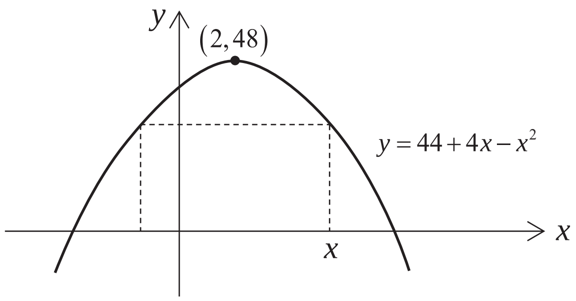
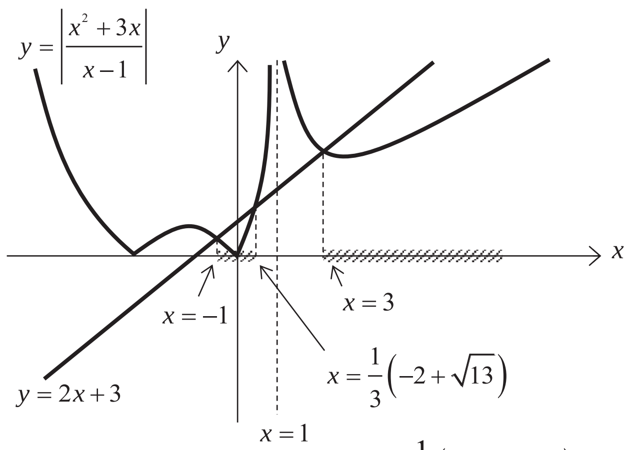
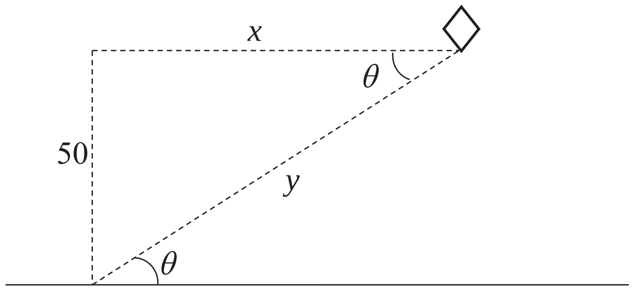
2019 ACJC H2 Math Prelim P1 Marker's Report

Qn	Solutions	Comments
1	$y=\mathrm{f}(x)\xrightarrow[\text{by factor }\frac{1}{a}]{\text{scaling // }x\text{-axis}}y=\mathrm{f}\left(ax\right)$ $\xrightarrow[\text{the positive }x\text{ direction}]{\text{translate }b\text{ units in}}y=\mathrm{f}\left(a(x-b)\right)$ $A:(2,-3)\to\left(\dfrac{2}{a},-3\right)\to\left(\dfrac{2}{a}+b,-3\right)=(7,-3)$ $B:(-3,1)\to\left(-\dfrac{3}{a},1\right)\to\left(-\dfrac{3}{a}+b,1\right)=(-1,1)$ $\left.\begin{array}{l}\dfrac{2}{a}+b=7\\ -\dfrac{3}{a}+b=-1\end{array}\right\}$ solving gives $a=\dfrac{5}{8},\ b=\dfrac{19}{5}$	**Badly done:** Common errors: (1) f(a(x – b)) is taken as translate b units in the positive x-direction then scale // x-axis by factor 1/a. Thus (2+b)/a = 7 and (-3 + b)/a = -1 were commonly seen. (2) Equations a(2-b) = 7 and a(-3-b) = -1 commonly seen. The correct equations should be a(7-b) = 2 and a(-1-b) = -3
2	 Area of rectangle, $A=2\left(x-2\right)y$ $=2\left(x-2\right)\left(44+4x-x^2\right)$ $\dfrac{\mathrm{d}A}{\mathrm{d}x}=0\ \Rightarrow\ 2\left(x-2\right)\left(4-2x\right)+\left(44+4x-x^2\right)(2)=0$ i.e. $-6x^2+24x+72=0$ i.e. $x^2-4x-12=0$ Hence $x=-2$ or 6. Check that $\left.\dfrac{\mathrm{d}^2A}{\mathrm{d}x^2}\right\|_{x=6}=-48<0$, therefore A is maximum when $x=6$. Maximum $A=2\left(6-2\right)\left(44+24-36\right)=256$ sq. units.	**Badly done** (1) Many assume that the area of the rectangle is xy or 2xy. (2) Some even differentiate y = 44 + 4x – x^2 to find the maximum value of y = 48 Thus area = 48 x 2 = 96 (3) Quite a number of candidates did not check that the area is maximum by checking the 2nd derivative
3	$\left\|\dfrac{x^2+3x}{x-1}\right\|=2x+3$ Note that for the equation to have any solution, $2x+3\geq 0\Rightarrow x\geq-\dfrac{3}{2}$.	Squaring both sides would lead to tedious working unless students were able to apply a^2-b^2. Another tedious method was to consider 4 different regions according to x = −3, 0, 1.

