ANNEX

H2 MA 2019 JC2 Prelim (Paper 1 and Paper 2)

Paper 1

Select topic from the dropdown list. If the question consists of multiple topics, choose 1 topic.

QN	TOPIC (H2) Paper 1	ANSWERS (Exclude graphs and text answers)
1	Graphs & Transformations	(a) $-\frac{9}{8}<k<-1$ (b) $\left(3,-\frac{9}{8}\right)$, $x=-1$, $y=-1$
2	Integration & Applications	(ii) 20
3	Functions	(ii) $\mathrm{f}^{-1}(x)=1-\frac{3}{x-1}=\frac{x-4}{x-1}$, $\frac{97}{100}$ (iii) $(-\infty,1)$
4	Maclaurin & Binomial Series	(i) $y=\sqrt{\mathrm{e}}-\frac{\sqrt{\mathrm{e}}}{4}x^2+\ldots$ (ii) $1+\frac{1}{4}x^2+\ldots$ (iii) 1.650
5	Integration & Applications	(ii) $y=2ap-p(x-ap^2)$ (iii) $\frac{16a^2}{p^4}(p^2+1)^3$
6	Complex Numbers	(a)(i) $2\mathrm{e}^{\mathrm{i}\left(-\frac{2\pi}{3}\right)}$ (ii) $n=2$ and $n=5$ (b) $w=7+2\mathrm{i}$ and $z=1-5\mathrm{i}$
7	Vectors	(i) 3 (iv) 0.730, $t=0.910, 1.45,\ 4.05,\ 4.59$
8	APGP	(a) r = 0.882854, 222.38 (b)(i) $\frac{\pi}{3}$ (ii) 16 (iii) $\frac{8}{7}$ mm
9	Integration & Applications	(a)(i) $\frac{225}{4}\pi$ (a)(ii) $\left(450-\frac{75}{2}\pi\right)$ cm^2 (b) $\left(32500\pi-2250\pi^2\right)$ cm^3

10	Differentiation & Applications	(i) 0.0327 (ii) 2.14 units^2, $(-0.449, 2.73)$ (iii) $P(-0.785, 2.55)$
11	H2 Prelim P1 Q11 Topic	Nil
12	H2 Prelim P1 Q12 Topic	Nil
13	H2 Prelim P1 Q13 Topic	Nil
14	H2 Prelim P1 Q14 Topic	Nil

Paper 2

Select topic from the dropdown list. If the question consists of multiple topics, choose 1 topic.

QN	TOPIC (H2) Paper 2	ANSWERS (Exclude graphs and text answers)
1	Graphs & Transformations	(i) $a = 4, \quad b = 4$
2	Differential Equations	(i) $V = \frac{1}{a}(k-(k-a)e^{-at})$
3	Vectors	(ii) $a \in \mathbb{R}, a \neq -2$. (iii) $l_{BA'}: \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 7 \\ 5 \end{pmatrix}, \ \mu \in \mathbb{R}$. (iv) 31.5° (v) $x+z=9$ or $x+z=-1$
4	Sigma Notation & MOD	(a) $\frac{n+1}{2^{n+1}} - \frac{1}{2}$ (b)(ii) 6
5	Normal Distribution	0.737, 0.943
6	PnC & Probability	(i) 1242 (ii) 0.210
7	DRV	(i) $\frac{n+1}{2}, \frac{n^2-1}{12}$ (ii) 0.339
8	PnC & Probability	(i) $\frac{5}{16}, \frac{825}{4096}$ (ii) $-9 + \frac{3600}{360+n}$
9	Correlation & Regression	(ii) (a) 0.96346 (ii) (b) 0.98710 (iii) second model (iv) 121
10	Binomial Distribution	(ii) 0.401 (iii) 0.535 (iv) 0.423 (v) $36p(1-p)^{35}$

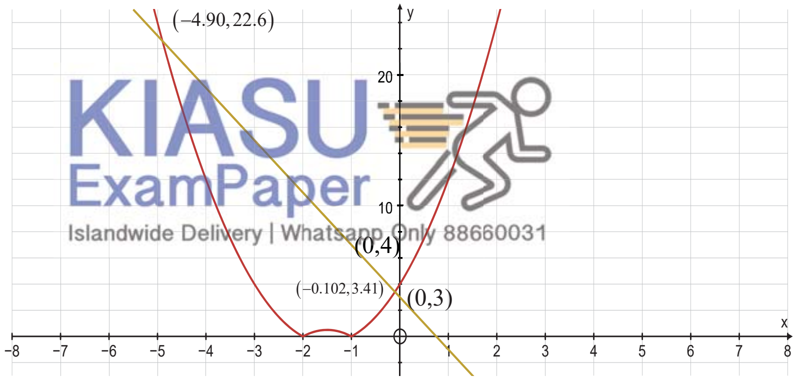
		(vi) $p=\dfrac{1}{36}$
11	Hypothesis Testing	(i) 32.1, 25 (iii) p-value = 0.0179 , reject H_0. (iv) $0<\overline{t}<31.6$. (v) $\sigma>7.30$
12	H2 Prelim P2 Q12 Topic	Nil
13	H2 Prelim P2 Q13 Topic	Nil
14	H2 Prelim P2 Q14 Topic	Nil

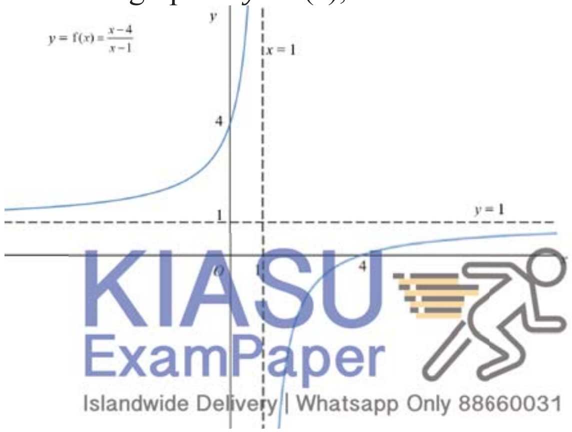
2019 H2 Math Prelim Paper 1 Solutions

<table>
<tr><th>Qn</th><th>Solutions</th></tr>
<tr><td>1(a)</td><td>$\mathrm{f}(3-x)=k$

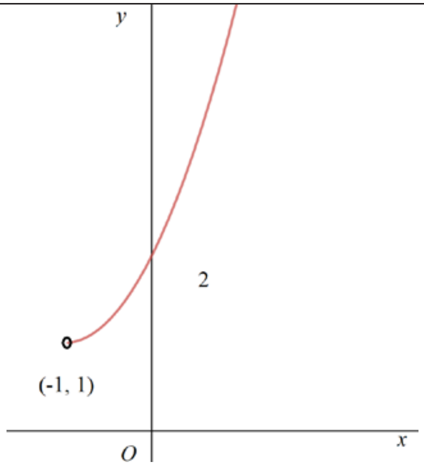
Range of values of k is $-\frac{9}{8}<k<-1$.</td></tr>
<tr><td>1(b)</td><td>The graph of $y=2\mathrm{f}(3-x)$ undergoes
1) A reflection in the y-axis followed by
2) A scaling of ½ unit parallel to the y-axis
$A\left(-3,-\frac{9}{4}\right)\xrightarrow{\text{(1) reflection in the } y\text{-axis}}A_1\left(3,-\frac{9}{4}\right)$
$\xrightarrow{\text{(2) scaling of a factor } \frac{1}{2} \text{ parallel to } y\text{-axis}}A_2\left(3,-\frac{9}{8}\right)$

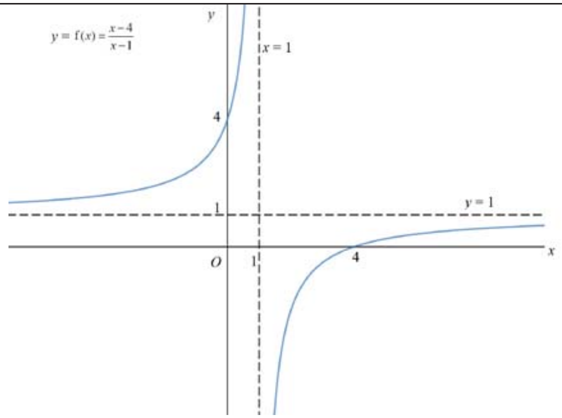
Coordinates of the minimum point on the graph of $y=\mathrm{f}(3+x)$ are $\left(3,-\frac{9}{8}\right)$.

