

Qtn	Solutions
1(i)	<p>Funds transferred into Account A: <math>0.05y + 0.1z</math></p> <p>Funds transferred from Account A: <math>0.375x + 0.125x = 0.5x</math></p> <p>So we have <math>0.5x - (0.05y + 0.1z) = 16</math>  i.e. <math>0.5x - 0.05y - 0.1z = 16</math> ----(1)</p>
(ii)	<p>Similarly, for Account B, we have  <math>-0.375x + 0.1y - 0.2z = -19</math> ----(2)</p> <p>We also know <math>x + y + z = 90</math> ----(3)</p> <p>Solving (1), (2), (3) using GC, we have  <math>x = 40, y = 20, z = 30</math></p>
2	$(2 + px)^{-q}$ $= 2^{-q} \left( 1 + \frac{px}{2} \right)^{-q}$ $= 2^{-q} \left( 1 + (-q) \left( \frac{px}{2} \right) + \dots \right)$ $= 2^{-q} \left( 1 - \frac{pqx}{2} + \dots \right)$ $\approx \frac{1}{4} - x$ $\Rightarrow 2^{-q} = \frac{1}{4} \text{ ---(1) \& } \frac{1}{4} \left( \frac{-2p}{2} \right) = -1 \text{ ---(2)}$ $q = 2, p = 4$
	$(2 + 4x)^{-2}$ $= \frac{1}{4} (1 + 2x)^{-2}$ $= \frac{1}{4} \left( 1 + (-2)(2x) + \frac{(-2)(-3)}{2!} (2x)^2 + \frac{(-2)(-3)(-4)}{3!} (2x)^3 + \dots \right)$ <p><math>x^n</math> coefficient</p> $= \frac{1}{4} \left( \frac{(-2)(-3)(-4) \dots (-(n+1))}{n!} \right) (2)^n$ $= \frac{1}{4} (-1)^n (n+1) 2^n = (-1)^n (n+1) 2^{n-2}$
3(i)	Vol of water at end of Day 1

$$= 0.9(8500)$$

Vol of water at end of Day 2

$$= 0.9(500 + 0.9(8500)) = 0.9(500) + 0.9^2(8500)$$

Vol of water at end of Day 3

$$= 0.9(500) + 0.9^2(500) + 0.9^3(8500)$$

$$= 7051.5 \text{ litres}$$

(ii)

Vol of water at end of Day  $n$ ,  $V$

$$= 0.9(500) + 0.9^2(500) + \dots + 0.9^{n-1}(500) + 0.9^n(8500)$$

$$= 500(0.9 + 0.9^2 + \dots + 0.9^{n-1}) + 0.9^n(8500)$$

$$= 500 \left[ \frac{0.9(1 - 0.9^{n-1})}{1 - 0.9} \right] + 0.9^n(8500)$$

$$= 4500[1 - 0.9^{n-1}] + 0.9^n(8500)$$

For  $V < 5000$ ,

$$4500[1 - 0.9^{n-1}] + 0.9^n(8500) < 5000$$

From G.C,

$n$	$V$
18	5025.3
19	4972.8
20	4925.5

Least  $n = 19$

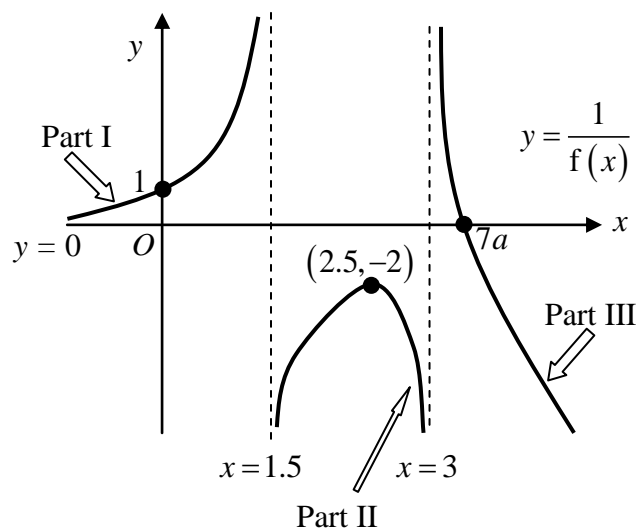
Least number of days = 19.