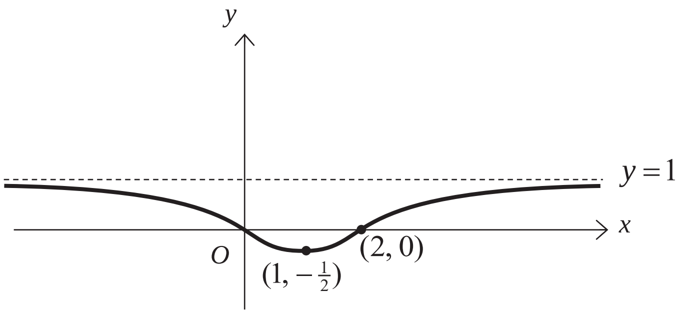
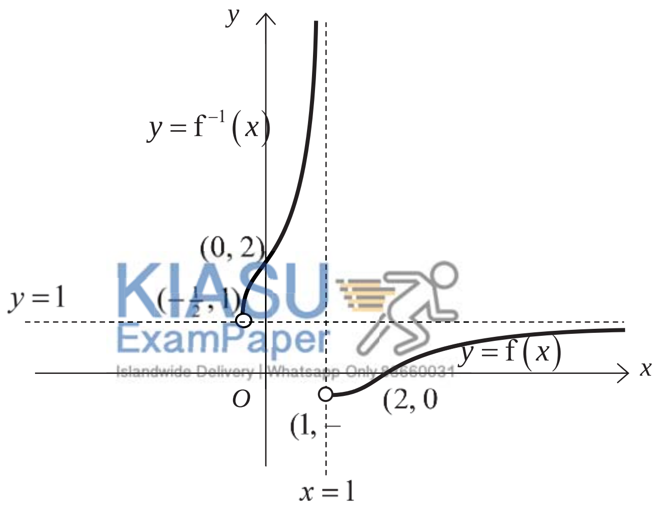
	$\dfrac{x^2+3x}{x-1}=2x+3$ $x^2+3x=(2x+3)(x-1)$ $x^2-2x-3=0$ $(x-3)(x+1)=0$ $\therefore x=3,-1$ $\dfrac{x^2+3x}{x-1}=-(2x+3)$ $x^2+3x=-(2x+3)(x-1)$ $3x^2+4x-3=0$ $x=\dfrac{-4\pm\sqrt{16-4(3)(-3)}}{2(3)}$ $=\dfrac{1}{3}\left(-2\pm\sqrt{13}\right)$ Reject $\dfrac{1}{3}\left(-2-\sqrt{13}\right)$(since $\approx -1.87<-1.5$) $\therefore x=-1, \dfrac{1}{3}\left(-2+\sqrt{13}\right)$ or 3.	Final answers need to be simplified: $\dfrac{\sqrt{52}}{6}=\dfrac{\sqrt{13}}{3}$ & $\dfrac{4}{6}=\dfrac{2}{3}$. Many students were not aware of the need to reject $\dfrac{1}{3}\left(-2-\sqrt{13}\right)$. Of those who rejected this negative root, few were able to provide reason that $x\geq -\dfrac{3}{2}$.
	 From graph, solution is $-1<x<\dfrac{1}{3}\left(-2+\sqrt{13}\right)$ or $x>3$	"Hence, **by sketching appropriate graphS**, solve…" was not followed: • $y=\left\|\dfrac{x^2+3x}{x-1}\right\|-2x-3$ was drawn instead. • Sign test was seen instead. • No graphs were seen in some scripts. Missing asymptote of $x = 1$ resulted in a pointed graph as shown in the GC.
4	 Given that $\dfrac{dx}{dt}=5$.	**Very badly done:** - wrong understanding of question. The rate of change given in the question is not the length of the string but the horizontal distance of the kite and the person. - many assume that the length of the string is constant at 100 and resulted in $\cos\theta=\dfrac{x}{100}$ and similar expressions.
(i)	From the diagram, $\tan\theta=\dfrac{50}{x}$. Hence, $x=\dfrac{50}{\tan\theta}=50\cot\theta$. Differentiating with respect to θ,	- There were many who did average rate of change, rather than instantaneous rate of change - when doing differentiation or integration, the angle is always in radians, a significant number left the answer in $^\circ$/sec.

