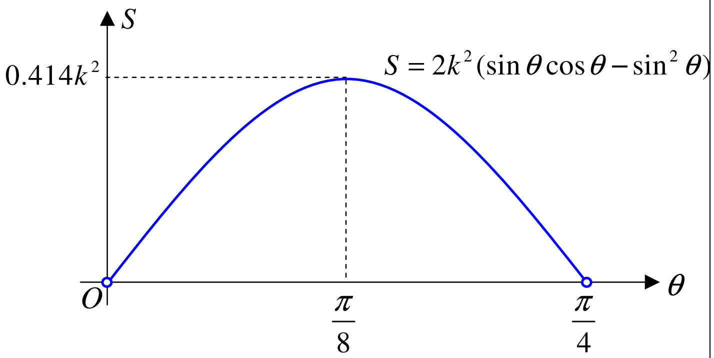
	i.e. $\sum_{r=2}^{k}\ln(\frac{r^2-1}{r^2})=\ln(\frac{k+1}{2k})$ Need to prove that $P(k+1)$ is true i.e. $\sum_{r=2}^{k+1}\ln(\frac{r^2-1}{r^2})=\ln(\frac{k+2}{2k+2})$ $\text{LHS}=\sum_{r=2}^{k+1}\ln(\frac{r^2-1}{r^2})$ $=\sum_{r=2}^{k}\ln(\frac{r^2-1}{r^2})+\ln\frac{\left(k+1\right)^2-1}{(k+1)^2}$ $=\ln(\frac{k+1}{2k})+\ln\frac{\left(k+1\right)^2-1}{(k+1)^2}$ $=\ln(\frac{k+1}{2k})+\ln\frac{k\left(k+2\right)}{(k+1)^2}$ $=\ln\left[\frac{k+1}{2k}\cdot\frac{k\left(k+2\right)}{(k+1)^2}\right]$ $=\ln(\frac{k+2}{2k+2})=\text{RHS}$ Thus $P(k)$ is true implies $P(k+1)$ is true. (*)Since $P(2)$ is true and $P(k)$ is true implies $P(k+1)$ is true, by mathematical induction, $P(n)$ is true for all $n\in\mathbb{Z}^+$, $n\geq 2$. $\sum_{r=5}^{15}\ln[\frac{2(r^2-1)}{r^2}]$ $=\sum_{r=5}^{15}\ln 2+\sum_{r=5}^{15}\ln[\frac{(r^2-1)}{r^2}]$ $=11\ln 2+\sum_{r=2}^{15}\ln[\frac{(r^2-1)}{r^2}]-\sum_{r=2}^{4}\ln[\frac{(r^2-1)}{r^2}]$ $=11\ln 2+\ln[\frac{(15+1)}{2(15)}]-\ln[\frac{(4+1)}{2(4)}]$ $=11\ln 2+\ln[\frac{8}{15}]-\ln[\frac{5}{8}]$ $=11\ln 2+\ln 2^3-\ln 3-\ln 5-\ln 5+\ln 2^3$ $=17\ln 2-\ln 3-2\ln 5$ $\therefore\ A=17,\quad B=-1,\ C=-2$	Many students committed the basic logarithmic error: $\sum_{r=5}^{15}\ln[\frac{2(r^2-1)}{r^2}]\neq 2\sum_{r=5}^{15}\ln[\frac{(r^2-1)}{r^2}]$ Some express $\sum_{r=5}^{15}\ =\sum_{r=2}^{15}\ -\sum_{r=2}^{5}$ which is also quite a common error found in students' solution Some write $\sum_{r=5}^{15}\ln 2=10\ln 2$ or $\sum_{r=5}^{15}\ln 2=\ln 2$. These are common errors made during the summation process.
8(i)	Jan 2004: $U_1=2000(12)=\$24000$	

	Jan 2005: $U_2 = \left(2000+0.5(2000)\right)(12) = \$36000$ Jan 2006: $U_3 = \left(2000+0.5(2000)+0.5^2(2000)\right)(12) = \$42000$ $U_n = [2000+0.5(2000)+0.5^2(2000)+...$ $+0.5^{n-1}(2000)](12)$ $= 2000\left[1+0.5+0.5^2+...+0.5^{n-1}\right](12)$ $= 2000\left[\dfrac{1-0.5^n}{0.5}\right](12)$ $= 48000(1-0.5^n)$ (shown) $\sum_{r=1}^{n} U_r = 48000\sum_{r=1}^{n}(1-0.5^r)$ $= 48000\left(n-\sum_{r=1}^{n}0.5^r\right)$ $= 48000\left(n-\dfrac{0.5\left(1-0.5^n\right)}{0.5}\right)$ $= 48000\left(n-1+0.5^n\right)$	Most students calculated in months instead of year Most students used $a = 1$ instead of 0.5, thus many GP formula were wrong due to the starting term.
