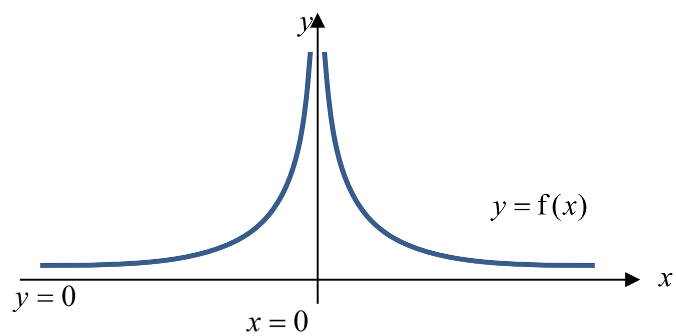
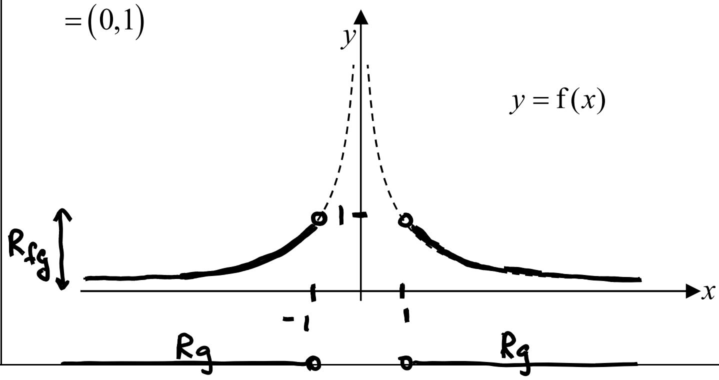
RAFFLES INSTITUTION
2019 YEAR 6 PRELIMINARY EXAMINATION

MATHEMATICS 9758/02 Suggested Solution

SOLUTION		COMMENTS
1(i) **[1]**		Students should be careful in showing asymptotic behavior and in the labeling of the two asymptotes $x=0$ (y-axis), $y=0$ (x-axis) .
1(ii) **[2]**	Least value of $k = 0$. If the domain of f is restricted to $x>0$, then for any horizontal line $y=h,\ \ h\in\mathbb{R}^+$, it cuts the graph of f at one and only one point, therefore f is a 1-1 function. Hence funcion f^{-1} exists.	For the horizontal line test to show that f is 1-1, many did not state that it is for any $y=h$, $h\in\mathbb{R}^+$ (which is the range of f) that cuts the graph of f at one and only one point.
1(iii) **[2]**	Consider $\mathrm{g}(-x)=-\mathrm{g}(x)$, with $x=1$. $$\frac{2}{3^{-1}-1}+m=-\left(\frac{2}{3-1}+m\right)$$ $$2m=3-1$$ $$m=1$$ NOTE : We may attempt this part with any specific value of x (in the domain of g).	Many did not see that since it is given that g is an odd function, then $\mathrm{g}(-x)=-\mathrm{g}(x)$ holds for any x in the domain. For those who tried to solve it for a general value of x, some did not simplify m to $m=\dfrac{-1}{3^x-1}-\dfrac{1}{3^{-x}-1}$ $=\dfrac{-1}{3^x-1}-\dfrac{3^x}{1-3^x}$. $=\dfrac{3^x-1}{3^x-1}=1$
1(iv) **[2]**	$R_g = (-\infty, -1)\cup(1,\infty)$.	R_g is found by looking at the asymptotic behavior of g as

	$\mathrm{R_{fg}} = \mathrm{R_f}$ with domain restricted to $(-\infty,-1)\cup(1,\infty)$ $=(0,1)$ 	$x\to\pm\infty$, which gives the y-values of g as $y<-1$ or $y>1$. Note that equivalent set notations for $(-\infty,-1)\cup(1,\infty)$ are $\mathbb{R}\setminus[-1,1]$, or $(-\infty,\infty)\setminus[-1,1]$. Note also that $\cup$ should not be written as "or", "and", "$\cap$", ",".