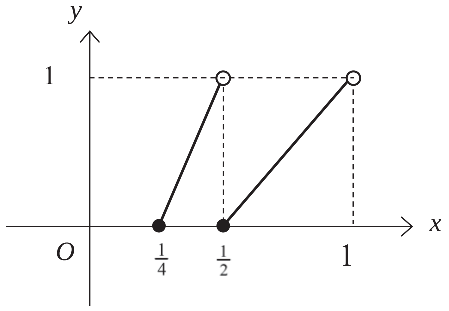
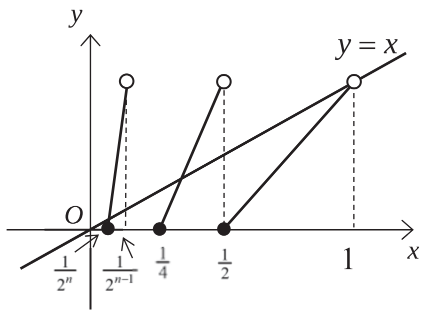
	$\frac{dx}{d\theta} = -50\operatorname{cosec}^2\theta$. When $y=100$, $\sin\theta = \frac{1}{2}$ and $\frac{dx}{dt}=5$, hence $\frac{dx}{dt} = \frac{dx}{d\theta}\cdot\frac{d\theta}{dt} \Rightarrow 5 = \left(-50\operatorname{cosec}^2\theta\right)\cdot\frac{d\theta}{dt}$ $\Rightarrow 5 = \left(-50\cdot(2)^2\right)\cdot\frac{d\theta}{dt}$ $\Rightarrow \frac{d\theta}{dt} = \frac{5}{-50(4)} = -\frac{1}{40}$ rad s^{-1}. ALTERNATIVELY, $\tan\theta = \frac{50}{x}$ At the instant when $y=100$, $x = \sqrt{100^2-50^2} = \sqrt{7500} = 50\sqrt{3}$, and $\theta = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$. Differentiating (*) with respect to *t*, $\sec^2\theta\frac{d\theta}{dt} = -\frac{50}{x^2}\frac{dx}{dt}$. When $x=50\sqrt{3}$, $\theta=\frac{\pi}{6}$ and $\frac{dx}{dt}=5$, $\sec^2\left(\frac{\pi}{6}\right)\cdot\frac{d\theta}{dt} = -\frac{50}{\left(50\sqrt{3}\right)^2}\cdot(5)$ $\Rightarrow \frac{d\theta}{dt} = -\frac{250}{(7500)\left(\frac{2}{\sqrt{3}}\right)^2} = -\frac{1}{40}$ rad s^{-1}.	- there are some who used $\theta = \cos^{-1}\frac{x}{\sqrt{x^2+50^2}}$ or $\theta = \sin^{-1}\frac{50}{\sqrt{x^2+50^2}}$ which will result in very tedious differentiation. - many did not use chain rule when differentiating $\theta = \tan^{-1}\frac{50}{x}$ and similar expressions, resulting in $\frac{d\theta}{dx} = \frac{1}{1+\left(\frac{50}{x}\right)^2}$ which is wrong - many students still do not know how to differentiate sec, cot, cosec directly, resulting in unnecessary working
(ii)	Want to find $\frac{dy}{dt}$ when $y=100$. Now $x^2+50^2=y^2 \Rightarrow y=\sqrt{x^2+50^2}$ $\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(x^2+50^2\right)^{-\frac{1}{2}}(2x) = \frac{x}{\sqrt{x^2+50^2}}$ By chain rule, $\frac{dy}{dt} = \frac{dy}{dx}\cdot\frac{dx}{dt}$. Hence when $y=100$, $\frac{dy}{dt} = \frac{dy}{dx}\cdot\frac{dx}{dt}$ $\frac{dy}{dt} = \left(\frac{x}{\sqrt{x^2+50^2}}\right)\cdot 5 = \left(\frac{50\sqrt{3}}{\sqrt{\left(50\sqrt{3}\right)^2+50^2}}\right)\cdot 5 = \frac{5\sqrt{3}}{2}$ cm s^{-1}.	This part was left empty in most scripts. For those who did this, many did not simplify the final answer.

5	$y=\tan\left(1-e^{3x}\right) \quad \Rightarrow \quad \tan^{-1} y = 1-e^{3x}$. Differentiating with respect to x, $\frac{1}{1+y^2}\frac{dy}{dx} = -3e^{3x}$ $\frac{dy}{dx} = -3e^{3x}\left(1+y^2\right)$. Hence $k=-3$. Differentiating again with respect to x, $\frac{d^2y}{dx^2} = -3\left(2y\frac{dy}{dx}\right)e^{3x} - 3\left(1+y^2\right)\left(3e^{3x}\right)$ $= -6y\frac{dy}{dx}e^{3x} - 9e^{3x}\left(1+y^2\right)$ $= -3e^{3x}\left(2y\frac{dy}{dx}+3+3y^2\right)$ $\frac{d^3y}{dx^3} = -3e^{3x}\left(2y\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^2+6y\frac{dy}{dx}\right) - 9e^{3x}\left(2y\frac{dy}{dx}+3+3y^2\right)$ When $x=0$, $y=0$, $\frac{dy}{dx}=-3$, $\frac{d^2y}{dx^2}=-9$ and $\frac{d^3y}{dx^3}=-81$. Hence, $y=\tan\left(1-e^{3x}\right)$ $= 0-3x+\frac{(-9)}{2!}x^2+\frac{(-81)}{3!}x^3+\ldots$ $\approx -3x-\frac{9}{2}x^2-\frac{27}{2}x^3$.	Most approach it this way: differentiate $\tan\left(1-e^{3x}\right)$ to get $-3e^{3x}\sec^2\left(1-e^{3x}\right)$ and use trigo identity to show k. A lot of complete and accurate work, as many of them make careless mistakes/slips: • $\frac{d}{dx}\left(1+y^2\right)=1+2y\frac{dy}{dx}$ • $\frac{d}{dx}\left(y\frac{dy}{dx}\right)=y\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)$ And some did not use product rule: • $\frac{d}{dx}\left(y\frac{dy}{dx}\right)=\frac{dy}{dx}\frac{d^2y}{dx^2}$ Clearly taught to avoid quotient rule for implicit differentiation but some students still proceed in that direction.
	$\frac{x}{a+bx}=x\left(a+bx\right)^{-1}$ $=\frac{x}{a}\left(1+\frac{bx}{a}\right)^{-1}$ $=\frac{x}{a}\left(1-\frac{bx}{a}+\left(\frac{bx}{a}\right)^2+\ldots\right)$ $\approx\frac{x}{a}-\frac{bx^2}{a^2}$ Hence $-3x-\frac{9}{2}x^2=\frac{x}{a}-\frac{bx^2}{a^2}$. Comparing coefficients, x: $\quad -3=\frac{1}{a} \quad \Rightarrow \quad a=-\frac{1}{3}$ x^2: $\quad -\frac{9}{2}=-\frac{b}{a^2} \quad \Rightarrow \quad b=\frac{1}{2}$	Instead of using binomial series to expand, more students differentiated twice, use Maclaurins again, compared coefficient of x with $f'(0)$, but many made mistake in comparing the coefficient of x^2 where they equated $f''(0)=-\frac{9}{2}$.