Equation of vertical asymptote: $x=-1$
Equation of horizontal asymptote : $y=-1$</td></tr>
<tr><td>2(i)</td><td>
</td></tr>
</table>

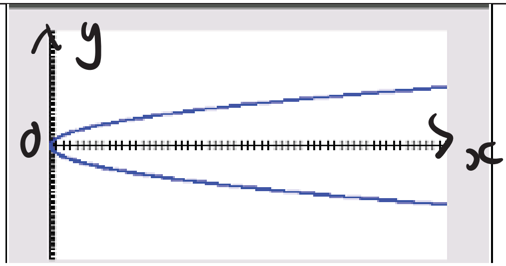
(ii)	$\int_{-3}^{-1}\left(3-4x-\lvert 2x^2+6x+4\rvert\right)\mathrm{d}x$ $=\int_{-3}^{-2}\left(3-4x-2x^2-6x-4\right)\mathrm{d}x+\int_{-2}^{-1}\left(3-4x+2x^2+6x+4\right)\mathrm{d}x$ $=\int_{-3}^{-2}\left(-2x^2-10x-1\right)\mathrm{d}x+\int_{-2}^{-1}\left(2x^2+2x+7\right)\mathrm{d}x$ $=\left[-\frac{2}{3}x^3-5x^2-x\right]_{-3}^{-2}+\left[\frac{2}{3}x^3+x^2+7x\right]_{-2}^{-1}$ $=(\frac{16}{3}-20+2)-(18-45+3)+(-\frac{2}{3}+1-7)-(-\frac{16}{3}+4-14)$ $=20.$
3(i)	$\mathrm{f}(x)=\frac{x-4}{x-1}=1-\frac{3}{x-1}$ From the graph of $y=\mathrm{f}(x)$,  Since every horizontal line $y=h, h\in\mathbb{R}, h\neq 1$ cuts the graph of f at exactly one point, f is an one-one function. Therefore f^{-1} exists.
(ii)	Let $y=1-\frac{3}{x-1}$

	$\frac{3}{x-1}=1-y$ $x-1=\frac{3}{1-y}$ $x=1+\frac{3}{1-y}$ $x=1-\frac{3}{y-1}$ Since $y=\mathrm{f}(x),\ x=\mathrm{f}^{-1}(y)$. $\therefore\ \mathrm{f}^{-1}(y)=1-\frac{3}{y-1}$ Hence $\mathrm{f}^{-1}(x)=1-\frac{3}{x-1}=\frac{x-4}{x-1},\ x\in\mathbb{R}, x\neq 1$ Since the $D_{\mathrm{f}^{-1}}=R_{\mathrm{f}}=(-\infty,1)\cup(1,\infty)=D_{\mathrm{f}}$ Since $\mathrm{f}^{-1}=\mathrm{f}$, $\mathrm{f}^{2}(x)=\mathrm{ff}^{-1}(x)=x,\ldots,\mathrm{f}^{100}(x)=x$ $\mathrm{f}^{101}(101)=\mathrm{f}\left(\mathrm{f}^{100}(101)\right)$ $=\mathrm{f}(101)$ $=\frac{101-4}{101-1}=\frac{97}{100}$
(iii)	$\mathrm{g}: x\mapsto x^2+2x+2,\ x\in\mathbb{R}, x>-1$ $\mathrm{g}(x)=(x+1)^2+1$ From the graph of g,

	 Range of g = $(1, \infty)$ Domain of f = $\mathbb{R} \setminus \{1\}$ Since $(1, \infty) \subseteq \mathbb{R} \setminus \{1\}$ i.e. Range of g $\subseteq$ Domain of f Therefore the composite function fg exists. To find range of composite function fg:

	 $(-1,\infty) \xrightarrow{g} (1,\infty) \xrightarrow{f} (-\infty,1)$ Range of fg is $(-\infty,1)$.
4	$y=\sqrt{e^{\cos x}}$ --- (1) $y^2 = e^{\cos x}$ Differentiate with respect to *x*, $2y\frac{dy}{dx} = (-\sin x)e^{\cos x}$ $y\left(2\frac{dy}{dx} + y\sin x\right) = 0$ Since $y=\sqrt{e^{\cos x}} > 0$, $2\frac{dy}{dx} + y\sin x = 0$ (Shown) --- (1) Differentiate with respect to x, $2\frac{d^2y}{dx^2} + (\sin x)\frac{dy}{dx} + y\cos x = 0$

	When $x = 0$, $y = \sqrt{e}$ from (1) $\frac{dy}{dx} = 0$ from (2) $\frac{d^2y}{dx^2} = -\frac{\sqrt{e}}{2}$ $\therefore y = \sqrt{e} - \frac{\sqrt{e}}{4}x^2 + \dots$
(ii)	$e^{\sin^2\left(\frac{x}{2}\right)} = e^{\frac{1-\cos x}{2}}$ $= \frac{e^{\frac{1}{2}}}{e^{\frac{1}{2}\cos x}}$ $\approx \frac{e^{\frac{1}{2}}}{e^{\frac{1}{2}} - \frac{e^{\frac{1}{2}}}{4}x^2}$ $= \frac{1}{1-\frac{1}{4}x^2}$ $= \left(1-\frac{1}{4}x^2\right)^{-1}$ $= 1+\frac{1}{4}x^2+\dots$
(iii)	$\int_0^{\sqrt{2}} e^{\sin^2\left(\frac{x}{2}\right)} dx \approx \int_0^{\sqrt{2}} \left(1+\frac{1}{4}x^2\right) dx$ $=1.649915$ $=1.650$

5(i)	
(ii)	$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{1}{\left(\frac{dx}{dt}\right)}=\frac{2a}{2at}=\frac{1}{t}$ Gradient of normal = -*t*. Equation of normal at a point *P* is given by $\frac{y-2ap}{x-ap^2}=-p$ $\Rightarrow y=2ap-p(x-ap^2)$
(iii)	If the normal at point *P* meets *C* again at point *Q*,

$$2aq = 2ap - p(aq^2 - ap^2)$$

$$pq^2 + 2q - (2p + p^3) = 0$$

$$q = \frac{-2 \pm \sqrt{4 + 4p(2p + p^3)}}{2p}$$

$$= \frac{-2 \pm \sqrt{4 + 8p^2 + 4p^4}}{2p}$$

$$= \frac{-2 \pm \sqrt{(2p^2 + 2)^2}}{2p}$$

$$= \frac{-2 + (2p^2 + 2)}{2p} \text{ or } \frac{-2 - (2p^2 + 2)}{2p}$$

$$= p \text{ (rejected as it is the point } P\text{)} \quad \text{or } -p - \frac{2}{p}$$

Therefore Q will meet C again with $q = -p - \dfrac{2}{p}$.