SOLUTION		**COMMENTS**
2(a) **[4]**	When $x=0$, $\dfrac{dy}{dx}=\left(\dfrac{81}{3}-15(0)\right)^{\frac{1}{3}}=3$ $\dfrac{d^2y}{dx^2}=\dfrac{1}{3}\left(\dfrac{1}{3}y-15x\right)^{-\frac{2}{3}}\left(\dfrac{1}{3}\dfrac{dy}{dx}-15\right)$ When $x=0$, $\dfrac{d^2y}{dx^2}=\dfrac{1}{3}\left(\dfrac{81}{3}-15(0)\right)^{-\frac{2}{3}}\left(\dfrac{1}{3}(3)-15\right)=-\dfrac{14}{27}$ Hence $y=81+3x-\dfrac{7}{27}x^2+\ldots$	Students should not present the following: $\dfrac{dy}{dx}=\left(\dfrac{y}{3}-15x\right)^{\frac{1}{3}}$ $=\left(\dfrac{81}{3}-15(0)\right)^{\frac{1}{3}}\cdots(*)$ (*) is only true when $\dfrac{dy}{dx}$ is evaluated at (0, 81). The presentation of $y=81+3x-\dfrac{7}{27}x^2+\ldots$ is not ideal for a significant number of students. Students should use $\approx$ or include $+\ldots$ in their answer.
2(b) **(i)** **[2]**	$\dfrac{4-3x+x^2}{(1+x)(1-x)^2}=\dfrac{A}{1+x}+\dfrac{1}{1-x}+\dfrac{B}{(1-x)^2}$ Therefore, $4-3x+x^2=A(1-x)^2+(1+x)(1-x)+B(1+x)$ When $x=1$: $2=2B \Rightarrow B=1$ When $x=-1$: $8=4A \Rightarrow A=2$	Quite a number of students tried to solve partial fractions in the form $\dfrac{A}{1+x}+\dfrac{C}{1-x}+\dfrac{B}{(1-x)^2}$, which involves solving 3 unknowns and is not necessary at all. A handful of students used

		$\frac{A}{1+x}+\frac{B}{1-x}+\frac{C}{(1-x)^2}$, i.e. the same unknowns as the ones given in the question, which is not accepted.
2(b) (ii) [3]	$\mathrm{g}(x)=\frac{4-3x+x^2}{(1+x)(1-x)^2}$ $=\frac{2}{1+x}+\frac{1}{1-x}+\frac{1}{(1-x)^2}$ $=2(1-x+x^2-x^3+...)+(1+x+x^2+x^3+...)$ $+(1+2x+3x^2+4x^3+...)$ $=4+x+6x^2+3x^3+...$ Hence $c_0=4, c_1=1, c_2=6,$ and $c_3=3$.	A good number of students differentiated $\mathrm{g}(x)$ repeatedly and obtained the following: $c_0=\mathrm{g}(0), c_1=\mathrm{g}'(0),$ $c_2=\frac{\mathrm{g}''(0)}{2!}$ and $c_3=\frac{\mathrm{g}'''(0)}{3!}$. They will arrive at the same answer, however, the working seems significantly longer. Please note that $(1+x)^{-1}$ and $(1-x)^{-1}$ could be obtained easily from MF26. Also note that, $\frac{1}{(1-x)^2}=\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{1-x}\right)$.
2(b) (iii) [1]	From above, $c_r=2(-1)^r+1+(r+1)=2(-1)^r+r+2$.	Students should note that c_r only refers to the coefficient of the x^r term.

SOLUTION		COMMENTS
General Comments: The question is well done in general, but many parts (ii)(b), (iii)(a), (iv) require detailed and clear explanations as they are show questions or the question explicitly required clear working. For example, in such show questions, scalar products should be expanded in full. For (iv), there are many correct numerical answers but not all obtained full credit. For students who obtained $2\cos^{-1}\left(\frac{7}{\sqrt{62}}\right) = 54.5^\circ$, the angle in question must be clearly defined or labelled in a diagram for full credit to be awarded.		
3(i) **[1]**	$(\mathbf{a}-\mathbf{b})\times(\mathbf{c}-\mathbf{b}) = \begin{pmatrix}-3\\1\\1\end{pmatrix}\times\begin{pmatrix}-1\\-2\\1\end{pmatrix} = \begin{pmatrix}3\\2\\7\end{pmatrix}$.	When the position vectors/coordinates are given, there is no need to evaluate using properties of cross product and end up doing more work.
3(ii) **(a)** **[1]**	Hence exact area of triangle ABC is $\frac{1}{2}\left\|\begin{pmatrix}3\\2\\7\end{pmatrix}\right\| = \frac{\sqrt{62}}{2}$.	Do not forget the $\frac{1}{2}$.
3(ii) **(b)** **[1]**	Hence the equation of the plane ABC is given by $\mathbf{r}\cdot\begin{pmatrix}3\\2\\7\end{pmatrix} = \begin{pmatrix}0\\1\\2\end{pmatrix}\cdot\begin{pmatrix}3\\2\\7\end{pmatrix} = 2+14=16$. Hence the cartesian equation of the plane is $3x+2y+7z=16$.	Choose a point on the plane with more 0's in the coordinates– the corresponding scalar product is easier to evaluate.
3(iii) **(a)** **[3]**	$\begin{pmatrix}1\\2\\-1\end{pmatrix}\cdot\begin{pmatrix}3\\2\\7\end{pmatrix} = 3+4-7=0$ Since the line is perpendicular to the normal vector of the plane, the line and the plane are parallel. Since $\begin{pmatrix}1\\1\\6\end{pmatrix}\cdot\begin{pmatrix}3\\2\\7\end{pmatrix} = 3+2+42=47\neq 16$, a point on the line does not lie on the plane, and since the line and the plane are parallel, the line does not lie on the plane.	The scalar product being 0 only shows that the line and the normal vector of the plane are perpendicular. This relation has to be clearly stated. $\begin{pmatrix}1\\1\\6\end{pmatrix}\cdot\begin{pmatrix}3\\2\\7\end{pmatrix}\neq 16$ only shows the point does not lie on the plane.