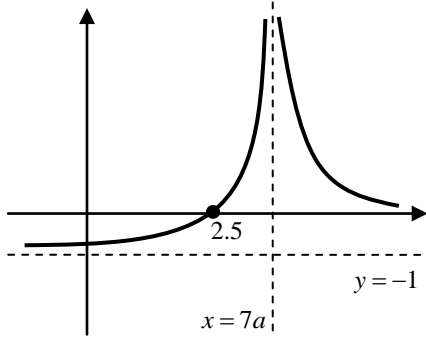
As  $n \rightarrow \infty$ ,  $V \rightarrow 4500$

(iii)

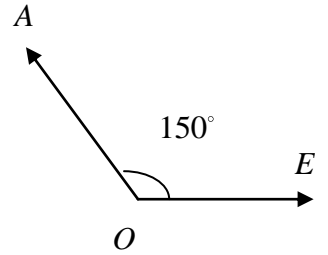
Therefore, water tank will never dry up.

4i



ii	 <p>A graph of a function on a Cartesian coordinate system. The x-axis and y-axis are shown. A vertical dashed line represents an asymptote at <math>x = 7a</math>. A horizontal dashed line represents an asymptote at <math>y = -1</math>. The curve approaches the vertical asymptote as <math>y \rightarrow \infty</math> and the horizontal asymptote as <math>x \rightarrow \infty</math>. The curve crosses the x-axis at a point labeled 2.5.</p>
5	<p>(i) Since <math>l</math> is the limit,  As <math>n \rightarrow \infty</math>, <math>u_n \rightarrow l</math>, <math>u_{n+1} \rightarrow l</math></p> $\therefore \frac{l+a}{l+b} = \frac{a}{b}$ $\Rightarrow b(l+a) = a(l+b)$ $\Rightarrow bl = al$ $\Rightarrow l(b-a) = 0 \quad (\because a \neq b)$ $\Rightarrow l = 0$ <p>(ii)</p> $\frac{u_{n+1}+a}{u_n+b} = \frac{a}{b}$ $\Rightarrow b(u_{n+1}+a) = a(u_n+b)$ $\Rightarrow bu_{n+1} = au_n$ $\Rightarrow u_{n+1} = \frac{a}{b}u_n$ <p>Hence <math>\{u_n\}</math> is a GP with ratio <math>\frac{a}{b}</math> and since <math>u_1 = a</math>,</p> $u_n = a\left(\frac{a}{b}\right)^{n-1}$

	<p>(ii) Since <math>S</math> exists, <math> r  &lt; 1 \Rightarrow \left  \frac{a}{b} \right  &lt; 1</math></p> $S = \frac{a}{1 - \frac{a}{b}}$ $= \frac{ab}{b - a}$
6(i)	$\frac{du}{dx} = 8x$ $\int x^3 \sqrt{9 + 4x^2} \, dx = \int \frac{1}{8} x^2 (8x) (9 + 4x^2)^{1/2} \, dx$ $= \frac{1}{8} \int \left( \frac{u-9}{4} \right) \left( \frac{du}{dx} \right) (u)^{1/2} \, dx$ $= \int \frac{1}{32} u^{3/2} - \frac{9}{32} u^{1/2} \, du$ $= \frac{1}{80} u^{5/2} - \frac{3}{16} u^{3/2} + C$ $= \frac{1}{80} (9 + 4x^2)^{5/2} - \frac{3}{16} (9 + 4x^2)^{3/2} + C$
(ii)	$\int_0^1 x^2 \tan^{-1} x \, dx = \left[ \left( \frac{1}{3} x^3 \right) \tan^{-1} x \right]_0^1 - \int_0^1 \left( \frac{1}{3} x^3 \right) \left( \frac{1}{1+x^2} \right) \, dx$ $= \left[ \left( \frac{1}{3} x^3 \right) \tan^{-1} x \right]_0^1 - \frac{1}{3} \int_0^1 \left( x - \frac{x}{1+x^2} \right) \, dx$ $= \left[ \left( \frac{1}{3} x^3 \right) \tan^{-1} x - \frac{1}{3} \left( \frac{1}{2} x^2 - \frac{1}{2} \ln(1+x^2) \right) \right]_0^1$ $= \left( \frac{1}{3} \right) \left( \frac{\pi}{4} \right) - \frac{1}{3} \left( \frac{1}{2} - \frac{1}{2} \ln(2) \right)$ $= \frac{\pi}{12} - \frac{1}{6} (1 - \ln 2)$