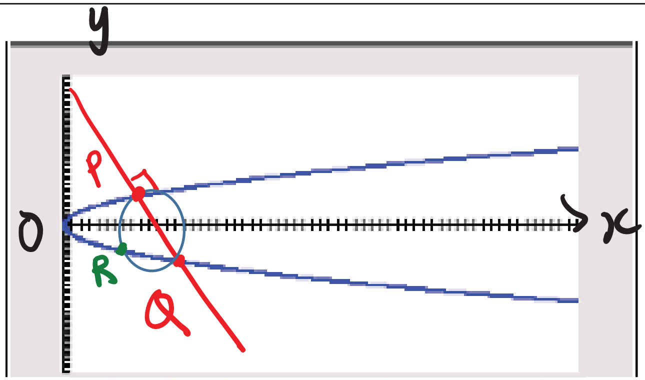
Coordinates of $P = (ap^2, 2ap)$

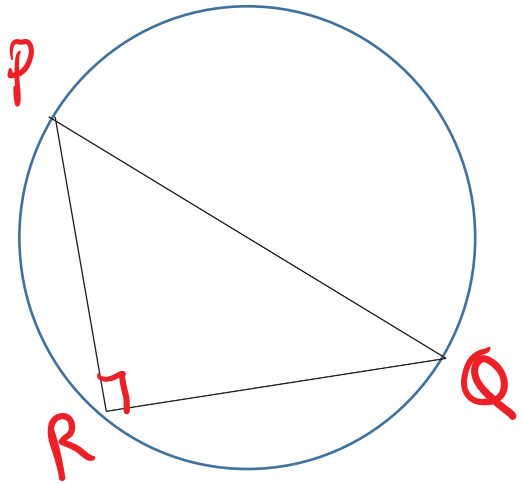
Coordinates of $Q = \left(a(p + \dfrac{2}{p})^2, -2a\left(\dfrac{p^2+2}{p}\right)\right)$

$|PQ|^2$

$$= (ap^2 - a((p + \frac{2}{p})^2))^2 + (2ap + 2a(\frac{p^2+2}{p}))^2$$

$$= (ap^2 - a(p^2 + 4 + \frac{4}{p^2}))^2 + (4ap + \frac{4a}{p})^2$$

	$=16a^2(1+\frac{1}{p^2})^2+16a^2(\frac{p^2+1}{p})^2$ $=16a^2(\frac{(p^2+1)^2}{p^4}+(\frac{p^2+1}{p})^2)$ $=\frac{16a^2}{p^4}((p^2+1)^2+p^2(p^2+1)^2)$ $=\frac{16a^2}{p^4}(p^2+1)^3$
	



Note that PR and QR are perpendicular to each other (angle PRQ is 90^0 – angle in a semi-circle).

Gradient of PR $= \dfrac{2ap-2ar}{ap^2-ar^2} = \dfrac{2}{p+r}$

Gradient of QR $= \dfrac{-2a\left(\dfrac{p^2+2}{p}\right)-2ar}{a\left(\dfrac{p^2+2}{p}\right)^2-ar^2} = -\dfrac{2}{\left(\dfrac{p^2+2}{p}\right)-r}$

$$-\frac{2}{\left(\dfrac{p^2+2}{p}\right)-r} \bullet \frac{2}{p+r} = -1$$

	$(p+r)(p+\frac{2}{p}-r)=4$ $p^2-r^2+\frac{2r}{p}=2$. (Shown).
6(a) (i)	$p=-1-\sqrt{3}\mathrm{i}$ $=2\mathrm{e}^{\mathrm{i}\left(-\frac{2\pi}{3}\right)}$
(ii)	$\frac{(p^*)^n}{\mathrm{i}p}=\frac{2^n\mathrm{e}^{\mathrm{i}\left(\frac{2n\pi}{3}\right)}}{2\mathrm{e}^{\mathrm{i}\left(-\frac{2\pi}{3}+\frac{\pi}{2}\right)}}=2^{n-1}\mathrm{e}^{\mathrm{i}\left(\frac{2n\pi}{3}+\frac{\pi}{6}\right)}$ For $\frac{(p^*)^n}{ip}$ to be purely imaginary, $\frac{2n\pi}{3}+\frac{\pi}{6}=(2k+1)\frac{\pi}{2}$ where $k\in\mathbb{Z}$ $\frac{2n\pi}{3}+\frac{\pi}{6}=k\pi+\frac{\pi}{2}$ $\frac{2n\pi}{3}=k\pi+\frac{\pi}{3}$ $2n=3k+1$ $n=\frac{3k+1}{2}$ For the two smallest $n\in\mathbb{Z}^+$, $k=1$ and $k=3$. $n=2$ and $n=5$

6 (b)	$z = w - 6 - 7\mathrm{i}$ --- (1) $2w - \mathrm{i}z^* - 19 - 3\mathrm{i} = 0$--- (2) Substitute (1) into (2): $2w - \mathrm{i}(w - 6 - 7\mathrm{i})^* - 19 - 3\mathrm{i} = 0$ $2w - \mathrm{i}(w^* - 6 + 7\mathrm{i}) - 19 - 3\mathrm{i} = 0$ Let $w = x + \mathrm{i}y$, $2(x + \mathrm{i}y) - \mathrm{i}((x - \mathrm{i}y) - 6 + 7\mathrm{i}) - 19 - 3\mathrm{i} = 0$ $2(x + \mathrm{i}y) - \mathrm{i}((x - \mathrm{i}y) - 6 + 7\mathrm{i}) - 19 - 3\mathrm{i} = 0$ $2x + 2y\mathrm{i} - \mathrm{i}x - y + 6\mathrm{i} + 7 - 19 - 3\mathrm{i} = 0$ $2x - y - 12 + \mathrm{i}(2y - x + 3) = 0$ Compare real and imaginary components, $2x - y = 12 - - - - - - - - - - (1)$ $2y - x = -3 - - - - - - - - - - (2)$ From (2), $x = 2y + 3 - - - - - - - - (3)$ Substituting (3) into (1) $2(2y + 3) - y = 12$ $3y = 6$ $y = 2$ When $y = 2$, $x = 2(2) + 3 = 7$ $w = 7 + 2\mathrm{i}$ and $z = 7 + 2\mathrm{i} - 6 - 7\mathrm{i} = 1 - 5\mathrm{i}$
7(i)	$\overrightarrow{OP} = (\cos t)\,\mathbf{i} + (\sin t)\,\mathbf{j}$ $\lvert\overrightarrow{OP}\rvert = \sqrt{\cos^2 t + \sin^2 t} = 1$

	$\lvert\overrightarrow{OQ}\rvert^2=\frac{9}{4}\cos^2(t+\frac{\pi}{4})+9\sin^2(t+\frac{\pi}{4})+\frac{27}{4}\cos^2(t+\frac{\pi}{4})$ $=9\cos^2(t+\frac{\pi}{4})+9\sin^2(t+\frac{\pi}{4})$ $=9$ $\Rightarrow\lvert\overrightarrow{OQ}\rvert=3.$
(ii)	$x=\cos t \quad y=\sin t \quad 0\le t\le 2\pi$ The cartesian equation is given by $x^2+y^2=1$. Since $z=0$, P lies on a circle centre at O and radius 1 unit in the x-y plane.
(iii)	Using scalar product, $\overrightarrow{OP}\bullet\overrightarrow{OQ}=\lvert\overrightarrow{OP}\rvert\lvert\overrightarrow{OQ}\rvert\cos\theta$ $\Rightarrow 3\cos\theta=\begin{pmatrix}\cos t\\ \sin t\\ 0\end{pmatrix}\bullet\begin{pmatrix}\frac{3}{2}\cos\left(t+\frac{\pi}{4}\right)\\ 3\sin\left(t+\frac{\pi}{4}\right)\\ \frac{3\sqrt{3}}{2}\cos\left(t+\frac{\pi}{4}\right)\end{pmatrix}$ $=\frac{3}{2}\cos t\cos\left(t+\frac{\pi}{4}\right)+3\sin t\sin\left(t+\frac{\pi}{4}\right)$