	Alternatively, $\left[\begin{pmatrix}1\\1\\6\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\-1\end{pmatrix}\right]\cdot\begin{pmatrix}3\\2\\7\end{pmatrix}=47\neq 16$ for all real λ so there is no intersection between the line and the plane. This means the line and the plane have to be parallel, and the line does not lie on the plane.	You need to show that the line and plane are parallel first before concluding that the the line is not contained in the plane.
3(iii) (b) [2]	The distance between the line and the plane is $\dfrac{\left\lvert\left[\begin{pmatrix}1\\1\\6\end{pmatrix}-\begin{pmatrix}0\\1\\2\end{pmatrix}\right]\cdot\begin{pmatrix}3\\2\\7\end{pmatrix}\right\rvert}{\sqrt{7^2+3^2+2^2}}=\dfrac{31}{\sqrt{62}}=\sqrt{\dfrac{31}{2}}=\dfrac{\sqrt{62}}{2}.$ **Alternate method** (more tedious and not encouraged since question did not ask for foot of perpendicular) Let F be the foot of perpendicular from $P(1, 1, 6)$ to the plane. Then $\overrightarrow{OF}=\begin{pmatrix}1\\1\\6\end{pmatrix}+\mu\begin{pmatrix}3\\2\\7\end{pmatrix}$ for some $\mu\in\mathbb{R}$. Then since F lies on the plane, $\left[\begin{pmatrix}1\\1\\6\end{pmatrix}+\mu\begin{pmatrix}3\\2\\7\end{pmatrix}\right]\bullet\begin{pmatrix}3\\2\\7\end{pmatrix}=16\Rightarrow\mu=-\dfrac{1}{2}$. Therefore the distance is $\lvert\overrightarrow{PF}\rvert=\lvert\overrightarrow{OF}-\overrightarrow{OP}\rvert=\left\lvert\mu\begin{pmatrix}3\\2\\7\end{pmatrix}\right\rvert=\dfrac{1}{2}\sqrt{62}.$	Please be careful when transferring the values to your work. Many ‘6’ morphed to ‘0’ and the scalar products were not carefully evaluated. There are many students who went to find the foot of perpendicular from (1, 1, 6) to the plane. This is unnecessary and often showed conceptual errors such as letting $\overrightarrow{OF}$ be $\overrightarrow{PF}$.
3(iv) [3]	The angle θ between the line and the plane is given by $\sin\theta=\dfrac{\left\lvert\begin{pmatrix}0\\0\\1\end{pmatrix}\cdot\begin{pmatrix}3\\2\\7\end{pmatrix}\right\rvert}{\sqrt{62}}=\dfrac{7}{\sqrt{62}}\Rightarrow\theta=62.748^\circ$. Hence the angle between the line and its reflection in the plane is $2(62.748)=125.496^\circ$ and the required acute angle is thus $180^\circ-125.496^\circ=54.5^\circ$ (1 d.p) **Alternatively**, The angle ϕ between the line and the normal of the plane is given by	As mentioned earlier in the comments, you will need clearly defined angles or diagrams with angles indicated to gain full credit for this question even if the numerical answer is correct. Common errors include angle between line and plane is $\cos^{-1}\left(\dfrac{7}{\sqrt{62}}\right)$

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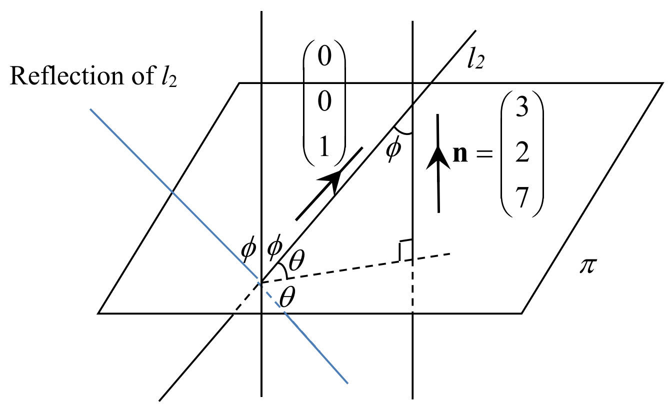
$$\cos\phi = \frac{\left|\begin{pmatrix}0\\0\\1\end{pmatrix}\cdot\begin{pmatrix}3\\2\\7\end{pmatrix}\right|}{\sqrt{62}} = \frac{7}{\sqrt{62}} \Rightarrow \phi = 27.252^\circ$$

The angle between the line and its reflection in the plane ABC is thus 2 times of the above angle, and since $2(27.252^\circ) = 54.5^\circ < 90^\circ$, this is the required angle.