8(ii)	$(2000)0.5^{n-1} < 10$ $0.5^{n-1} < 0.005$ $(n-1)\ln 0.5 < \ln 0.005$ $n-1 > 7.64$ $n > 8.64$ $\therefore \quad n = 9$ He will quit in 2012.	Most students used $(1000)0.5^{n-1} < 10$ or $(2000)0.5^n < 10$, which led to the wrong years of 2011 and 2013.
9	$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA}$ $= \lambda\mathbf{a} + (1-\lambda)\mathbf{b} - \mathbf{a}$ $= (\lambda-1)\mathbf{a} + (1-\lambda)\mathbf{b}$ $= (1-\lambda)(\mathbf{b}-\mathbf{a})$ $= (1-\lambda)\overrightarrow{AB}$ Therefore, $\overrightarrow{AP}$ is parallel to $\overrightarrow{AB}$ and since A is a common point, A, B and P are collinear.	Students should not be using ratio theorem to prove collinearity. To show parallel we want to prove $\overrightarrow{AP} = k\overrightarrow{AB}$. Thus, $\overrightarrow{AP} = k\overrightarrow{AB}$ should not be used in the proof. Note: vectors cannot be divided.

	$\angle APO = 90° \Rightarrow \overrightarrow{OP} \perp \overrightarrow{AB}$  $\Rightarrow \overrightarrow{OP} \cdot \overrightarrow{AB} = 0$ $\Rightarrow [\lambda\mathbf{a} + (1-\lambda)\mathbf{b}] \cdot (\mathbf{b} - \mathbf{a}) = 0$ $\Rightarrow \lambda\mathbf{a} \cdot \mathbf{b} + (1-\lambda)\mathbf{b} \cdot \mathbf{b} - \lambda\mathbf{a} \cdot \mathbf{a} - (1-\lambda)\mathbf{b} \cdot \mathbf{a} = 0$ $\Rightarrow (1-\lambda)\|\mathbf{b}\|^2 - \lambda\|\mathbf{a}\|^2 = 0 \quad (\because \mathbf{a} \cdot \mathbf{b} = 0 = \mathbf{b} \cdot \mathbf{a})$ $\Rightarrow (1-\lambda)\|\mathbf{b}\|^2 = \lambda\|\mathbf{a}\|^2$ (shown)	Many students did not mention the requirement of a common point. The question requires students to use scalar product. Some students picked the wrong vectors for the scalar product.
	Area of $\Delta OBP = \frac{\sqrt{2}}{6}\|\mathbf{b}\|^2$ Area of $\Delta OBP = \frac{1}{2}\left\|\overrightarrow{OB} \times \overrightarrow{OP}\right\|$ $= \frac{1}{2}\left\|\mathbf{b} \times \left(\lambda\mathbf{a} + (1-\lambda)\mathbf{b}\right)\right\|$ $= \frac{1}{2}\left\|\lambda\mathbf{b} \times \mathbf{a} + (1-\lambda)\mathbf{b} \times \mathbf{b}\right\|$ $= \frac{1}{2}\lambda\|\mathbf{b}\|\|\mathbf{a}\| \quad (\because \mathbf{b} \times \mathbf{b} = \mathbf{0})$ $\therefore \frac{1}{2}\lambda\|\mathbf{b}\|\|\mathbf{a}\| = \frac{\sqrt{2}}{6}\|\mathbf{b}\|^2$ $\Rightarrow \frac{1}{2}\lambda\|\mathbf{b}\| \times \sqrt{\frac{1-\lambda}{\lambda}}\|\mathbf{b}\| = \frac{\sqrt{2}}{6}\|\mathbf{b}\|^2$ $\Rightarrow \sqrt{\lambda(1-\lambda)} = \frac{\sqrt{2}}{3}$ $\Rightarrow 9\lambda(1-\lambda) = 2$ $\Rightarrow 9\lambda^2 - 9\lambda + 2 = 0$ $\Rightarrow (3\lambda - 1)(3\lambda - 2) = 0$ $\Rightarrow \lambda = \frac{1}{3}$ or $\lambda = \frac{2}{3}$	If students state the area formula without making any progress, no marks will be awarded. In ΔOBP, none of the angle are known. Thus, using the cross product formula immediately at the first line results in no progress in the proof.