6	Total area of n rectangles $=\frac{1}{n}\left[(2^{\frac{1}{n}}+1)+(2^{\frac{2}{n}}+1)+...(2^{\frac{n-1}{n}}+1)+(2^{1}+1)\right]$ $=\frac{1}{n}\left[2^{\frac{1}{n}}+2^{\frac{2}{n}}+...2^{\frac{n-1}{n}}+2^{1}\right]+\frac{1}{n}[1+1+...1+1]$ $=\frac{1}{n}\sum_{r=1}^{n}2^{\frac{r}{n}}+1$ $=\frac{1}{n}\left(\frac{2^{\frac{1}{n}}(1-(2^{\frac{1}{n}})^{n})}{1-2^{\frac{1}{n}}}\right)+1=\frac{2^{\frac{1}{n}}(-1)}{n(1-2^{\frac{1}{n}})}+1=\frac{2^{\frac{1}{n}}}{n(2^{\frac{1}{n}}-1)}+1$	Most students could identify the area of the r^th rectangle having width $\frac{1}{n}$ and length $(2^{\frac{r}{n}}+1)$. Many fail to recognize that they should **split** the following sum: $\frac{1}{n}\sum_{r=1}^{n}(2^{\frac{r}{n}}+1)$ and proceed with S_{GP} and sum of constant. As this is an answer given question, students have to show the **full details** when applying the GP formula.
	$\lim_{n\to\infty}S_n=\int_0^1 2^x+1\,\mathrm{d}x=\left[\frac{2^x}{\ln 2}+x\right]_0^1=\left(\frac{2^1}{\ln 2}+1\right)-\frac{1}{\ln 2}=\frac{1}{\ln 2}+1$	Most students aren't aware that the limit of the sum of the area of n rectangles is the area under curve. Many students concluded that $n(2^{\frac{1}{n}}-1)$ tends to zero when the table in the GC indicates that $n(2^{\frac{1}{n}}-1)$ tends to 0.69339. But since the question asked for exact answers, students can't use this method.
7(a)	$\int\sin px\cos qx\,\mathrm{d}x=\frac{1}{2}\int\sin(p+q)x+\sin(p-q)x\,\,\mathrm{d}x$ $=-\frac{\cos(p+q)x}{2(p+q)}-\frac{\cos(p-q)x}{2(p-q)}+c$	Half of the cohort not aware the need to use factor formula. They attempted to solve by parts.
(b)	$\int x\sin nx\,\mathrm{d}x=x(-\frac{\cos nx}{n})-\int-\frac{\cos nx}{n}\,\,\mathrm{d}x$ $=-\frac{x\cos nx}{n}+\int\frac{\cos nx}{n}\,\,\mathrm{d}x=-\frac{x\cos nx}{n}+\frac{\sin nx}{n^2}+c$	Most able to use by parts correctly.
(b)(i)	$\int_0^{\pi}x\sin nx\,\,\mathrm{d}x=\left[-\frac{x\cos nx}{n}+\frac{\sin nx}{n^2}\right]_0^{\pi}=-\frac{\pi\cos n\pi}{n}=\pm\frac{\pi}{n}$ $\therefore k=\pm1$	Many have no idea what to do with $\cos n\pi$.