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, when it should be $\sin^{-1}\left(\frac{7}{\sqrt{62}}\right)$ or $90^\circ - \cos^{-1}\left(\frac{7}{\sqrt{62}}\right)$.

Again, there were a handful of students who went to calculate the foot of perpendicular from a point on the line, before finding the reflection of the point about the plane. This is totally inefficient and unnecessary, and often resulted in algebraic and conceptual errors mentioned in (iii).

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SOLUTION		COMMENTS
4(a) (i) [1]	$b=\frac{a+c}{2}$	Many students did this in a long-winded manner. It may be noted that a, b, c are also consecutive terms of an AP. Hence, $b-a=c-b$, and the answer can be quoted right away.
4(a) (ii) [1]	Common difference, $d=\frac{b-a}{2}$	
4(a) (iii) [2]	Sum of first 10 terms $=\frac{10}{2}\left(2a+(10-1)\left(\frac{b-a}{2}\right)\right)$ $=5\left(2a+\frac{9}{2}\left(\frac{a+c}{2}-a\right)\right)$ $=\frac{5}{4}(9c-a)$	Generally well done. Reminders ♦ To write 9 and a more distinctly different. ♦ Clear steps are expected (as the result is given in the question).
4(a) (iv) [2]	Let r be the common ratio. $r=\frac{4^{\text{th}}\text{ term}}{3^{\text{rd}}\text{ term}}=\frac{a}{b}$. Also, 3^{rd} term $=cr^2=b$. $\Rightarrow (2b-a)\left(\frac{a}{b}\right)^2=b$ $\Rightarrow \quad (2b-a)a^2=b^3$	There are so many approaches to do this part. If your solution is more than 4 or 5 lines, please take a look at this suggested solution.
4(b) (i) [4]	Common ratio, $r=2\sin^2\alpha$ For the series to be convergent, $\lvert r\rvert<1$. That is, $\lvert 2\sin^2\alpha\rvert<1$. $\Rightarrow \quad \lvert\sin\alpha\rvert<\frac{1}{\sqrt{2}}$ $\Rightarrow \quad -\frac{1}{\sqrt{2}}<\sin\alpha<\frac{1}{\sqrt{2}}$	Many students went on to consider u_n/u_{n-1} to find r. For this question, you don't have to, really. Other common **mistakes** : ♦ $\sin\alpha<\frac{1}{\sqrt{2}}$ (No lower bound.)

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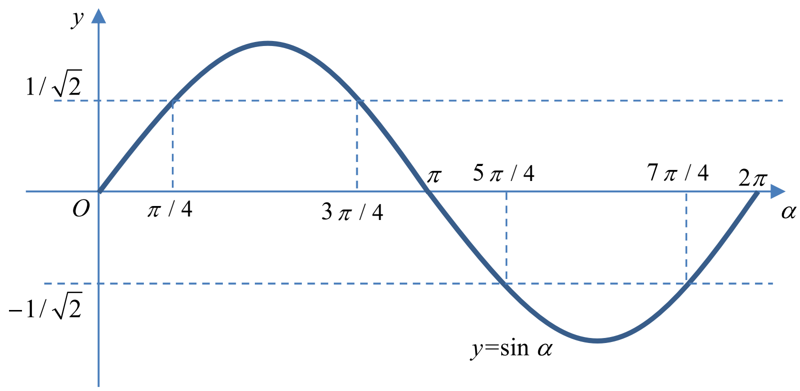

However, for this particular question, $r = 0$ makes the first term 0^0, which is not defined.
Therefore, $2\sin^2\alpha \neq 0$.
$\Rightarrow \quad \sin\alpha \neq 0$
$\Rightarrow \quad \alpha \neq 0,\ \pi,\ 2\pi$

Hence,
$0 < \alpha < \frac{\pi}{4}$ or $\frac{3\pi}{4} < \alpha < \pi$ or $\pi < \alpha < \frac{5\pi}{4}$ or $\frac{7\pi}{4} < \alpha < 2\pi$.
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♦ $0 < \sin\alpha < \frac{1}{\sqrt{2}}$
(Claiming $\sin\alpha$ cannot take negative values.)
♦ $\sin\alpha < \pm\frac{1}{\sqrt{2}}$
(This makes no sense.)
♦ $-\frac{1}{4} < \sin\alpha < \frac{1}{4}$

Many struggled to solve the inequality involving trigo expression. Advice : Sketch the graphs out!