7(a)	$AB \text{ line} \Rightarrow \vec{r} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ 1 \\ 3 \end{pmatrix}, \lambda \in \mathbb{R}$ $\overrightarrow{OP} = \begin{pmatrix} 2-5\lambda \\ \lambda \\ -1+3\lambda \end{pmatrix}$ $\overrightarrow{OD} = \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix}$ <p><math>C, P, D</math> are collinear.</p> $\overrightarrow{DP} = k\overrightarrow{CD}$ $\begin{pmatrix} 2-5\lambda-a \\ \lambda \\ -1+3\lambda \end{pmatrix} = k \begin{pmatrix} a-1 \\ 2 \\ 4 \end{pmatrix}$ $\Rightarrow \lambda = 1, k = \frac{1}{2}, a = -\frac{5}{3}$ $\overrightarrow{OD} = \begin{pmatrix} -\frac{5}{3} \\ 0 \\ 0 \end{pmatrix}$
(b)	$E(a, b, 0)$ $\overrightarrow{OE} = \begin{pmatrix} a \\ b \\ 0 \end{pmatrix}$ $a^2 + b^2 = 1$ $\cos 150^\circ = \frac{\overrightarrow{OA} \cdot \overrightarrow{OE}}{ \overrightarrow{OA} (1)}$ $-\frac{\sqrt{3}}{2} = \frac{\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ 0 \end{pmatrix}}{\sqrt{5}}$ $-\frac{\sqrt{3}}{2} = \frac{2a}{\sqrt{5}} \Rightarrow a = -\frac{\sqrt{15}}{4} \text{ or } -0.968 \text{ (3 s.f.)}$ 

	$\frac{15}{16} + b^2 = 1 \Rightarrow b = \pm \frac{1}{4}$ $E\left(-\frac{\sqrt{15}}{4}, \frac{1}{4}, 0\right) \text{ or } E\left(-\frac{\sqrt{15}}{4}, -\frac{1}{4}, 0\right)$
8	<p>(i)</p> $\frac{2}{r(r+1)(r+3)} \equiv \frac{A}{r} + \frac{B}{r+1} + \frac{C}{r+3}$ $2 \equiv A(r+1)(r+3) + Br(r+3) + Cr(r+1)$ $r=0, \quad A = \frac{2}{3} \quad r=-1, \quad B = -1 \quad r=0, \quad C = \frac{1}{3}$ $\therefore \frac{2}{r(r+1)(r+3)} \equiv \frac{2}{3r} - \frac{1}{r+1} + \frac{1}{3(r+3)}$ $\frac{1}{4} \sum_{r=1}^n \frac{2}{r(r+1)(r+3)} = \frac{1}{4} \sum_{r=1}^n \left( \frac{2}{3r} - \frac{1}{r+1} + \frac{1}{3(r+3)} \right)$ $= \frac{1}{4} \left[ \frac{2}{3} - \frac{1}{2} + \frac{1}{12} \right. \\ + \frac{2}{6} - \frac{1}{3} + \frac{1}{15} \\ + \frac{2}{9} - \frac{1}{4} + \frac{1}{18} \\ + \frac{2}{12} - \frac{1}{5} + \frac{1}{21} \\ + \frac{2}{15} - \frac{1}{6} + \frac{1}{24} \\ \vdots \\ + \frac{2}{3(n-3)} - \frac{1}{n-2} + \frac{1}{3n} \\ + \frac{2}{3(n-2)} - \frac{1}{n-1} + \frac{1}{3(n+1)} \\ + \frac{2}{3(n-1)} - \frac{1}{n} + \frac{1}{3(n+2)} \\ + \left. \frac{2}{3n} - \frac{1}{n+1} + \frac{1}{3(n+3)} \right]$