	$\Rightarrow 3\cos\theta = \frac{3}{4}\left[\cos\left(2t+\frac{\pi}{4}\right)+\cos\frac{\pi}{4}\right]-\frac{3}{2}\left[\cos\left(2t+\frac{\pi}{4}\right)-\cos\frac{\pi}{4}\right]$ $\Rightarrow 3\cos\theta = -\frac{3}{4}\cos\left(2t+\frac{\pi}{4}\right)+\frac{3\sqrt{2}}{8}+\frac{3\sqrt{2}}{4}$ $\Rightarrow \cos\theta = \frac{3\sqrt{2}}{8}-\frac{1}{4}\cos\left(2t+\frac{\pi}{4}\right)$ (Shown).
(iv)	The length of projection of $\overrightarrow{OQ}$ onto $\overrightarrow{OP}$ is $\lvert\overrightarrow{OQ}\bullet\overrightarrow{OP}\rvert$ since $\overrightarrow{OP}$ is a unit vector. Given that the length of projection of $\overrightarrow{OQ}$ onto $\overrightarrow{OP}$ is $\sqrt{5}$ units, $\lvert\overrightarrow{OQ}\bullet\overrightarrow{OP}\rvert=\sqrt{5}$ $\Rightarrow\lvert\overrightarrow{OQ}\rvert\cos\theta=\sqrt{5}$ $\Rightarrow \cos\theta=\frac{\sqrt{5}}{3}$ $\Rightarrow \theta=0.730$ (since θ is acute). Solving $\frac{3\sqrt{2}}{8}-\frac{1}{4}\cos\left(2t+\frac{\pi}{4}\right)=\frac{\sqrt{5}}{3}$ $\cos(2t+\frac{\pi}{4})=-0.86010$ $2t+\frac{\pi}{4}=0.53532$ (basic angle) $2t+\frac{\pi}{4}=\pi-0.53532, \pi+0.53532, \pi-0.53532+2\pi, \pi+0.53532+2\pi$ $t=0.910, 1.45, 4.05, 4.59$

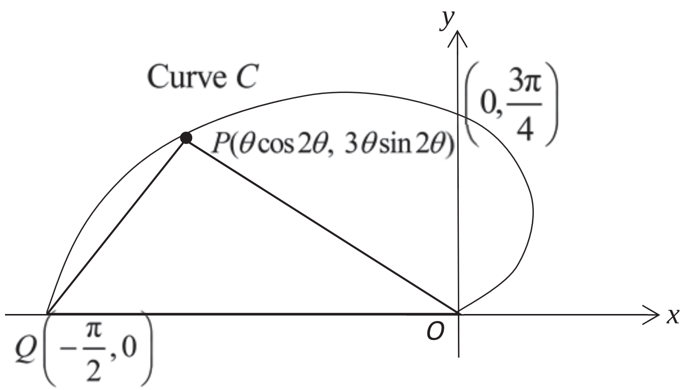
8(a)	Let the length of the first screwdriver be b, and the common ratio be r. $b+br+br^2=3\left(br^{12}+br^{13}+br^{14}+br^{15}+br^{16}\right)$ $1+r+r^2-3r^{12}-3r^{13}-3r^{14}-3r^{15}-3r^{16}=0$ Using the GC, r = 0.882854 (6 s.f.) since $0<r<1$. There are 9 odd numbered screwdrivers, with common ratio of lengths being r^2. Total length = $\dfrac{b\left(1-\left(r^2\right)^9\right)}{1-r^2}=120$ $\dfrac{b\left(1-r^{18}\right)}{1-r^2}=120$ Using the GC, b = 29.6122 (6 s.f.) The total length is $\dfrac{b\left(1-r^{17}\right)}{1-r}=222.38$ cm $\quad$(to 2 d.p.)
8 (b)(i)	$\cos\left(u_{n+1}-u_n\right)=\cos u_{n+1}\cos u_n+\sin u_{n+1}\sin u_n$ $=\dfrac{1}{2}\cos^2 u_n-\dfrac{\sqrt{3}}{2}\sin u_n\cos u_n$ $+\dfrac{1}{2}\sin^2 u_n+\dfrac{\sqrt{3}}{2}\sin u_n\cos u_n$ $=\dfrac{1}{2}$ $\therefore u_{n+1}-u_n=\cos^{-1}\dfrac{1}{2}=\dfrac{\pi}{3}$ (since the difference is less than π) is a constant independent of n. Hence $\{u_n\}$ is an arithmetic progression.

	The common difference is $\frac{\pi}{3}$. **<u>Alternatively</u>** $\cos u_{n+1} = \frac{1}{2}\cos u_n - \frac{\sqrt{3}}{2}\sin u_n$ $= \cos u_n \cos\frac{\pi}{3} - \sin u_n \sin\frac{\pi}{3}$ $= \cos\left(u_n + \frac{\pi}{3}\right)$ $\therefore u_{n+1} = u_n + \frac{\pi}{3}$ since the difference is less than π. $u_{n+1} - u_n$ is a constant, therefore $\{u_n\}$ is an arithmetic progression. The common difference is $\frac{\pi}{3}$.
(ii)	The total angle rotated over n twists is $\frac{n}{2}\left(2\left(\frac{2\pi}{3}\right)+(n-1)\frac{\pi}{3}\right)$. $\frac{n}{2}\left(\frac{4\pi}{3}+(n-1)\frac{\pi}{3}\right) \geq 25\times(2\pi)$ $\frac{2n\pi}{3} + \frac{\pi}{6}n(n-1) - 50\pi \geq 0$ $4n + n(n-1) - 300 \geq 0$ $n^2 + 3n - 300 \geq 0$ Using the GC, <table><tr><td>n</td><td>$n^2+3n-300$</td></tr><tr><td>15</td><td>-30</td></tr></table>

	<table><tr><td>16</td><td>4</td></tr><tr><td>17</td><td>40</td></tr></table> The minimum number of buttons presses required is 16.
(iii)	Total angle rotated after 21 button presses is $\frac{21}{2}\left(2\left(\frac{2\pi}{3}\right)+(20)\frac{\pi}{3}\right)$ $=14\pi+70\pi$ $=84\pi$ $84\pi k=144,$ where k is a positive real constant. The first button press rotates the screw by $\frac{2\pi}{3}$. $d_1=ku_1=\frac{144}{84\pi}\left(\frac{2\pi}{3}\right)=\frac{8}{7}$ The first button press drills the screw in by $\frac{8}{7}$ mm (1.14mm).
9(i)	$x=15\sin\theta+15$ $\frac{dx}{d\theta}=15\cos\theta$ $\int_0^{15}\sqrt{15^2-(x-15)^2}\,dx$

	$\begin{aligned} &= \int_{-\frac{\pi}{2}}^{0}\left(\sqrt{15^2-(15\sin\theta)^2}\right)(15\cos\theta)\,\mathrm{d}\theta \\ &= 225\left[\int_{-\frac{\pi}{2}}^{0}(\cos\theta)(\cos\theta)\,\mathrm{d}\theta\right] \\ &= \frac{225}{2}\left[\int_{-\frac{\pi}{2}}^{0}\cos^2\theta\ \mathrm{d}\theta\right] \\ &= \frac{225}{2}\left[\int_{-\frac{\pi}{2}}^{0}\left(\cos 2\theta+1\right)\ \mathrm{d}\theta\right] \\ &= \frac{225}{2}\left[\frac{\sin 2\theta}{2}+\theta\right]_{-\frac{\pi}{2}}^{0} \\ &= \frac{225}{2}\left(\frac{\pi}{2}\right) \\ &= \frac{225}{4}\pi \end{aligned}$

9(ii) (a)	Required area $=\int_0^{15} \mathrm{g}(x)\mathrm{d}x$ $=\int_0^{15}\left(30-\frac{2}{3}\sqrt{15^2-(x-15)^2}\right)\mathrm{d}x$ $=[30x]_0^{15}-\frac{2}{3}\int_0^{15}\sqrt{15^2-(x-15)^2}\mathrm{d}x$ $=450-\frac{2}{3}\left(\frac{225}{4}\pi\right)$ from (i) $=\left(450-\frac{75}{2}\pi\right)$ cm^2
(b)	Required Volume $=\pi(30)^2(20)+\pi\int_0^{15} y^2\mathrm{d}x$ $=\pi(30)^2(20)+\pi\int_0^{15}\left(30-\frac{2}{3}\sqrt{15^2-(x-15)^2}\right)^2 \mathrm{d}x$ $=\pi(30)^2(20)+$ $\pi\int_0^{15}\left(900-30(2)\left(\frac{2}{3}\right)\sqrt{15^2-(x-15)^2}+\left(\frac{4}{9}\right)\left(225-(x-15)^2\right)\right)\mathrm{d}x$ $=18000\pi+\pi\int_0^{15}\left(1000-40\sqrt{15^2-(x-15)^2}-\left(\frac{4}{9}\right)(x-15)^2\right)\mathrm{d}x$ $=18000\pi+\pi\left[1000x-\left(\frac{4}{9}\right)\frac{(x-15)^3}{3}\right]_0^{15}-40\pi\left(\frac{225}{4}\pi\right)$ from (i) $=\left(32500\pi-2250\pi^2\right)$ cm^3.