Very few students considered $r \neq 0$.
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<td>4(b)
(ii)
[2]</td>
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Sum to infinity $= \frac{1}{1-2\sin^2\alpha}$
$= \frac{1}{\cos 2\alpha}$
$= \sec 2\alpha$
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Reminders :
♦ To write α and a more distinctly different.
♦ $\frac{1}{\cos 2\alpha} \neq \cos^{-1} 2\alpha$
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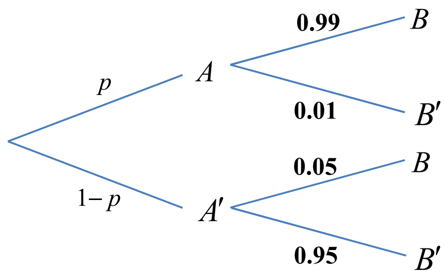
SOLUTION		COMMENTS
5(a) **[5]**	Let A be the event that a patient has a particular disease, and B be the event that a test for the disease gives a positive result.  $\mathrm{f}(p)=\mathrm{P}(A\|B)=\dfrac{0.99p}{0.99p+0.05(1-p)}$ $=\dfrac{99p}{94p+5}$ $\mathrm{f}'(p)=\dfrac{99(94p+5)-99p(94)}{(94p+5)^2}$ $=\dfrac{495}{(94p+5)^2}>0$, since $(94p+5)^2>0$ for $0<p<1$. Hence, f is an increasing function for $0<p<1$. The greater the prevalence of the disease in the population (hence the higher the possibility of being infected), the higher the chance that a positive test means that the patient actually has the disease. Examples of answers which ***were not accepted***: As the probability of a patient being infected with a particular disease increases, i) probability of patient being tested positive increases (No reference to f(p) being conditional probability) ii) probability of a patient being diagnosed correctly increases (not clear whether the given condition refers to patients who have tested positive or negative or patients who have disease or no disease)	A tree diagram is best suited for this type of question involving different conditional probabilities. Using the GC to sketch the graph of f is not an acceptable method to show f is an increasing function. Also, f(p) >0 does not imply f is an increasing function. The terms p and f(p) and their relationship has to be explained in the context of the question.
5(b) **(i)** **[1]**	Since A and B are independent events, A and B' are also independent events. $\mathrm{P}(A\cap B')=\mathrm{P}(A)P(B')=\dfrac{7(1-k)}{10}$	

5(b) (ii) [2]	Since A and B are mutually exclusive events $B \subseteq A'$, $0 \le k = \mathrm{P}(B) \le \mathrm{P}(A') = 0.3$ $\mathrm{P}(B \mid A') = \dfrac{\mathrm{P}(B \cap A')}{\mathrm{P}(A')} = \dfrac{\mathrm{P}(B)}{\mathrm{P}(A')} = \dfrac{k}{0.3} = \dfrac{10}{3}k$.	You are encouraged to draw your own Venn diagram to visualize the relation between A and B. The condition that A and B are independent is only applicable in (bi) and should not be assumed in (bii) (see the word " instead" used in the question for bii).

SOLUTION		**COMMENTS**
6(i) [2]	We require $a \to A,\ b \to C$ or D or E and no restriction on c, d and e. Number of ways $= 1 \times 3 \times 3! = 18$	Students who did not get the final answer but presented the correct "thinking process" were given a mark here. As such, for P&C questions, do provide some explanation instead of just writing a string of numbers.
6(ii) [3]	**<u>Method 1</u>** : Number of ways without restriction = 5! = 120 **Case 1** : Exactly one of a or b placed correctly. Number of ways = $(\underbrace{1\times 3 \times 3!}_{\text{(from part (i))}}) \times 2 = 36$ **Case 2** : both a and b placed correctly. Number of ways $= 1 \times 1 \times 3! = 6$ Total number of ways = 120 – 36 – 6 = 78 **<u>Method 2</u>** : Number of ways without restriction = 5! = 120 Number of ways $a \to A$ $= 1 \times 4! = 24$ Number of ways $b \to B$ $= 1 \times 4! = 24$ Number of ways $a \to A$ and $b \to B$ $= 1 \times 1 \times 3! = 6$ Total number of ways = 120 – 2(24) + 6 = 78	There are at least 10 ways to tackle this part. 4 have been out-lined here. Method 1 is "direct" as it makes use of the answer in (i). Quite a number of students were not careful and thought the complement was just Case 2. We can see (ii) as $n(A' \cap B') = n(\Omega) - n(A \cup B)$ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

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Method 3 :

Cases	Number of ways
$a \to B$, $b \to A, C, D, E$	$1 \times 4 \times 3! = 24$
$a \to C$, $b \to A, D, E$	$1 \times 3 \times 3! = 18$
$a \to D$, $b \to A, C, E$	18 (as above)
$a \to E$, $b \to A, C, D,$	18 (as above)

Total number of ways $= 24 + 18 + 18 + 18 = 78$

Method 4 :
Case 1 : a is placed in B
Number of ways = $1 \times 4! = 24$

Case 2 : a is not placed in B
Number of ways = ${}^{3}C_{1} \times {}^{3}C_{1} \times 3! = 54$

Total number of ways $= 24 + 54 = 78$
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A number of students did not realize that the 1st two cases are not mutually exclusive.