	$= \frac{1}{4} \left[ \frac{7}{18} - \frac{1}{n+1} + \frac{1}{3(n+1)} + \frac{1}{3(n+2)} + \frac{1}{3(n+3)} \right]$ $= \frac{1}{12} \left[ \frac{7}{6} - \frac{2}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} \right]$ <p>(iii)</p> $\sum_{r=5}^{\infty} \frac{1}{2r(r-2)(r-3)}$ <p>Replace <math>r</math> by <math>r+3</math>,</p> $= \sum_{r=2}^{\infty} \frac{1}{2r(r+1)(r+3)}$ $= \sum_{r=1}^{\infty} \frac{1}{2r(r+1)(r+3)} - \frac{1}{2(1)(2)(4)}$ $= \lim_{n \rightarrow \infty} \left( \frac{1}{12} \left[ \frac{7}{6} - \frac{2}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} \right] \right) - \frac{1}{16}$ $= \frac{7}{72} - \frac{1}{16}$ $= \frac{5}{144}$
9	<p>(See alternative solution below)</p> <p>Let <math>P(n)</math> be the statement</p> <p>“ <math>1 + (1+2) + (1+2+3) + (1+2+3+\dots+n) = \frac{1}{6}n(n+1)(n+2)</math> , <math>n \in \mathbb{Z}^+</math> ”</p> <p>When <math>n=1</math>, LHS of <math>P(1) = 1</math>,</p> <p>RHS of <math>P(1) = \frac{(1)(2)(3)}{6} = 1</math></p> <p>Since LHS = RHS, <math>P(1)</math> is true.</p> <p>Assume <math>P(k)</math> is true for some <math>k \in \mathbb{Z}^+</math>,</p> <p>i.e. <math>1 + (1+2) + (1+2+3) + (1+2+3+\dots+k) = \frac{1}{6}k(k+1)(k+2)</math></p> <p>To show <math>P(k+1)</math> is true,</p> <p>i.e. <math>1 + (1+2) + (1+2+3) + (1+2+3+\dots+k+k+1) = \frac{1}{6}(k+1)(k+2)(k+3)</math></p> <p>LHS of <math>P(k+1)</math></p> $= 1 + (1+2) + (1+2+3) + (1+2+3+\dots+k) + (1+2+3+\dots+k+k+1)$ $= \frac{1}{6}k(k+1)(k+2) + (1+2+3+\dots+k+k+1)$

$$= \frac{1}{6}k(k+1)(k+2) + \frac{1}{2}(k+1)(k+2)$$

$$= \frac{1}{6}(k+1)(k+2)(k+3)$$

= RHS of  $P(k+1)$

Since  $P(1)$  is true, and  $P(k)$  is true  $\Rightarrow P(k+1)$  is true, by mathematical induction,  
 $P(n)$  is true for  $n \in \mathbb{Z}^+$ .

### Alternative Solution:

Let  $P(n)$  be the statement " $\sum_{r=1}^n U_r = \frac{1}{6}n(n+1)(n+2)$ , where  $U_r = 1+2+3+\dots+r$ ,  
 $n \in \mathbb{Z}^+$ "

When  $n=1$ , LHS of  $P(1) = \sum_{r=1}^1 U_r = U_1 = 1$ ,

$$\text{RHS of } P(1) = \frac{6}{6} = 1$$

Since LHS = RHS,  $P(1)$  is true.

Assume  $P(k)$  is true for some  $k \in \mathbb{Z}^+$ ,

$$\text{i.e. } \sum_{r=1}^k U_r = \frac{1}{6}k(k+1)(k+2)$$

To show  $P(k+1)$  is true,

$$\text{i.e. } \sum_{r=1}^{k+1} U_r = \frac{1}{6}(k+1)(k+2)(k+3)$$

LHS of  $P(k+1)$

$$= \sum_{r=1}^{k+1} U_r$$

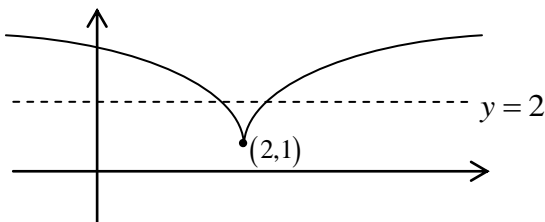
$$= \sum_{r=1}^k U_r + U_{k+1}$$

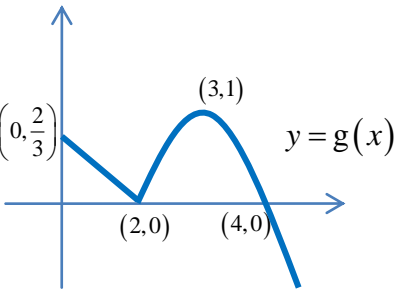
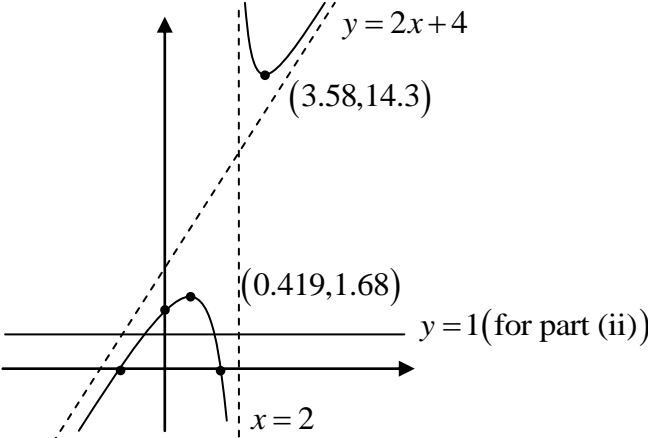
$$= \frac{1}{6}k(k+1)(k+2) + (1+2+3+\dots+k+k+1)$$