10	$y = \frac{x^2 - 2x + 2}{x-1} = x - 1 + \frac{1}{x-1}$ -----------(1)	

		
	Required area = $\int_2^3 y\ \mathrm{d}x - 1\times 2$ ----------(1) $= \int_2^3 \left(x - 1 + \frac{1}{x-1} \right) \mathrm{d}x - 2$ $= \left[\frac{x^2}{2} - x + \ln\lvert x-1 \rvert \right]_2^3 - 2$ $= \left(\frac{9}{2} - 3 + \ln 2 - 2 + 2 - \ln 1 \right) - 2$ $= \ln 2 - \frac{1}{2} \quad \text{unit}^2$	**Common mistakes:** • did not perform long division to get oblique asymptote • did not show max pt on the y-axis. • Some students did not give exact answer.
	$y = x - 1 + \frac{1}{x-1}$ $y(x-1) = x^2 - 2x + 2$ $x^2 - (2+y)x + 2 + y = 0$ $x = \frac{2 + y \pm \sqrt{(2+y)^2 - 4(1)(2+y)}}{2} = \frac{2 + y \pm \sqrt{y^2 - 4}}{2}$ ...(*) Required volume $= \pi(3^2)(0.5) - \pi \int_2^{2.5} \left(\frac{2 + y + \sqrt{y^2 - 4}}{2} \right)^2 \mathrm{d}y$ $= 3.308 \quad \text{unit}^3 \quad \text{(3 d.p.)}$	Many students did not know how to solve for x from $y(x-1) = x^2 - 2x + 2$ Some students used $\pi \int y^2 \mathrm{d}x$ for rotation about y-axis.

11 (i) (ii)	$\sin\theta = \dfrac{BP}{OB} = \dfrac{BP}{k}$ Hence $BP = k\sin\theta$ $OR = AR = BP = k\sin\theta$ $\cos\theta = \dfrac{OP}{OB} = \dfrac{OP}{k}$ Hence $OP = k\cos\theta$ $\therefore\ PR = OP - OR$ $= k\cos\theta - k\sin\theta$ $= k(\cos\theta - \sin\theta)$ (shown)  	For part (ii), the proof must be clearly stated. Many students did not explain the reason for $OR = k\sin\theta$ properly.
(iii)	$BC = 2BQ = 2k\sin\theta$ $S = AB \times BC = k(\cos\theta - \sin\theta) \times 2k\sin\theta$ $= 2k^2(\sin\theta\cos\theta - \sin^2\theta)$ $\dfrac{dS}{d\theta} = 2k^2(\cos^2\theta - \sin^2\theta - 2\sin\theta\cos\theta)$ $= 2k^2(\cos 2\theta - \sin 2\theta)$ When $\dfrac{dS}{d\theta} = 0$, $2k^2(\cos 2\theta - \sin 2\theta) = 0$ $\cos 2\theta - \sin 2\theta = 0$ $\tan 2\theta = 1$ $2\theta = \dfrac{\pi}{4}$ [since $0 < \theta < \dfrac{\pi}{4} \Rightarrow 0 < 2\theta < \dfrac{\pi}{2}$] $\therefore\ \theta = \dfrac{\pi}{8}$ $\dfrac{d^2S}{d\theta^2} = 2k^2(-2\sin 2\theta - 2\cos 2\theta)$ When $\theta = \dfrac{\pi}{8}$, $\dfrac{d^2S}{d\theta^2} = 2k^2(-2\sin 2(\dfrac{\pi}{8}) - 2\cos 2(\dfrac{\pi}{8}))$ $= -4\sqrt{2}k^2 < 0$ Hence area S is maximum when $\theta = \dfrac{\pi}{8}$. Maximum $S = 2k^2(\sin\dfrac{\pi}{8}\cos\dfrac{\pi}{8} - \sin^2\dfrac{\pi}{8})$ $= 0.4142135624k^2$ $\approx 0.414k^2$	**Common Mistakes:** * Did not notice k is a constant. * Forgot derivative test to verify the maximum * Applied double angle formula wrongly * Applied integration instead of differentiation * Careless mistakes in Constant/Coefficient * Final answer with a combination of 3 s.f. and exact numbers, which is not acceptable.

(iv)	 The range of values of θ are $0 < \theta < \frac{\pi}{4}$, and the maximum point $\left(\frac{\pi}{8}, 0.414k^2\right)$ of the curve corresponds to the maximum area $0.414k^2$ square metres of S at $\theta = \frac{\pi}{8}$.	Most students did not sketch the graph within the correct domain. Students should find the maximum point from the graph and check against part (iii). Also, "the results found in part (iii)" means both the value of θ and the maximum value of *S*, so the *x*, *y*-coordinates must be labelled properly in the graph.