Method 3 is straightforward, although seemingly long. A simple table can help to make the solution less cumbersome however. When listing cases, it is important to ensure that you have considered all possibilities, and check if the cases are mutually exclusive.

Notice that Method 4 is similar to method 3. It considers cases 2 to 4 in method 3 as one case.
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<td>6(iii)
[3]</td>
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Case 1: 2 correct, 3 incorrect placings
Number of ways to choose 2 objects to be placed correctly = ${}^{5}C_{2} = 10$
Suppose $a \to A$ and $b \to B$.
Then $c \to D$, $d \to E$, $e \to C$
or $c \to E$, $d \to C$, $e \to D$
Number of ways = 10 × 2 = 20

Case 2: 3 correct, 2 incorrect placings
Number of ways to choose 3 objects to be placed correctly = ${}^{5}C_{3} = 10$
Suppose $a \to A$, $b \to B$ and $c \to C$.
Then $d \to E$, $e \to D$
Number of ways = 10 × 1 = 10

Case 3: 5 correct placings
Number of ways = 1

Total number of ways = 20 + 10 + 1 = 31
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Note that it is not possible to have 4 correct placings and 1 incorrect placing.

Many students thought that the answer is ${}^{5}C_{2} \times 3!$. This is actually the number of ways of getting at least 2 correct placings with repetitions. Try listing out the cases to convince yourself.

Although there are only 2 cases to consider if using the complement method, it is not so straightforward. The answer is
$5! - {}^{5}C_{1} \times {}^{3}C_{1} \times 3 - {}^{4}C_{1}\left(2 + 3 \times 3\right)$.
Try figuring it out if you are keen.
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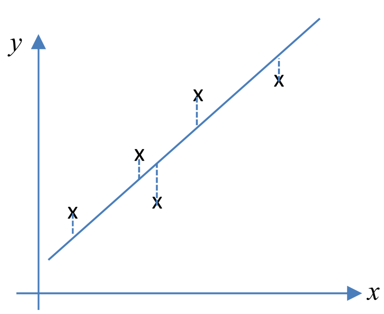
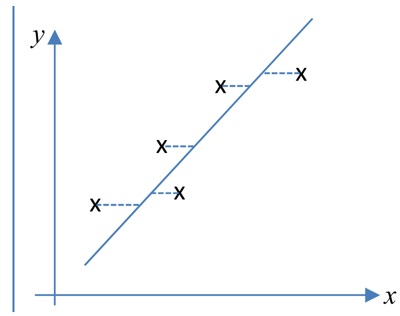
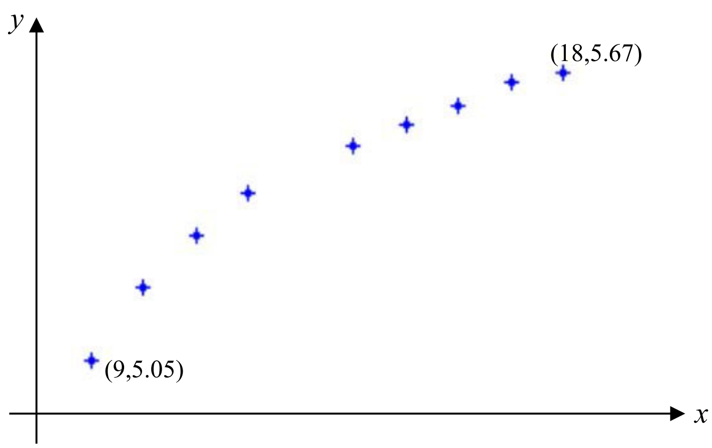
SOLUTION		COMMENTS
7(i) **[2]**	Let X be the time in minutes of a medical consultation. If $X \sim \mathrm{N}(15, 10^2)$, then $\mathrm{P}(X < 0) = 0.066807$ (5 s.f.) $= 0.0668$ (3 s.f.) This means that about 7 patients out of every 100 take a "negative" amount of time which is not possible.	Most students did not mention that time cannot be negative. Proper justification needs to be given in order to get full credit.
[1]	$\mathrm{H}_0: \ \mu = 15$ vs $\mathrm{H}_1: \ \mu > 15$ where μ denotes the population mean consultation time in minutes.	
7(ii) **[3]**	Under H_0, since $n = 30$ is large, $\bar{X} \sim \mathrm{N}\left(15, \frac{10^2}{30}\right)$ approximately by Central Limit Theorem. We also assume that the population standard deviation is unchanged. $\mathrm{P}(\bar{X} > 18) = 0.050174$ (5 s.f.) $= 0.0502$ (3 s.f.) The smallest level of significance is 5.02%	Most students were able to pen down the sample mean distribution with correct use of CLT. Students must remember that to reject the null hypothesis, the level of significance must be greater or equals to the p-value.
7(iii) **[4]**	$\sum (y - 15) = -50$, $\sum (y-15)^2 = 555$. An unbiased estimate of the population mean is, $\bar{y} = 15 + \frac{-50}{80} = \frac{115}{8}$ An unbiased estimate of the population variance is, $s^2 = \frac{1}{79}\left(555 - \frac{(50)^2}{80}\right) = \frac{2095}{316}$ $\mathrm{H}_0: \ \mu = 15$ vs $\mathrm{H}_1: \ \mu < 15$ Under H_0, since $n = 80$ is large, $\bar{Y} \sim \mathrm{N}\left(15, \frac{\frac{2095}{316}}{80}\right)$ approximately by Central Limit Theorem. Since p-value $= \mathrm{P}\left(\bar{Y} < \frac{115}{8}\right) = 0.0149623748 = 0.0150$ (3 sf) < 0.05. We reject H_0, and conclude that there is sufficient evidence, at the 5% significance level, to support the administrator's claim that the average consultation time at private clinics is less than 15 minutes.	This part was very well done. Students who have studied this topic were able to find the unbiased estimate of the population mean and variance, set up the null and alternate hypothesis, correct sample mean distribution with CLT, and lastly, correct p-value. A handle of students were penalized for their tardy presentation and incomplete conclusion.