(b)(ii)	$\int_0^{\frac{\pi}{2}} \lvert x\sin 3x\rvert \,\mathrm{d}x = \int_0^{\frac{\pi}{3}} x\sin 3x \,\mathrm{d}x - \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} x\sin 3x \,\mathrm{d}x$ $= \left[-\frac{x\cos 3x}{3}+\frac{\sin 3x}{9}\right]_0^{\frac{\pi}{3}} - \left[-\frac{x\cos 3x}{3}+\frac{\sin 3x}{9}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ $= -\frac{\pi\cos\pi}{9} - \left(-\frac{\pi\cos\frac{3\pi}{2}}{6}+\frac{\sin\frac{3\pi}{2}}{9}\right)+\left(-\frac{\pi\cos\pi}{9}\right)$ $=\frac{2\pi+1}{9}$	Some students used 1 instead of $\frac{\pi}{3}$. They gotten the number from GC.
8(a)	From $w-2z=9$, $w=9+2z$. Substitute into $3w-wz^*=17-30\mathrm{i}$: $3(9+2z)-(9+2z)z^*=17-30\mathrm{i}$ $\Rightarrow\ 27+6z-9z^*-2zz^*=17-30\mathrm{i}$ Let $z=a+b\mathrm{i}$, then $27+6(a+b\mathrm{i})-9(a-b\mathrm{i})-2(a+b\mathrm{i})(a-b\mathrm{i})=17-30\mathrm{i}$ i.e. $27+6a-9a-2a^2-2b^2+6b\mathrm{i}+9b\mathrm{i}=17-30\mathrm{i}$ i.e. $27-3a-2a^2-2b^2+15b\mathrm{i}=17-30\mathrm{i}$. Comparing coefficients, Imaginary: $15b=-30 \ \Rightarrow\ b=-2$ Real: $27-3a-2a^2-2b^2=17 \ \Rightarrow\ 2-3a-2a^2=0$ solving for a, $a=\frac{1}{2}$ or -2. Since $\mathrm{Re}(z)<0$, $a=-2$. Therefore, $z=-2-2\mathrm{i}$, and $w=9+2(-2-2\mathrm{i})=5-4\mathrm{i}$.	It is good practice to work to eliminate one variable when solving simultaneous equations before substituting $z=a+b\mathrm{i}$. Many students who started by using $z=a+b\mathrm{i}$ and $w=c+d\mathrm{i}$ made careless mistakes in their computation and were not successful in arriving at the correct answer.
8(b)(i)	$-\mathrm{i}$ is a root of the equation $z^3+kz^2+\left(8+2\sqrt{2}\,\mathrm{i}\right)z+8\,\mathrm{i}=0$ hence $(-\mathrm{i})^3+k(-\mathrm{i})^2+\left(8+2\sqrt{2}\,\mathrm{i}\right)(-\mathrm{i})+8\,\mathrm{i}=0$ $\mathrm{i}-k-8\mathrm{i}+2\sqrt{2}+8\,\mathrm{i}=0$ $\mathrm{i}-k+2\sqrt{2}=0$ $\therefore\ k=2\sqrt{2}+\mathrm{i}$. $z^3+\left(2\sqrt{2}+\mathrm{i}\right)z^2+\left(8+2\sqrt{2}\,\mathrm{i}\right)z+8\,\mathrm{i}=0$ $\Rightarrow\ (z+\mathrm{i})\left(z^2+bz+8\right)=0$ Comparing coefficients of z, $8+2\sqrt{2}\,\mathrm{i}=8+b\mathrm{i}$. Hence $2\sqrt{2}=b$. $(z+\mathrm{i})\left(z^2+2\sqrt{2}\,z+8\right)=0$ $\therefore\ z=-\mathrm{i}$ or $z=\frac{-2\sqrt{2}\pm\sqrt{8-32}}{2} \quad =-\sqrt{2}\pm\sqrt{6}\,\mathrm{i}$. The other roots are $-\sqrt{2}+\sqrt{6}\,\mathrm{i}$ and $-\sqrt{2}-\sqrt{6}\,\mathrm{i}$.	The given equation is NOT a real polynomial as the coefficients are not all real numbers. Hence conjugate of $-\mathrm{i}$ is NOT a root. k is a constant does not mean it is a real number. In this case it is a complex constant. So when $\mathrm{i}-k+2\sqrt{2}=0$ $\Rightarrow\ k=2\sqrt{2}+\mathrm{i}$ Instead, some students **wrongly** proceeded to equate real and imaginary parts.

(b)(ii)	$\mathrm{i}z^3 + kz^2 + \left(2\sqrt{2} - 8\,\mathrm{i}\right)z - 8\,\mathrm{i} = 0$ $\Rightarrow \quad -(\mathrm{i}z)^3 - k(\mathrm{i}z)^2 - \left(8 + 2\sqrt{2}\,\mathrm{i}\right)(\mathrm{i}z) - 8\,\mathrm{i} = 0$ i.e. $\quad (\mathrm{i}z)^3 + k(\mathrm{i}z)^2 + \left(8 + 2\sqrt{2}\,\mathrm{i}\right)(\mathrm{i}z) + 8\,\mathrm{i} = 0$. Hence from **(i)**, $\mathrm{i}z = -\mathrm{i}, \quad -\sqrt{2} + \sqrt{6}\,\mathrm{i}, \quad$ or $\quad -\sqrt{2} - \sqrt{6}\,\mathrm{i}$ $\therefore \quad z = -1, \quad -\sqrt{6} + \sqrt{2}\,\mathrm{i}, \quad$ or $\quad \sqrt{6} + \sqrt{2}\,\mathrm{i}$.	Not well done. A substitution is needed here.
(b)(iii)	$z_0 = -\sqrt{2} + \sqrt{6}\,\mathrm{i}$ $\arg z_0 = \dfrac{2\pi}{3}$ For $(\mathrm{i}z_0)^n$ to be purely imaginary, $\arg(\mathrm{i}z_0)^n = \pm\dfrac{\pi}{2} \quad \Rightarrow \quad n\arg(\mathrm{i}z_0) = \pm\dfrac{k\pi}{2}$ where k is odd i.e. $\quad n\left[\arg \mathrm{i} + \arg z_0\right] = \pm\dfrac{k\pi}{2}$ i.e. $\quad n\left[\dfrac{\pi}{2} + \dfrac{2\pi}{3}\right] = \pm\dfrac{k\pi}{2}$ i.e. $\quad n\left[-\dfrac{5\pi}{6}\right] = \pm\dfrac{k\pi}{2}$ Hence smallest positive integer value of n is 3.	Not well done. Some errors in the method to find argument of a complex number.
9(ai)		Generally well done, except some students who totally do not know how to sketch reciprocal graph. Common errors are - both tails tend to infinity - left tail tends to infinity - wrong y-value for minimum point
(a)(ii)	$k = 1$ 	Some did not get the mark $k = 1$ even though their (i) is correct. Weird! Common mistakes for $y = \mathrm{f}^{-1}(x)$ graph - draw $y = \mathrm{f}'(x)$ instead - the left end points are on the respective asymptotes - missing labelling of points, especially the y-intercept - the point $(1, -\frac{1}{2})$ becomes $(-1, \frac{1}{2})$ - missing vertical asymptote