12 (i)	$\frac{dx}{d\theta} = p(-2\sin 2\theta + 2\sin\theta)$ $\frac{dy}{d\theta} = \frac{3}{2}p\cos\frac{3\theta}{2}$ $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{3}{2}p\cos\frac{3\theta}{2}}{p(-2\sin 2\theta + 2\sin\theta)}$ $= \left(-\frac{3}{4}\right)\frac{\cos\frac{3\theta}{2}}{\sin 2\theta - \sin\theta}$ $= \left(-\frac{3}{4}\right)\frac{\cos\frac{3\theta}{2}}{2\cos\frac{3\theta}{2}\sin\frac{\theta}{2}}$ $= -\frac{3}{8}\operatorname{cosec}\frac{\theta}{2}$ (shown)	There were a sizeable number of students who found $\frac{dx}{dp}$ and $\frac{dy}{dp}$ instead. Notice that these are undefined because *p* is a constant. Some students performed $\cos\frac{3\theta}{2} = \cos\theta\cos\frac{\theta}{2} - \sin\theta\sin\frac{\theta}{2}$ on the numerator of $\frac{dy}{dx}$ instead of using factor formula on the denominator, which resulted in a long detour to the answer. Some got lost in this unnecessary detour.
(ii)	When $x = -p$, $-p = p(\cos 2\theta - 2\cos\theta)$ $\cos 2\theta - 2\cos\theta + 1 = 0$ $2\cos^2\theta - 2\cos\theta = 0$ $\cos\theta(\cos\theta - 1) = 0$ $\cos\theta = 0$ or $\cos\theta = 1$ $\therefore\ \theta = \frac{\pi}{2}$ or $\theta = 0$ or 2π [rejected since $0 < \theta < \pi$]	It is generally not accepted that the G.C. provides an answer of $\frac{\pi}{2}$ from the equation $\cos 2\theta - 2\cos\theta = -1$, unless you are talking about more powerful softwares with inbuilt CAS but definitely not among those permitted for examinations.

	When $\theta=\dfrac{\pi}{2}$, $\dfrac{dy}{dx}=-\dfrac{3}{8}\operatorname{cosec}\dfrac{\pi}{4}=-\dfrac{3}{8}\left(\dfrac{1}{\sin\frac{\pi}{4}}\right)=-\dfrac{3\sqrt{2}}{8}$ $\therefore$ gradient of normal $=\dfrac{8}{3\sqrt{2}}=\dfrac{4\sqrt{2}}{3}$ $y=p\sin\dfrac{3\left(\frac{\pi}{2}\right)}{2}=\dfrac{p}{\sqrt{2}}$ Required equation of normal is $y-\dfrac{p}{\sqrt{2}}=\dfrac{4\sqrt{2}}{3}(x+p)$ $y=\dfrac{4\sqrt{2}}{3}x+\dfrac{11\sqrt{2}}{6}p$	
(iii)	Substitute $y=-\dfrac{\sqrt{2}p}{x}$ into $y=\dfrac{4\sqrt{2}}{3}x+\dfrac{11\sqrt{2}}{6}p$, $-\dfrac{\sqrt{2}p}{x}=\dfrac{4\sqrt{2}}{3}x+\dfrac{11\sqrt{2}}{6}p$ $\dfrac{4}{3}x^2+\dfrac{11}{6}px+p=0$ $8x^2+11px+6p=0$ When the normal and the curve with equation $y=-\dfrac{\sqrt{2}p}{x}$ intersect at exactly one point, $(11p)^2-4(8)(6p)=0$ $p(121p-192)=0$ $\therefore\ p=\dfrac{192}{121}$ or $p=0$ [rejected since p is positive]	Quite a number forgot a p, i.e., they wrote $(11p)^2-4(8)(6)=0$ instead of $(11p)^2-4(8)(6p)=0$, which resulted in them obtaining $p=\sqrt{\dfrac{192}{121}}$ instead.
(iv)	Given $\dfrac{dy}{dt}=-1.5$ $\dfrac{d}{dt}(x+y)=\dfrac{dx}{dt}+\dfrac{dy}{dt}$ Now $\dfrac{dy}{dx}=\dfrac{dy}{dt}\cdot\dfrac{dt}{dx}$ When $\theta=1$, $\dfrac{dx}{dt}=\dfrac{\frac{dy}{dt}}{\frac{dy}{dx}}=\dfrac{-1.5}{-\frac{3}{8}\operatorname{cosec}\frac{1}{2}}=4\sin\dfrac{1}{2}$ $\therefore\ \dfrac{d}{dt}(x+y)=\dfrac{dx}{dt}+\dfrac{dy}{dt}$ $=4\sin\dfrac{1}{2}-1.5$ $=0.4177021544$ $\therefore\ x+y$ changes at a rate of 0.4177 unit per second.	A standard mistake would be writing $\dfrac{dy}{dt}=1.5$ instead of $\dfrac{dy}{dt}=-1.5$. Again, finding $\dfrac{dx}{dp}$ and $\dfrac{dy}{dp}$ in this context is not correct because p is a constant.