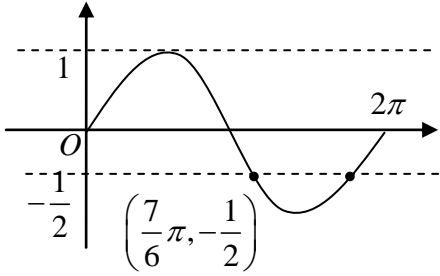
$$= \frac{1}{6}k(k+1)(k+2) + \frac{1}{2}(k+1)(k+2)$$

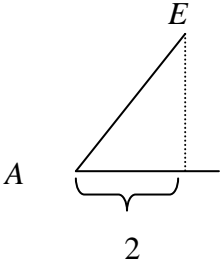
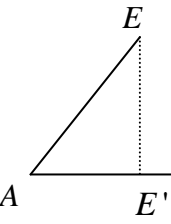
$$= \frac{1}{6}k(k+1)(k+2)(k+3)$$

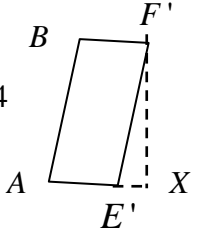
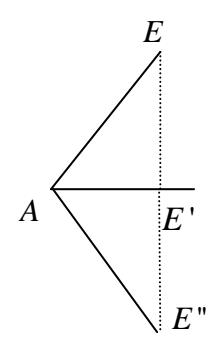


	<p>= RHS of <math>P(k+1)</math></p> <p>Since <math>P(1)</math> is true, and <math>P(k)</math> is true <math>\Rightarrow P(k+1)</math> is true, by mathematical induction, <math>P(n)</math> is true for <math>n \in \mathbb{Z}^+</math>.</p> <p>(i)</p> $3 + (3+6) + (3+6+9) + \dots + (3+6+9+\dots+(6n-3))$ $= 3[1 + (1+2) + (1+2+3) + \dots + (1+2+3+\dots+(2n-1))] = 3\left[\frac{1}{6}(2n-1)(2n)(2n+1)\right]$ $= n(2n-1)(2n+1)$ <p>(ii)</p> $3 \times (3 \times 9) \times (3 \times 9 \times 27) \times \dots \times (3 \times 9 \times 27 \times 81 \times \dots \times 3^n)$ $= 3 \times (3^{1+2}) \times (3^{1+2+3}) \times \dots \times (3^{1+2+3+\dots+n})$ $= 3^{1+(1+2)+(1+2+3)+\dots+(1+2+3+\dots+n)}$ $= 3^{\frac{n(n+1)(n+2)}{6}}$
10 (i)	<p><math>f(x) = \sqrt{2-x} + 1, x \in \mathbb{R}</math></p>  <p>The horizontal line <math>y = 2</math> cuts the curve at more than one point, hence <math>f</math> is not one-to-one and <math>f^{-1}</math> does not exist.</p> <p><u>OR</u> <math>f(1) = f(3) = 2</math>, hence <math>f</math> is not one-to-one and <math>f^{-1}</math> does not exist.</p>
(ii)	<p>The minimum value is <math>k = 2</math>.</p> <p>Let <math>y = f(x) = \sqrt{2-x} + 1 = \sqrt{x-2} + 1 (\because x \geq 2)</math></p> $\Rightarrow x = 2 + (y-1)^2$ $D_{f^{-1}} = R_f = [1, \infty) \quad \therefore f^{-1}(x) = 2 + (x-1)^2, x \geq 1$