(b)(i)	$g(x)=\begin{cases}\boxed{4x-1}, & \frac{1}{4}\le x<\frac{1}{2}\\ \boxed{2x-1}, & \frac{1}{2}\le x<\frac{1}{4}\end{cases}$ 	Most students get the 2 marks if they attempt it Common mistakes in graph - the *y*-value of both end points are at 1, so they should reach the same height - scale on the x-axis, there were a significant number who drew the same width for both pieces - swap the rule for the domains
(b)(ii)	$g(x)=2^n x-1,\ \frac{1}{2^n}\le x<\frac{1}{2^{n-1}}$ $g\left(\frac{x}{2}\right)=2^n\left(\frac{x}{2}\right)-1,\ \frac{1}{2^n}\le\frac{x}{2}<\frac{1}{2^{n-1}}$ $=2^{n-1}x-1,\frac{1}{2^{n-1}}\le x<\frac{1}{2^{n-2}}$ $=2^k x-1,\frac{1}{2^k}\le x<\frac{1}{2^{k-1}}$ (optional) $=g(x)$	About half did not attempt this. Most who attempted it got partial marks. Partial marks were given if - Obtain the rule for the general case in simplified form - obtain the rule for the case in **(i)** with the correct domain
(b)(iii)	Consider $\frac{1}{2^n}<0.001<\frac{1}{2^{n-1}}$ $n>\frac{\ln 0.001}{\ln\left(\frac{1}{2}\right)}=9.97$ $\therefore \frac{1}{2^{10}}<0.001<\frac{1}{2^9}$  When $n=10$, $g(x)=2^{10}x-1,\ \frac{1}{2^{10}}\le x<\frac{1}{2^9}$. Solving $g(x)=x$ when $n=10$, $2^{10}x-1=x \Rightarrow x=0.000978<0.001$. Hence there is no solution when $n=10$. There are therefore **8** solutions (since *n* =1 also has has no solution).	Most students assume that the solution is in the region of the graph drawn in **b(i)**. Few who attempted this got the correct answer, which is fine.
10(i)	As there are 9 papers, there are 8 durations in between the papers.	Very few students manage to write down **both** inequalities correctly. Many students wrote $S_8\le 90$ as they may have miss out on the fact that the 1st practice paper is

	$S_8=\frac{8}{2}\left(2a+(8-1)(-d)\right)<90$ $\Rightarrow 8a-28d<90$ $\Rightarrow a<11.25+3.5d$ $T_8=a+(8-1)(-d)>0$ $\Rightarrow a>7d$ For the last paper to be as close to the exam date as possible, a and S_8 must be as large as possible, By trial and error, $d=1,\ a<14.75\Rightarrow a=14(>7(1))$ and $S_8=84$ $d=2,\ a<18.25\Rightarrow a=18(>7(2))$ and $S_8=88$ $d=3,\ a<21.75\Rightarrow a=21(\not>7(3))$ Therefore $d=2,\ a=18$.	already attempted on the 1st day. By writing $S_8\le 90$, this implies that the 9th practice paper could be on the 91st day (examination day) which is not what the question wants. Quite a number of student also wrote $T_8\ge 0$. This is incorrect as $T_8\ge 0$ means that it is possible for the 8th and 9th practice paper to be done on the same day which is also not what the question wants. Even fewer students realised of the need to determine the values of a and d by trial and error. Quite a number of students attempt to "solve" the 2 inequalities by treating them as "equations" and attempting to "solve" them "simultaneously" which is incorrect.
10(ii)	92 is the highest (theoretical) mark that he will get even if he practise many many times. OR 92 is the highest (theoretical) mark that he will get based on his aptitude and ability.	This part was quite well attempted as students generally know the significance of the number 92. However, their answer can be improved on by being clearer in stating the reason why 92 is the highest mark David will get.
10(iii)	$m=\frac{1}{9}\sum_{n=1}^{9}u_n=\frac{1}{9}\sum_{n=1}^{9}\left(92-65(b^n)\right)$ $=\frac{1}{9}\left(92(9)-65\sum_{n=1}^{9}b^n\right)$ $=92-\frac{65}{9}\left(\frac{b(1-b^9)}{1-b}\right)$	Many students assume wrongly that $\sum_{n=1}^{9}u_n$ is an AP and took the first term as $\left(92-65(b)\right)$ and the last term as $\left(92-65(b^9)\right)$ which are incorrect. Quite a number of student could not recall the S_n of GP correctly.
10(iv)	As he scored higher than m from his 4th paper onwards,	Many students could write the inequality but could not solve as they did not realise that GC can be used to help them solve.