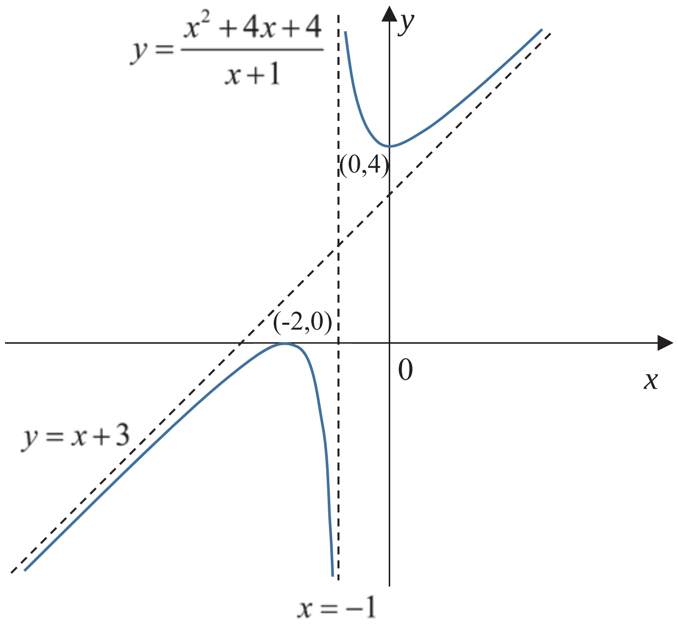
10(i)	 Point P has coordinates $(\theta\cos 2\theta,\ 3\theta\sin 2\theta)$. Let area of triangle OPQ be A. $A = \frac{1}{2}\left(\frac{\pi}{2}\right)(3\theta\sin 2\theta) = \frac{3\pi}{4}(\theta\sin 2\theta)$ $\frac{dA}{d\theta} = \frac{3\pi}{4}(2\theta\cos 2\theta + \sin 2\theta)$ $\frac{dA}{dt} = \frac{dA}{d\theta} \times \frac{d\theta}{dt}$ $= \frac{3\pi}{4}(2\theta\cos 2\theta + \sin 2\theta)(0.01)$ When $\theta = \frac{\pi}{6}$, $\frac{dA}{dt}$ = 0.0327 units^2/s (3 s.f.)

(ii)	When $\frac{\mathrm{d}A}{\mathrm{d}\theta}=0$, $\frac{3\pi}{4}\left(2\theta\cos 2\theta+\sin 2\theta\right)=0$ Since $\frac{3\pi}{4}\neq 0$, $2\theta\cos 2\theta+\sin 2\theta=0$ Using GC, $\theta=1.0144$ (5 s.f.) $=1.01$ (3 s.f.) $\frac{\mathrm{d}^2A}{\mathrm{d}\theta^2}=\frac{3\pi}{4}\left(-4\theta\sin 2\theta+2\cos 2\theta+2\cos 2\theta\right)$ When $\theta=1.0144$, $\frac{\mathrm{d}^2A}{\mathrm{d}\theta^2}=-12.7<0$ $\therefore\ \theta=1.0144$ will result in maximum A. When $\theta=1.0144$, $A=\frac{3\pi}{4}(1.0144)\sin(2\times 1.0144)$ $=2.14$ units^2 (3 s.f.)

	When $\theta = 1.0144$, $x = 1.0144\cos\left[2(1.0144)\right] = -0.449$ $y = 3(1.0144)\sin\left[2(1.0144)\right] = 2.73$ $\therefore$ Location of the camera is at a point with coordinates $(-0.449, 2.73)$
(iii)	For triangle OPQ to be an isosceles triangle, $x = -\frac{\pi}{2} \div 2 = -\frac{\pi}{4}$ $-\frac{\pi}{4} = \theta\cos 2\theta$ Using GC, $\theta = 1.1581$ $y = 3(1.1581)\sin(2\times 1.1581) = 2.55$ $\therefore$ coordinates of $P(-0.785, 2.55)$

2019 SAJC H2 Math Paper 2 Solutions

Qn	Solution
1(i)	Since $x=-1$ is a vertical asymptote, $c=1$ $y=\dfrac{x^2+ax+b}{x+1}$ $y(x+1)=x^2+ax+b$ $x^2+(a-y)x+(b-y)=0$ The values that y can take satisfy the inequality: $(a-y)^2-4(b-y)\geq 0$ $y^2+(4-2a)y+(a^2-4b)\geq 0$ Since $y\leq 0$ or $y\geq 4$: $y=0$ and $y=4$ are roots to the equation $y^2+(4-2a)y+(a^2-4b)=0$ ....(1) Substituting $y=0$ and $y=4$ into (1) and solving: $a^2-4b=0$ $16+(4-2a)(4)=0$ $a=4,\quad b=4$
(ii)	$y=\dfrac{x^2+4x+4}{x+1}=x+3+\dfrac{1}{x+1}$

Qn	Solution
	
(iii)	Point of intersection: $(-1,2)$ For all $k\in\mathbb{R}$, the line $y=k(x+1)+2$ passes through the point (-1,2). Hence the line will cut C for $k>1$.
2(i)	Let the volume of water in the tank be V cubic metres at t seconds. $\dfrac{dV}{dt}=\dfrac{dV_{IN}}{dt}-\dfrac{dV_{OUT}}{dt}$ $=k-aV$ $=k\left(1-\dfrac{a}{k}V\right)$. (Shown) where $k>0$, and $a>0$.

Qn	Solution

$$\int \frac{1}{1-\frac{a}{k}V}\,\mathrm{d}V = \int k\,\mathrm{d}t$$

$$-\frac{k}{a}\ln\left|1-\frac{a}{k}V\right| = kt + C$$

$$\ln\left|1-\frac{a}{k}V\right| = -at - \frac{aC}{k}$$

$$1-\frac{a}{k}V = \pm e^{-\frac{ac}{k}}e^{-at} = Ae^{-at}$$

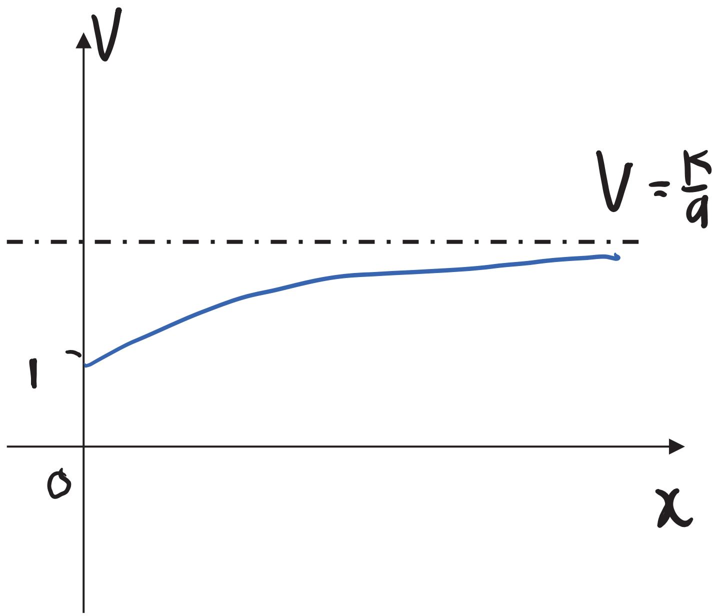
$$V = \frac{k}{a}\left(1 - Ae^{-at}\right)$$

