



CANDIDATE
 NAME

CG

INDEX NO

MATHEMATICS

9758/02

Paper 2

19 SEPT 2019

3 hours

Candidates answer on the Question Paper.
 Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your CG and name on the work you hand in.
 Write in dark blue or black pen.
 You may use a HB pencil for any diagrams or graphs.
 Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
 Write your answers in the spaces provided in the question paper.
 Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
 The use of an approved graphing calculator is expected, where appropriate.
 Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
 Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
 You are reminded of the need for clear presentation in your answers.
 The number of marks is given in brackets [] at the end of each question or part question.
 The total number of marks for this paper is 100.

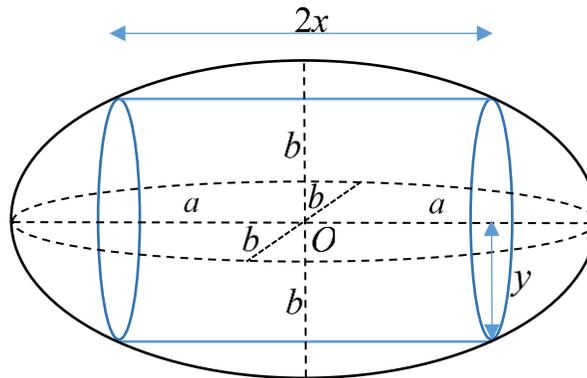
For Examiners' Use

Question	1	2	3	4	5	6
Marks						

Question	7	8	9	10	Total marks	
Marks						

Section A: Pure Mathematics [40 marks]

1



The above diagram shows a hollow ellipsoid with centre O , enclosing a fixed volume of $\frac{4}{3}\pi ab^2$. A solid cylinder of length $2x$ and base radius y is inscribed in the ellipsoid. It is

given that $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a and b are positive constants with $a > b$.

Use differentiation to find, in terms of a and b , the maximum volume of the cylinder, proving that it is a maximum. Hence determine the ratio of the maximum volume of the cylinder to that of the volume enclosed by the ellipsoid. [6]

2 (i) Find $\int 2 \sin(k+1)x \sin kx \, dx$. [2]

- (ii) Hence, determine in terms of k , the value of $\int_0^{\frac{\pi}{2}} (\sin(k+1)x - \sin kx)^2 dx$,
where k is an even integer. [5]

- 3** The plane p contains the point A with coordinates $(5, -1, 2)$ and the line l_1 with equation $\frac{x-3}{2} = y, z = 1$.
- (i) The point B has coordinates $(c, 2, 2)$. Given that the shortest distance from B to l_1 is $\frac{\sqrt{205}}{5}$, find the possible values of c . [3]

- (ii) Find a cartesian equation of p . [3]

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$, $\mu \in \mathbb{R}$.

- (iii) Find the coordinates of the point at which l_2 intersects p . [3]

The line l_3 has equation $\mathbf{r} = \begin{pmatrix} a \\ 2a \\ a \end{pmatrix} + t \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$, $t \in \mathbb{R}$, where a is a constant.

(iv) Show that p is parallel to l_3 . [1]

(v) Given that l_3 and p have no point in common, what can be said about the value of a ? [1]

- (vi) It is given instead that $a = 1$, find the distance between l_3 and p , leaving your answer in exact form. [2]

4 Do not use a calculator in answering this question.

- (a) The equation $2z^3 - 3z^2 + kz + 26 = 0$, where k is a real constant, has a root $z = 1 + ai$, where a is a positive real constant. Find the other roots of the equation and the values of a and k . [6]

- (b) (i) Given that $(x+iy)^2 = 15+8i$, determine the possible values of the real numbers x and y . [3]

- (ii) The roots of the equation $z^2 - (2 + 7i)z = 15 - 5i$ are z_1 and z_2 , with $\arg(z_1) < \arg(z_2)$. Find an exact expression for z_2 , giving your answer in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. [3]

- (iii) Find the argument of $z_1^2 z_2^*$ in exact form. [2]

Section B: Probability and Statistics [60 marks]

- 5 An investigation was carried out to determine the effect of rainfall on crop yield. The table below shows the average monthly rainfall, x mm, and the crop yield, y kg. The data is recorded during different months of a certain year.

x	150	163	172	175	180	187	196
y	48	70	87	92	95	89	80

- (i) Draw a scatter diagram for these values. State with a reason, which of the following equations, where a and b are constants, provides the most accurate model of the relationship between x and y .