SOLUTION		COMMENTS
8(i) **[3]**	Sum of all probabilities = 1, thus, $k+3k+5k+7k+11k+13k=1$ $40k=1$ $\therefore k=\dfrac{1}{40}$ $\mathrm{P}(X=3)=3k=\dfrac{3}{40}$.	This is a 3-mark show question. You need to show your detailed working.
8(ii) **[3]**	$\mathrm{E}(X)=k+3(3k)+5(5k)+7(7k)+11(11k)+13(13k)$ $=374k$ $=\dfrac{374}{40}$ $\mathrm{E}(X^2)=k(1+3^3+5^3+7^3+11^3+13^3)$ $=4024k$ $=\dfrac{4024}{40}$ $\mathrm{Var}(X)=\mathrm{E}(X^2)-\left[\mathrm{E}(X)\right]^2$ $=\dfrac{4024}{40}-\left(\dfrac{374}{40}\right)^2$ $=\dfrac{5271}{400}$	GC can give you the answers for this part. However, Question asked for exact value. You have to show the working on how you get the values of $\mathrm{E}(X)$ and $\mathrm{E}(X^2)$.
8(iii) **[2]**	$R \sim \mathrm{B}(15,0.4)$ $P(R\geq 5)=1-P(R\leq 4)$ $=0.783$	R is defined in the question. You NEED to state the distribution.
8(iv) **[2]**	Let W be the random variable denoting the number of times that a score less than 10 is observed in 14 throws. Then $W \sim \mathrm{B}(14, 0.4)$. Required probability = $\mathrm{P}(W=7)\times 0.4=0.0630$ (Explanation: "W = 7" represent 7 out of the first 14 throws have score less than 10 follow by the 15$^{\text{th}}$ throw having score less than 10) Alternatively (without defining new random variable, BUT using the same argument AND the correct formula), Required probability = ${}^{14}C_7\left(0.6\right)^7\left(0.4\right)^7\left(0.4\right)=0.0630$	Remember to define your random variable AND state its distribution.