(iii)	<p>If there exists a solution for <math>f^{-1}(x) = f(x)</math></p> <p><math>\Rightarrow</math> there exists a solution for <math>f^{-1}(x) = x</math></p> <p><math>\Rightarrow 2 + (x-1)^2 = x</math></p> <p><math>\Rightarrow x^2 - 3x + 3 = 0</math></p> <p><math>\Rightarrow \left(x - \frac{3}{2}\right)^2 + \frac{3}{4} = 0</math></p> <p><math>\Rightarrow</math> no solution for <math>x</math></p> <p><math>\Rightarrow f^{-1}(x) = f(x)</math> has no solution.</p>
(iv)	 <p><math>[2, \infty) \xrightarrow{f} [1, \infty) \xrightarrow{g} (-\infty, 1] \therefore R_{gf} = (-\infty, 1]</math></p>
(v)	<p><math>gf(x) = 1</math></p> <p><math>f(x) = 3</math></p> <p><math>\sqrt{x-2} + 1 = 3</math></p> <p><math>\sqrt{x-2} = 2</math></p> <p><math>x - 2 = 4</math></p> <p><math>x = 6</math></p>
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	<p><u>3 axial intercepts</u>  <math>\left(0, \frac{3}{2}\right), \left(\pm\sqrt{\frac{3}{2}}, 0\right)</math> OR <math>(0, 1.5), (\pm 1.22, 0)</math></p> <p><u>2 turning points</u>  <math>(0.419, 1.68), (3.58, 14.3)</math></p> <p><u>2 asymptotes</u>  <math>x = 2, y = 2x + 4</math></p>
(a)	<p>Using the graph, the intersections of the curve with the line <math>y = 1</math> are <math>(-0.5, 1), (1, 1)</math>, so the solution is <math>-\frac{1}{2} \leq x \leq 1</math> or <math>x &gt; 2</math></p> <p><math>\frac{2\sin^2 x - 3}{\sin x - 2} \geq 1</math>          So the solution is <math>-\frac{1}{2} \leq \sin x \leq 1</math> or <math>\sin x &gt; 2</math> (rej)</p>  <p><math>\therefore 0 \leq x \leq \frac{7}{6}\pi</math> or <math>\frac{11}{6}\pi \leq x \leq 2\pi</math></p>
(b)	<p><math>y = \frac{2x^2 - 3}{x - 2} = 2x + 4 + \frac{5}{x - 2}</math></p> <p>Translation of 2 units in the positive <math>x</math>-direction, followed by translation of 8 units in the positive <math>y</math>-direction.</p>

12 (i)	$l_{EF} : \vec{r} = \begin{pmatrix} 5 \\ -14 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 7 \\ 0 \end{pmatrix}, \lambda \in \mathbb{R}$ $l_{AE} : 3x = z + 15$ $\frac{x-0}{1} = \frac{z-(-15)}{3}, y=0$ $l_{AE} : \vec{r} = \begin{pmatrix} 0 \\ 0 \\ -15 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}, \mu \in \mathbb{R}$ $\lambda = 2$ $\overrightarrow{OE} = \begin{pmatrix} 5 \\ -14 \\ 6 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ 7 \\ 0 \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \\ 6 \end{pmatrix}$
(ii)	$\vec{n} = \begin{pmatrix} 1 \\ 7 \\ 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 21 \\ -3 \\ -7 \end{pmatrix}$ $\begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 21 \\ -3 \\ -7 \end{pmatrix} = 105$ $\vec{r} \cdot \begin{pmatrix} 21 \\ -3 \\ -7 \end{pmatrix} = 105$
(iii)	<p>Method 1: By Observation, Projection vector of <math>\overrightarrow{AE}</math> onto <math>\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}</math></p>  <p>Method 2: Projection of <math>\overrightarrow{AE}</math> onto floor</p> $\overrightarrow{AE}' = \left( \begin{pmatrix} 2 \\ 0 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$ 