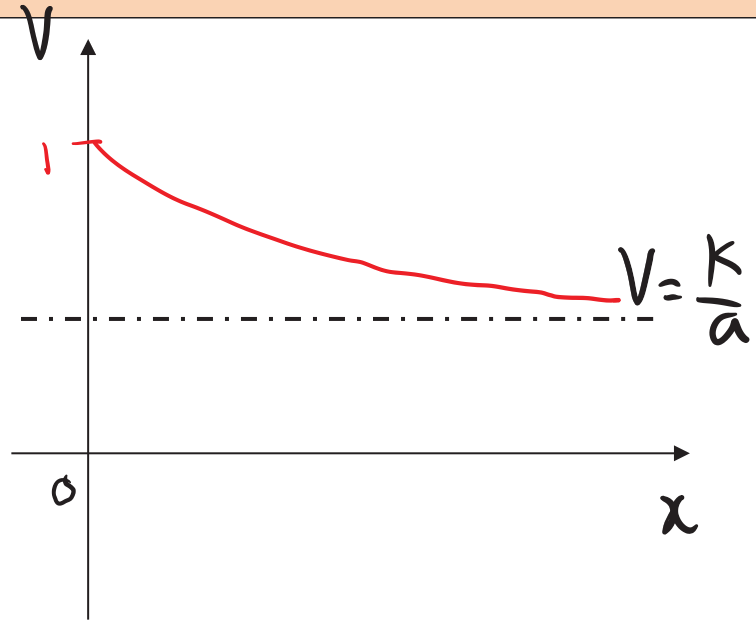
C is an arbitrary constant and $A = \pm e^{-\frac{aC}{k}}$

At $t = 0$, $V = 1$, $A = 1 - \frac{a}{k}$

$$V = \frac{k}{a}\left(1-(1-\frac{a}{k})e^{-at}\right) = \frac{1}{a}(k-(k-a)e^{-at})$$

Case (i) if $a < k$

Qn	Solution
	 The volume of water in the water tank, V, increases from one cubic meter and approach $\frac{k}{a}$ cubic meters eventually. **Case (ii) if $a > k$**

Qn	Solution
	 The volume of water in the water tank, V, decreases from one cubic meter and approach $\frac{k}{a}$ cubic meters eventually.

3(i)	$\overrightarrow{OB}\cdot\begin{pmatrix}1\\-1\\3\end{pmatrix}=1+0+9=10$ B lies on π_1.
(ii)	$\begin{pmatrix}2\\a\\2\end{pmatrix}\cdot\begin{pmatrix}1\\-1\\3\end{pmatrix}\neq 10$ $2-a+6\neq 10$ $a\neq -2$ The range of values is $a\in\mathbb{R}, a\neq -2$.
(iii)	Let the foot of perpendicular be F. The line through AF has vector equation $l_{AF}:\mathbf{r}=\begin{pmatrix}2\\9\\2\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\3\end{pmatrix},\ \lambda\in\mathbb{R}.$ Since F lies on l_{AF}, $\overrightarrow{OF}=\begin{pmatrix}2\\9\\2\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\3\end{pmatrix}$ for some fixed $\lambda\in\mathbb{R}$ Since F lies on π_1, $\overrightarrow{OF}\cdot\begin{pmatrix}1\\-1\\3\end{pmatrix}=10$ $\therefore \left[\begin{pmatrix}2\\9\\2\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\3\end{pmatrix}\right]\cdot\begin{pmatrix}1\\-1\\3\end{pmatrix}=10$ $2-9+6+11\lambda=10$ $\lambda=1$

$$\overrightarrow{OF}=\begin{pmatrix}2\\9\\2\end{pmatrix}+1\begin{pmatrix}1\\-1\\3\end{pmatrix}=\begin{pmatrix}3\\8\\5\end{pmatrix}$$

The coordinates of F are $(3,8,5)$.

Let the point of reflection of A about π_1 be A'.

$$\frac{\overrightarrow{OA}+\overrightarrow{OA'}}{2}=\overrightarrow{OF}$$

$$\overrightarrow{OA'}=2\overrightarrow{OF}-\overrightarrow{OA}$$

$$=\begin{pmatrix}6\\16\\10\end{pmatrix}-\begin{pmatrix}2\\9\\2\end{pmatrix}$$

$$=\begin{pmatrix}4\\7\\8\end{pmatrix}$$

$$\overrightarrow{BA'}=\overrightarrow{OA'}-\overrightarrow{OB}$$

$$=\begin{pmatrix}4\\7\\8\end{pmatrix}-\begin{pmatrix}1\\0\\3\end{pmatrix}=\begin{pmatrix}3\\7\\5\end{pmatrix}$$

Line of reflection, $l_{BA'}$, has vector equation

$$l_{BA'}:\mathbf{r}=\begin{pmatrix}1\\0\\3\end{pmatrix}+\mu\begin{pmatrix}3\\7\\5\end{pmatrix},\ \mu\in\mathbb{R}.$$

(iv)	Acute angle between π_1 and π_2 is $\cos^{-1}\left\|\dfrac{\begin{pmatrix}1\\-1\\3\end{pmatrix}\cdot\begin{pmatrix}1\\0\\1\end{pmatrix}}{\left\|\begin{pmatrix}1\\-1\\3\end{pmatrix}\right\|\left\|\begin{pmatrix}1\\0\\1\end{pmatrix}\right\|}\right\| = \cos^{-1}\left\|\dfrac{4}{\sqrt{11}\sqrt{2}}\right\| = 31.5° \ (1 \text{ d.p.})$
(v)	The desired planes have equation $\mathbf{r}\cdot\begin{pmatrix}1\\0\\1\end{pmatrix} = D$, where D is the constant to be determined. Distance between the planes is given by $\left\|\dfrac{D}{\sqrt{2}} - \dfrac{4}{\sqrt{2}}\right\| = \dfrac{5}{\sqrt{2}}$ $\dfrac{D}{\sqrt{2}} - \dfrac{4}{\sqrt{2}} = \pm\dfrac{5}{\sqrt{2}}$ $D = 4 \pm 5$ $D = 9$ or $D = -1$ The possible equations are $x + z = 9$ or $x + z = -1$. **<u>Alternative Solution</u>** Let a point D on the desired plane have coordinates (x, y, z). Then

	$\dfrac{\left\lvert \overrightarrow{AD}\cdot\begin{pmatrix}1\\0\\1\end{pmatrix}\right\rvert}{\sqrt{2}}=\dfrac{5}{\sqrt{2}}$ $\left\lvert\begin{pmatrix}x-1\\y\\z-3\end{pmatrix}\cdot\begin{pmatrix}1\\0\\1\end{pmatrix}\right\rvert=5$ $\lvert x-1+z-3\rvert=5$ $x+z-4=\pm5$ $x+z=4\pm5$ The possible equations are $x+z=9$ or $x+z=-1$.
4(a)	$\mathrm{f}(r+1)-\mathrm{f}(r)=\dfrac{r+1}{2^{r+1}}-\dfrac{r}{2^{r}}$ $=\dfrac{r+1-2r}{2^{r+1}}$ $=\dfrac{1-r}{2^{r+1}}$