(A) $y = a \ln(x - 100) + b$

(B) $y = a(x - 180)^2 + b$

(C) $y = \frac{a}{x - 130} + b$ [3]

- (ii) Using the model you chose in part (i), write down the equation for the relationship between x and y , giving the numerical values of the coefficients. State the product moment correlation coefficient for this model. [2]

- (iii) Calculate an estimate of the crop yield when the average monthly rainfall is 185 mm. Comment on the reliability of your estimate. [2]

- 6 A factory produces a large number of packets of cornflakes. On average, two in 7 packets contain a toy. The packets of cornflakes are sold in cartons of 12.
- (a) A carton is randomly chosen.
- (i) Find the probability that there are fewer than 2 toys. [1]

- (ii) Find the probability that there are more than 1 but at most 6 toys. [2]
- (b) Find the probability that in five randomly selected cartons, two of them contain exactly 4 toys and three of them contain exactly 2 toys. [2]
- (c) A mini-mart ordered 40 cartons of cornflakes from the factory. Find the probability that none of the cartons contains fewer than 2 toys. [2]

The factory also produces a large number of packets of oats. A random sample of n packets of oats is chosen. The number of packets of oats containing a toy in the sample is denoted by A . Assume that A has the distribution $B(n, p)$, where $p > 0.1$.

Given that $n = 25$ and $P(A = 2 \text{ or } 3) = 0.25$, write down an equation in terms of p and find p numerically. [2]

- 7 A box contains five balls numbered 1, 3, 5, 6, 8. Three balls are drawn at random from the box.
- (a) Find the probability that the sum of the three numbers drawn is an even number. [2]

- (b) The random variable S denotes the smallest of the three numbers drawn. [2]
- (i) Determine the probability distribution of S .

- (ii) Find $E(S)$ and $\text{Var}(S)$. [2]

- (iii) The mean of a random sample of 55 observations of S is denoted by \bar{S} . Find the probability that \bar{S} is within 0.5 of $E(S)$. [3]

- 8 (a) Events A and B are such that $P(A \cup B) = 0.6$ and $P(A|B) = 0.4$. Given that A and B are independent, find

(i) $P(B)$, [2]

(ii) $P(A'|B')$. [2]

- (b) 12 people are to be seated at 3 different coloured round tables.
- (i) Find the number of ways that there are at least 3 people at each table. [4]
- (ii) Find the probability that there are 4 people at each table given that there are at least 3 people at each table. [2]

- 9** A company produces car batteries. The life, in months, of a car battery of the regular type has the distribution $N(\mu, \sigma^2)$. The mean life of 4 randomly selected car batteries of the regular type is denoted by \bar{X} . It is given that $P(\bar{X} < 36.1) = P(\bar{X} > 49.1) = 0.03355$.

(i) State the value of μ and show that $\sigma \approx 7.10$, correct to 2 decimal places. [4]

(ii) Find the smallest integer value of k such that more than 90% of the car batteries of the regular type have a life less than k months. [2]

- (iii) Past experience shows that 25% of the car batteries of the regular type with lives less than 36 months are due to bad driving habits. A random sample of 100 car batteries of the regular type is selected. Find the expected number of these car batteries which will have lives each less than 36 months due to bad driving habits. [2]
- (iv) After research and experimentation, the company produces a premium type of car battery using an improved manufacturing process which is able to increase the life of each car battery by 10%. Find the probability that the total life of 5 randomly chosen car batteries of the premium type is more than the total life of 6 randomly chosen car batteries of the regular type. [4]

- 10** A previous study revealed that the average time taken to assemble a certain type of electrical component is at least 15 minutes. The manager wants to investigate if the results of the study is valid. A random sample of 40 components is taken and the times taken to assemble the components are summarised in the following table:

Time to assemble a component (min)	9	10	12	13	15	16	17	18
Number of components	1	6	3	8	5	7	8	2

- (i) Find unbiased estimates of the population mean and variance. [2]
- (ii) Test at the 5% level of significance whether the results of the study is valid. You should state your hypotheses and define any symbols you use. [5]

- (iii) Explain why the manager is able to conduct the test without knowing anything about the distribution of the times taken to assemble the electrical components.[1]
- (iv) Explain what is meant by the phrase “5% level of significance” in this context.[1]

The manager claims that the average time taken to assemble another type of electrical component is 30 minutes. A random sample of 50 components of this type is chosen and the time taken to assemble each component is recorded. The mean and standard deviation of the sample are 29.7 minutes and k minutes respectively. Find the range of possible values of k if a test at the 8% significance level shows that there is sufficient evidence that the manager’s claim is valid. [4]

~ END OF PAPER ~