SOLUTION		**COMMENTS**
9(i) **[1]**	Let R denote the working lifespan of Brand R tyre. $R \sim \mathrm{N}(64000, 8000^2)$ Required probability = $\mathrm{P}(R > 70000) = 0.227$ (to 3 s.f.).	In general, the letter used for a new random variable should always be defined with the distribution explicitly written down, same for part (iii) This part was well done as it is only 1 mark so not penalized for presentation
9(ii) **[2]**	Consider $\mathrm{P}(R > t) = 0.98$. From G.C., $t = 47570$ (to nearest whole number).	It is a 2 mark question so student should have a simple line stating what they are working with, or a normal distribution graph with region shaded, before writing down the value of t
9(iii) **[2]**	Let S denote the working lifespan of Brand S tyre. $\bar{S} \sim \mathrm{N}\left(68000, \frac{7500^2}{50}\right)$. Required probability $= \mathrm{P}(\bar{S} > 70000)$ $= 0.0297$ (correct to 3 s.f.).	Some students chose to do by total sum of 50 tyres instead which is fine too. Do note that the question already gave the lifespan to be normally distributed so there is no reason to bring in C.L.T.
9(iv) **[3]**	$(R_1 + R_2 + R_3) - 3S \sim \mathrm{N}(-12000\ ,12(8000^2))$ Required probability $= \mathrm{P}((R_1 + R_2 + R_3) < 3S)$ $= \mathrm{P}((R_1 + R_2 + R_3) - 3S < 0)$ $= 0.667$ (to 3 s.f.)	Generally well done, usual careless mistake of not taking the square root of the variance when entering into the GC.
9(v) **[3]**	Consider $\mathrm{P}(S > 50000) > \mathrm{P}(R > 50000)$ $\mathrm{P}\left(Z > \frac{50000 - 68000}{\alpha}\right) > 0.9599408865$ $\mathrm{P}\left(Z > \frac{-18000}{\alpha}\right) > 0.9599408865$ $\frac{-18000}{\alpha} < -1.750000503$ $\therefore \quad 0 < \alpha < 10286$ (to nearest whole number)	Easy 1 mark for writing down this inquality. Subsequently, students who do not sketch the normal curve would have to be careful to get the

	[or $0 < \alpha \leq 10285$]	correct direction for the inequality sign Unfortunately, many students could not get the last mark for this question, mostly either not stating the lower bound or stating wrongly ($\leq$ instead of $<$)
9(vi) **[1]**	Assume that the working lifespan of **<u>all</u>** the tyres are independent.	This is another part where many students lost mark by not being clear enough. It is much simpler to say "all the tyres" but some students solution only suggested that Brand S needs to be independent with Brand R

SOLUTION		COMMENTS
10(a) **(i)** **[2]**	Regression line y on x :  Regression line x on y :  The least squares regression line of y on x is the line with the minimum sum of the squared vertical distances for each point to the line while the the least squares regression line x on y is the line with the minimum sum of the squared horizontal distances from each point to the line.	Both the least squares regression line is related to the minimum sum of the squared distances and NOT the sum of the distances.
10(b) **(i)** **[2]**	 The scatter diagram shows that the rate of increase of y decreases as x increases. So the variables do not have a linear relation and hence the relationship is not likely to be well modelled by an equation of the form $y = a + bx$, where a and b are constants.	The points which defines the ranges of the values of x and y must be labelled. The scales of the axes should be carefully selected to "space out" the points so that the trend can be observed correctly. The scatter diagram should be used to explain the relationship and not using the product moment coefficient.
10(b) **(ii)** **[2]**	**(a)** product moment correlation coefficient between $\ln x$ and y , $r_1 = 0.986554$ (correct to 6 d.p.) **(b)** product moment correlation coefficient between x^2 and y , $r_2 = 0.945806$ (correct to 6 d.p.)	Note that the question has required the answers to be corrected to 6 decimal places and not 3 significant figures.

10(b) (iii) [1]	Since $\lvert r_2 \rvert < \lvert r_1 \rvert < 1$, the product moment correlation coefficient between $\ln x$ and y (= 0.986553) is closer to 1, hence $y = a + b\ln x$ is the better model.	It is must be noted that the better model is chosen based on the product moment coefficient being closer to 1 (or -1 in other cases) and NOT just being larger.
10(b) (iv) [3])	The equation of the suitable regression line is $y = 3.237876 + 0.854181\ln x$ $y = 3.24 + 0.854\ln x$ (3 s.f.) When y = 6.5, x = 45.55899 The population size first exceed 6.5 millions in the year 2045. Since the population size of 6.5 millions is not in the data range (from 5.05 to 5.67), the estimate is an extrapolation and so it is not reliable.	The regression line of ln x on y should NOT be used even though a similar answer is obtained. Need to observe carefully the required year is 2045 and NOT 2046 based on the context. It is not sufficient to mention that it is an extrapolation. It is necessary to state the given population size is not in the given data range.
10(b) (v) [1]	When x = 13, $y = 3.237876 + 0.854181\ln 13 = 5.428807$ The required estimate is 5.43 millions	It is necessary to state the estimate as 5.43 millions and not just 5.43.
10(b) (vi) [1]	5.31 millions is lower than 5.43 millions calculated in part (v) (and it does not follow the rising trend of the data given - outlier). There might be some form of deadly diseases spreading in part(s) of the communities in the country that caused the drop in actual figure that year. OR There might be a natural disaster in the country that caused the drop in actual figure that year.	Need to take note that the difference of estimated and actual population sizes is more than100,000 and the actual size is smaller than the previous year, so a decrease in birth rate is not enough to justify the difference.