	<p>Method 1:</p> <p><math>F'X = 7</math> (Deduce from <math>\overrightarrow{OC}</math>)</p> <p>Area = <math>(AE')(F'X) = 2 \times 7 = 14</math></p>  <p>Method 2:</p> $\text{Area} =  \overrightarrow{AB} \times \overrightarrow{AE'}  = \left  \begin{pmatrix} 1 \\ 7 \\ 0 \end{pmatrix} \times \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} \right  = \left  \begin{pmatrix} 0 \\ 0 \\ 14 \end{pmatrix} \right  = 14$
(iv)	<p>Let <math>E''</math> be the reflection of <math>E</math> about and plane <math>OABC</math>.</p> <p><math>\overrightarrow{OE} = \begin{pmatrix} 7 \\ 0 \\ 6 \end{pmatrix}, \overrightarrow{OE''} = \begin{pmatrix} 7 \\ 0 \\ -6 \end{pmatrix}</math>,</p> <p><math>\overrightarrow{AE''} = \overrightarrow{OE''} - \overrightarrow{OA} = \begin{pmatrix} 2 \\ 0 \\ -6 \end{pmatrix}</math></p> <p><math>l_3: \underline{r} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix}, \beta \in \mathbb{R}</math></p> 
(v)	<p>Let <math>\Pi</math> be plane <math>x + ay + bz = c</math>.</p> <p><math>EF</math> is <math>\parallel \Pi</math>.</p> <p><math>\begin{pmatrix} 1 \\ 7 \\ 0 \end{pmatrix}</math> is <math>\perp</math> to <math>\underline{n}_{\Pi}</math>.</p> <p><math>\begin{pmatrix} 1 \\ 7 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ a \\ b \end{pmatrix} = 0 \Rightarrow 1 + 7a = 0 \Rightarrow a = -\frac{1}{7}</math></p> <p><math>E</math> is on plane <math>\Pi</math>.</p> <p><math>\begin{pmatrix} 7 \\ 0 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ a \\ b \end{pmatrix} = c \Rightarrow 7 + 6b = c.</math></p>

<p>13 (a)</p>	$x^3 - y^3 = 3x - 3y$ $\frac{d}{dx}(x^3 - y^3) = \frac{d}{dx}(3x - 3y)$ $3x^2 - 3y^2 \frac{dy}{dx} = 3 - 3 \frac{dy}{dx}$ $3x^2 - 3 = 3y^2 \frac{dy}{dx} - 3 \frac{dy}{dx}$ $\frac{x^2 - 1}{y^2 - 1} = \frac{dy}{dx}$ <p>Substitute <math>x = -2</math> and <math>y = 1</math>,</p> $\frac{dy}{dx} = \frac{3}{0} \text{ (undefined)}$ <p>Therefore, the tangent is a vertical line. Thus, the tangent is <math>x = -2</math>.</p>
<p>(b)</p>	<p>Let the radius be <math>r</math>.</p> <p>We want to find <math>\frac{dS}{dt}</math>,</p> $\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt} \div \frac{dV}{dr}$ $= (8\pi r) \times (0.1) \div (4\pi r^2)$ $= \frac{1}{5r}$ <p>Sub <math>r = \frac{1}{2}</math> into <math>\frac{dS}{dt}</math>, we get <math>\frac{2}{5} \text{ m}^2/\text{s}</math>.</p>
<p>(c)</p>	<p>Let the side of each room be <math>x</math>.</p> <p>By cosine rule,</p> $(4x)^2 = a^2 + b^2 - 2ab \cos \theta$ <p>Total area, <math>A = \frac{1}{2}ab \sin \theta + 4x^2</math></p> $A = \frac{1}{2}ab \sin \theta + \frac{1}{4}(a^2 + b^2 - 2ab \cos \theta)$ $= \frac{1}{2}ab \sin \theta + \frac{1}{4}a^2 + \frac{1}{4}b^2 - \frac{1}{2}ab \cos \theta$ <p>To find max area, we let <math>\frac{dA}{d\theta} = 0</math>.</p>

$$\frac{dA}{d\theta} = \frac{d}{d\theta} \left( \frac{1}{2} ab \sin \theta + \frac{1}{4} a^2 + \frac{1}{4} b^2 - \frac{1}{2} ab \cos \theta \right)$$

$$= \frac{1}{2} ab \cos \theta + \frac{1}{2} ab \sin \theta$$

$$\frac{1}{2} ab \cos \theta + \frac{1}{2} ab \sin \theta = 0$$

$$\tan \theta = -1$$

$$\theta = \frac{3\pi}{4} \quad (\text{since } 0 < \theta < \pi)$$

Therefore, stationary point at  $\theta = \frac{3\pi}{4}$ .

$$\frac{d^2 A}{d\theta^2} = \frac{1}{2} ab \cos \theta - \frac{1}{2} ab \sin \theta$$

$$\left. \frac{d^2 A}{d\theta^2} \right|_{\theta = \frac{3\pi}{4}} < 0$$

Thus, the stationary point is maximum.