	$92-\frac{65}{9}\left(\frac{b(1-b^9)}{1-b}\right)<92-65(b^4)$ $\Rightarrow 1-b^9>9b^3(1-b)$ $\Rightarrow b^9-9b^4+9b^3-1<0$ From GC, $0<b<0.726$	
11	$\frac{d^2x}{dt^2}+k\left(\frac{dx}{dt}\right)^2=10$ Substitute $v=\frac{dx}{dt}$ and $\frac{dv}{dt}=\frac{d^2x}{dt^2}$ into DE, $\therefore \frac{dv}{dt}+kv^2=10$	Majority are able to get the differential equation. A handful of students left blank. Many students misinterpreted the question as $v=\frac{x}{t}$.
	when $v=\sqrt{10}$, $\frac{dv}{dt}=6$ $6+k(\sqrt{10})^2=10 \therefore k=0.4$ $\frac{dv}{dt}=10-0.4v^2$ $\int \frac{1}{10-0.4v^2}\,dv=\int dt$ $\frac{1}{0.4}\int \frac{1}{25-v^2}\,dv=t+c$ $\frac{1}{0.4}\left(\frac{1}{2(5)}\right)\ln\left\|\frac{5+v}{5-v}\right\|=t+c$ $\frac{1}{4}\ln\left\|\frac{5+v}{5-v}\right\|=t+c$ $\ln\left\|\frac{5+v}{5-v}\right\|=4t+d$, $d=4c$ $\frac{5+v}{5-v}=Ae^{4t}$, $A=\pm e^d$ $\frac{5-v}{5+v}=Be^{-4t}$, $B=\frac{1}{A}$ when $t=0$, $v=0$ $\therefore A=1$ $\frac{5-v}{5+v}=e^{-4t}$ $5-v=(5+v)e^{-4t}$ $\therefore v=\frac{5(1-e^{-4t})}{1+e^{-4t}}$ As $t\to\infty$, $v\to 5$ ms^{-1}	Those who managed to obtain $\frac{dv}{dt}+kv^2=10$, are able to get $k=0.4$ easily. With $\frac{dv}{dt}+kv^2=10$, majority knew how to separate the variables. However, only a handful include modulus. A common mistake made is $\int\frac{5}{50-2v^2}\,dv=\frac{5}{2\left(\sqrt{50}\right)}\ln\left\|\frac{\sqrt{50}+\sqrt{2}v}{\sqrt{50}-\sqrt{2}v}\right\|$. Students did not realise that coefficient of v^2 must be 1 if they are applying the same formula from MF26. Students have no idea why $\frac{5(e^{4t}-1)}{e^{4t}+1}=\frac{5(1-e^{-4t})}{1+e^{-4t}}$. Working must be shown explicitly. Students didn't read question. At least a quarter of the cohort missed this part. Those who attempted this part managed to answer correctly.