	$\displaystyle\sum_{r=1}^{n}\frac{1-r}{2^{r+1}}=\sum_{r=1}^{n}\left[\mathrm{f}(r+1)-\mathrm{f}(r)\right]$ $=\left[\begin{array}{l} \mathrm{f}(\cancel{2})-\mathrm{f}(1) \\ +\cancel{\mathrm{f}(3)}-\cancel{\mathrm{f}(2)} \\ +\cancel{\mathrm{f}(4)}-\cancel{\mathrm{f}(3)} \\ \quad\vdots \\ +\cancel{\mathrm{f}(n-1)}-\cancel{\mathrm{f}(n-2)} \\ +\cancel{\mathrm{f}(n)}-\cancel{\mathrm{f}(n-1)} \\ +\mathrm{f}(n+1)-\cancel{\mathrm{f}(n)} \end{array}\right]$ $=\mathrm{f}(n+1)-\mathrm{f}(1)$ $=\dfrac{n+1}{2^{n+1}}-\dfrac{1}{2}$
(b)(i)	$a_n=\dfrac{2^n n^x}{3^n}$ $\displaystyle\lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\frac{2^n n^x}{3^n}}$ $\displaystyle=\lim_{n\to\infty}\left(\frac{2\sqrt[n]{n^x}}{3}\right)$ $\displaystyle=\frac{2}{3}\lim_{n\to\infty}\left(\sqrt[n]{n^x}\right)$ $\displaystyle=\frac{2}{3}(1)\quad\text{since }\lim_{n\to\infty}\left(\sqrt[n]{n^x}\right)=1$ $=\dfrac{2}{3}<1$

	Since $\lim\limits_{n\to\infty}\sqrt[n]{a_n}=\frac{2}{3}<1$, by the Cauchy Test, the series converges for all real values of x
(b)(ii)	$\sum\limits_{r=0}^{\infty}\frac{2^r r}{3^r}=0+1\left(\frac{2}{3}\right)+2\left(\frac{2}{3}\right)^2+3\left(\frac{2}{3}\right)^3+4\left(\frac{2}{3}\right)^4+\ldots$ $=\frac{2}{3}\left[1+2\left(\frac{2}{3}\right)+3\left(\frac{2}{3}\right)^2+4\left(\frac{2}{3}\right)^3+\ldots\right]$ $=\frac{2}{3}\left(1-\frac{2}{3}\right)^{-2}$ since $1+2\left(\frac{2}{3}\right)+3\left(\frac{2}{3}\right)^2+4\left(\frac{2}{3}\right)^3+\ldots=\left(1-\frac{2}{3}\right)^{-2}$ with $y=\frac{2}{3}$ $=6$
5	Let X be the mass of a standard packet of sugar in grams. Let Y be the mass of a large packet of sugar in grams. $X\sim \mathrm{N}\left(520,8^2\right)$ $Y\sim \mathrm{N}\left(1030,11^2\right)$ $X_1+X_2-Y\sim \mathrm{N}\left(10,249\right)$ $\mathrm{P}\left(X_1+X_2>Y\right)=\mathrm{P}\left(X_1+X_2-Y>0\right)$ $=0.73687$ $=0.737$ (3 s.f.)
	$\frac{X_1+X_2+Y}{3}\sim \mathrm{N}\left(690,\frac{83}{3}\right)$ $\mathrm{P}\left(680<\frac{X_1+X_2+Y}{3}<700\right)=0.94272$ $=0.943$ (3 s.f.)
6(i)	Total number of ways to select 5 members $={}^{13}C_5$ Number of ways to select 5 members with 3 Biology students $={}^{10}C_2$ Number of ways to select 5 members with at most 2 Biology students $={}^{13}C_5-{}^{10}C_2=1242$

(ii)	Let the number of Biology, History, and Literature students be B, H, L respectively. $\mathrm{P}(\mathrm{H}>\mathrm{L} \mid \mathrm{B}\le 2)=\dfrac{\mathrm{P}((\mathrm{H}>\mathrm{L})\cap(\mathrm{B}\le 2))}{\mathrm{P}(\mathrm{B}\le 2)}$ $=\dfrac{\mathrm{n}((\mathrm{H}>\mathrm{L})\cap(\mathrm{H}+\mathrm{L}\ge 3))}{\mathrm{n}(\mathrm{B}\le 2)}$ Number of ways to select cast members with H > L when there are 3 humanities students = ${}^3C_2\times\left({}^4C_2\times{}^6C_1+{}^4C_3\times{}^6C_0\right)=120$ Number of ways to select cast members with H > L when there are 4 humanities students = ${}^3C_1\times\left({}^4C_3\times{}^6C_1+{}^4C_4\times{}^6C_0\right)=75$ Number of ways to select cast members with H > L when there are 5 humanities students = ${}^3C_0\times\left({}^4C_3\times{}^6C_2+{}^4C_4\times{}^6C_1\right)=66$ Required Probability $=\dfrac{\mathrm{n}((\mathrm{H}>\mathrm{L})\cap(\mathrm{H}+\mathrm{L}\ge 3))}{\mathrm{n}(\mathrm{B}\le 2)}$ $=\dfrac{120+75+66}{1242}$ $=\dfrac{261}{1242}=0.210\ \ (3\text{ s.f.})$
7(i)	$\mathrm{P}(X=1)=\mathrm{P}(X=2)=....=\mathrm{P}(X=n)=\dfrac{1}{n}$ $\mathrm{E}(X)=\sum_{x=1}^{n}x\mathrm{P}(X=x)=\sum_{x=1}^{n}x\cdot\left(\dfrac{1}{n}\right)$ $=\dfrac{1}{n}\left(\dfrac{n(n+1)}{2}\right)$ $=\dfrac{n+1}{2}$ $\mathrm{Var}(X)=\mathrm{E}(X^2)-(\mathrm{E}(X))^2$

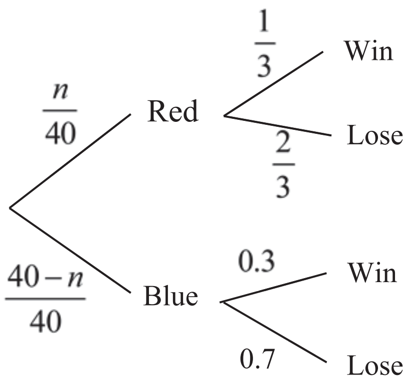
	$\mathrm{E}(X^2)=\sum_{x=1}^{n} x^2\cdot\left(\frac{1}{n}\right)$ $=\frac{1}{n}\left(\frac{n}{6}(n+1)(2n+1)\right)$ $=\frac{(n+1)(2n+1)}{6}$ $\mathrm{Var}(X)=\mathrm{E}(X^2)-\left(\mathrm{E}(X)\right)^2$ $=\frac{(n+1)(2n+1)}{6}-\left(\frac{n+1}{2}\right)^2$ $=\frac{(n+1)}{12}\left(2(2n+1)-3(n+1)\right)$ $=\frac{(n+1)(n-1)}{12}$ $=\frac{n^2-1}{12}$
(ii)	$\mathrm{E}(X)=\frac{7}{2}$ and $\mathrm{Var}(X)=\frac{6^2-1}{12}=\frac{35}{12}$ $\mathrm{P}\left(\lvert X-\mu\rvert>\sigma\right)=\mathrm{P}\left(\left\lvert X-\frac{7}{2}\right\rvert>\sqrt{\frac{35}{12}}\right)$ $=\mathrm{P}\left(X>\frac{7}{2}+\sqrt{\frac{35}{12}}\right)+\mathrm{P}\left(X<\frac{7}{2}-\sqrt{\frac{35}{12}}\right)$ $=\mathrm{P}(X>5.21)+\mathrm{P}(X<1.79)$ $=\mathrm{P}(X=6)+\mathrm{P}(X=1)$ $=\frac{2}{6}=\frac{1}{3}$

Let S be the random variable "no. of observations, out of 20, such that the total score is an outlier".

$S \sim \mathrm{B}(20, \frac{1}{3})$

$\mathrm{P}(S \geq 8) = 1 - \mathrm{P}(S \leq 7) = 0.339$

8(i)



P(a player wins the game)

$= \frac{15}{40} \times \frac{1}{3} + \frac{25}{40} \times \frac{3}{10}$

$= \frac{5}{16}$

P(exactly 2 of 3 players win)

$= {}^{3}C_{2} \left(\frac{5}{16}\right)^{2} \left(\frac{11}{16}\right)$

$= \frac{825}{4096}$

Alternatively,

	Let X be the random variable "the number of people who wins the game out of 3" $X \sim \mathrm{B}(3, \frac{5}{16})$ $\mathrm{P}(X = 2) = 0.201$ (to 3 s.f.)
(ii)	P(a player wins the game) $= \frac{n}{40} \times \frac{1}{3} + \frac{40-n}{40} \times \frac{3}{10}$ $= \frac{10n}{1200} + \frac{360-9n}{1200}$ $= \frac{360+n}{1200}$ $\mathrm{f}(n)$ = P(player draws blue | player wins) $= \frac{\mathrm{P}(\text{player draws blue and wins})}{\mathrm{P}(\text{player wins})}$ $= \frac{\frac{40-n}{40} \times \frac{3}{10}}{\frac{360+n}{1200}}$

$$= \frac{120-3n}{400} \times \frac{1200}{360+n}$$

$$= \frac{3(120-3n)}{360+n}$$

$$= \frac{360-9n}{360+n}$$

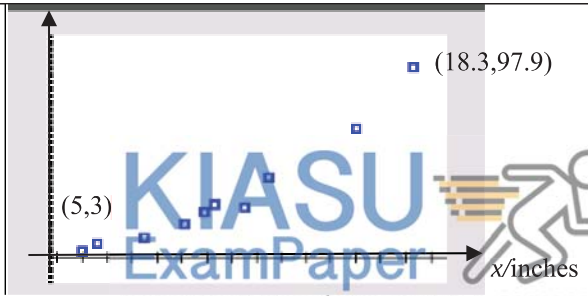
$$= \frac{-9(360+n)+3600}{360+n}$$

$$= -9 + \frac{3600}{360+n}$$

As n increases, $\frac{3600}{360+n}$ decreases, hence f(n) decreases.
Hence f is decreasing for all n, $0 \leq n \leq 40$.

This means that as the number of red counters increase, the probability that a winning player drew a blue counter decreases.

9(i)



(ii)

(a) product moment correlation coefficient, $r = 0.96346$

(b) product moment correlation coefficient, $r = 0.98710$

(iii)	The second model $\ln y = c + d\ln x$ is the better model because its product moment correlation coefficient is closer to one as compared to the product moment correlation coefficient of the first model. From the scatter plot, it can be seen that the data seems to indicate a non-linear (curvilinear) relationship between y and x. Hence the model $y = a + bx$ is not appropriate.
(iv)	We have to use the regression line $\ln y$ against $\ln x$. From GC, the equation is $\ln y = -2.5866 + 2.4665\ln x$ When $x = 20$, $\ln y = -2.5866 + 2.4665\ln 20$ $y = e^{4.8025} = 121.82 = 121$ $x = 20$ is outside the data range and hence the relationship $\ln y = c + d\ln x$ may not hold. Hence the estimate may not be reliable.
10(i)	Assumptions 1. Every eraser is equally likely to be blue. 2. The colour of a randomly selected eraser is independent of the colour of other erasers.
(ii)	Let Y be the number of blue erasers, out of 36. $Y \sim B(36, 0.20)$ $P(Y \le 6) = 0.40069 \approx 0.401$
(iii)	Let W be the number of boxes that contain at most six blue erasers, out of 200. $W \sim B(200, 0.40069)$ $P(W \ge 40\% \text{ of } 200) = P(W \ge 80) = 1 - P(W \le 79)$ $= 0.53477 \approx 0.535$
(iv)	Let T denote the number of cartons where each carton contains at least 40% of the boxes that contains at most six blue erasers per box. $T \sim B(150, 0.53477)$ $E(T) = 150 \times 0.53477 = 80.216$ $\text{Var}(T) = 150 \times 0.53477 \times (1 - 0.53477) = 37.319$

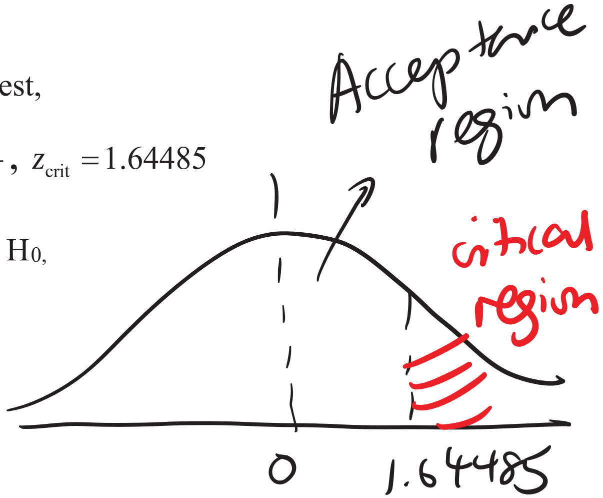
	Since n is large $(n=30)$, by the Central Limit Theorem, $\overline{T}=\dfrac{T_1+T_2+...+T_{30}}{30}\sim N(80.216,\dfrac{37.319}{30})$ approximately. $\mathrm{P}(\overline{T}<80)=0.423219\approx 0.423$ (3 sig. fig.)
(v)	Let R be the number of blue erasers, out of 36. $R\sim B(36, p)$ $\mathrm{P}(R=1)=\binom{36}{1}p^1(1-p)^{35}=36p(1-p)^{35}$
(vi)	$\mathrm{P}(R=2)=\binom{36}{2}p^2(1-p)^{34}=630p^2(1-p)^{34}$ $\mathrm{P}(R=1)=2\mathrm{P}(R=2)$ $36p(1-p)^{35}=2\times 630p^2(1-p)^{34}$ $36(1-p)=1260p$ $1-p=35p$ $1=36p$ $p=\dfrac{1}{36}$

11(i)	Let T be the random variable " time taken in seconds for a computer to boot up", with population mean μ. Unbiased estimate of the population mean, $\bar{t} = \frac{802.5}{25} = 32.1$ Unbiased estimate of the population variance, $s^2 = \frac{1}{24}\left[26360.25 - \frac{802.5^2}{25}\right] = 25$
(ii)	A statistic is said to be an unbiased estimate of a given parameter when the mean of the sampling distribution of the statistic can be shown to be equal to the parameter being estimated. For example, $\mathrm{E}(\bar{X}) = \mu$.
(iii)	Test H_0: $\mu = 30$ against $H_1 : \mu > 30$ at the 5% level of significance. Under H_0, $\bar{T} \sim N(30, \frac{25}{25})$. Using GC, $\bar{t} = 32.1$ gives rise to $z_{calc} = 2.1$ and p-value = 0.0179 Since p-value = 0.0179 ≤ 0.05, we reject H_0 and conclude that there is sufficient evidence at the 5% significance level that the specification is not being met (<u>or</u> the computer requires more than 30 seconds to boot up). "5% significance level" is the probability of wrongly concluding that the mean boot up time for the computer is more than 30 seconds when in fact it is not more than 30 seconds.
(iv)	The critical value for the test is 31.645. For the specification to be met, H_0 is not rejected. $\bar{t} < 31.6$ (3 s.f.) Since $\bar{t} > 0$, Answer is $0 < \bar{t} < 31.6$.
(v)	Under H_0, $\bar{Y} \sim N\left(30, \frac{\sigma^2}{25}\right)$

$$Z = \frac{\overline{Y}-(30)}{\frac{\sigma}{\sqrt{25}}} \sim N(0,1)$$

Using a 1 – tailed z test,

$$z_{\text{calc}} = \frac{32.4-30}{\frac{\sigma}{5}} = \frac{12}{\sigma}, \; z_{\text{crit}} = 1.64485$$



In order not to reject H_0,

$z_{\text{calc}} < 1.64485$

$\frac{12}{\sigma} < 1.64485$

$\sigma > 7.2955$

$\sigma > 7.30$