	$\frac{dx}{dt}=\frac{5(1-e^{-4t})}{1+e^{-4t}}$ $x=\int \frac{5(1-e^{-4t})}{1+e^{-4t}}\,dt$ $=5\int \frac{1}{1+e^{-4t}}-\frac{e^{-4t}}{1+e^{-4t}}\,dt=5\int \frac{e^{4t}}{1+e^{4t}}-\frac{e^{-4t}}{1+e^{-4t}}\,dt$ $=\frac{5}{4}\left[\ln(1+e^{4t})+\ln(1+e^{-4t})\right]+c$ when $t=0$, $x=0$ $\therefore c=-2.5\ln 2$ Hence, $x=\frac{5}{4}\left[\ln(1+e^{4t})+\ln(1+e^{-4t})\right]-\frac{5}{2}\ln 2$. From G.C., when $x = 10$, $t = 2.3465 \approx 2.35$ s Alternatively, $\int_0^t \frac{5(1-e^{-4t})}{1+e^{-4t}}\,dt=10$ From G.C., $t = 2.3465 \approx 2.35$ s	Badly done. Many students did not attempt this part. Those who attempted, majority have no idea that the question is solving for the particular solution of x. Many students differentiated $\frac{5(1-e^{-4t})}{1+e^{-4t}}$. There is still a handful of students who got $x=\int \frac{5(1-e^{-4t})}{1+e^{-4t}}\,dt$. However, most of them stuck at $\int \frac{1}{1+e^{-4t}}\,dt$, which many mistook as $\ln\left(1+e^{-4t}\right)$ or tangent inverse. Some students interpreted the question wrongly as "when $t=0,\ x=10$". There are some impressive solutions, but a handful of them didn't realise that they need to make use of G.C to solve.
12(i)	F_1: $\mathbf{r}=\mathbf{i}+2\mathbf{j}+3\mathbf{k}+\lambda(2\mathbf{i}-4\mathbf{j}+\mathbf{k})$ F_2: $\mathbf{r}=-2\mathbf{i}+\mathbf{j}+3\mathbf{k}+\mu(\mathbf{i}+m\mathbf{j}-7\mathbf{k})$ Since $\begin{pmatrix}2\\-4\\1\end{pmatrix}\neq k\begin{pmatrix}1\\m\\-7\end{pmatrix}$, the paths cannot be parallel. If the paths intersect, $\begin{pmatrix}1\\2\\3\end{pmatrix}+\lambda\begin{pmatrix}2\\-4\\1\end{pmatrix}=\begin{pmatrix}-2\\1\\3\end{pmatrix}+\mu\begin{pmatrix}1\\m\\-7\end{pmatrix}$ $2\lambda-\mu=-3$ .... (1) $\lambda+7\mu=0$ .... (2) $4\lambda+m\mu=1$ .... (3) Solving (1) and (2), $\lambda=-\frac{7}{5}$, $\mu=\frac{1}{5}$ From (3), $m = 33$ Since the paths do not intersect, they are skew lines $\therefore m\neq 33$	Shocking that a significant number of students did not know how or made mistakes/slips when converting from Cartesian to vector form. For non-intersecting lines, many considered parallel lines and concluded that m is not multiples of 4. A lot of students did not even consider case of skew lines.
(ii)	Signal is located at $S(0, 0, 3)$. <u>Method 1:</u> Let $A(1,2,3)$ be a point on F_1. Then $\overrightarrow{AS}=\begin{pmatrix}0\\0\\3\end{pmatrix}-\begin{pmatrix}1\\2\\3\end{pmatrix}=\begin{pmatrix}-1\\-2\\0\end{pmatrix}$	Many students did not write the position of signal correctly, <u>base</u> of control towel at (0, 0, 0) (info given earlier and found on a different page).

Perpendicular dist $= \left|\overrightarrow{AS} \times \hat{\mathbf{b}}\right|$

$$= \left|\begin{pmatrix}-1\\-2\\0\end{pmatrix} \times \frac{1}{\sqrt{21}}\begin{pmatrix}2\\-4\\1\end{pmatrix}\right| = \frac{1}{\sqrt{21}}\left|\begin{pmatrix}-2\\1\\8\end{pmatrix}\right|$$

$$= \frac{\sqrt{(-2)^2+1+8^2}}{\sqrt{21}} = \sqrt{\frac{23}{7}}$$

<u>Method 2:</u>

$$\left|\overrightarrow{AF}\right| = \left|\overrightarrow{AS}.\hat{\mathbf{b}}\right| = \left|\begin{pmatrix}-1\\-2\\0\end{pmatrix}.\frac{1}{\sqrt{21}}\begin{pmatrix}2\\-4\\1\end{pmatrix}\right| = \frac{6}{\sqrt{21}}$$

$$SF = \sqrt{AS^2 - AF^2} = \sqrt{(\sqrt{5})^2 - (\frac{6}{\sqrt{21}})^2} = \sqrt{\frac{23}{7}}$$

<u>Method 3:</u>

Let $\overrightarrow{OF} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}2\\-4\\1\end{pmatrix}$ for some λ.

$$\overrightarrow{SF} = \begin{pmatrix}1+2\lambda\\2-4\lambda\\3+\lambda\end{pmatrix} - \begin{pmatrix}0\\0\\3\end{pmatrix} = \begin{pmatrix}1+2\lambda\\2-4\lambda\\\lambda\end{pmatrix}$$

$$\overrightarrow{SF}.\mathbf{b} = 0$$

$$\begin{pmatrix}1+2\lambda\\2-4\lambda\\\lambda\end{pmatrix}.\begin{pmatrix}2\\-4\\1\end{pmatrix} = 0 \Rightarrow 2(1+2\lambda) - 4(2-4\lambda) + \lambda = 0 \therefore \lambda = \frac{2}{7}$$

$$\overrightarrow{SF} = \begin{pmatrix}1+2(2/7)\\2-4(2/7)\\2/7\end{pmatrix} = \frac{1}{7}\begin{pmatrix}11\\6\\2\end{pmatrix}$$

$$\left|\overrightarrow{SF}\right| = \frac{\sqrt{11^2+6^2+2^2}}{7} = \frac{\sqrt{161}}{7}$$

<u>Method 4:</u>

$$\left|\overrightarrow{SF}\right| = \left|\begin{pmatrix}1+2\lambda\\2-4\lambda\\3+\lambda\end{pmatrix} - \begin{pmatrix}0\\0\\3\end{pmatrix}\right| = \left|\begin{pmatrix}1+2\lambda\\2-4\lambda\\\lambda\end{pmatrix}\right|$$

$$= \sqrt{(1+2\lambda)^2 + (2-4\lambda)^2 + \lambda^2}$$

$$= \sqrt{5 - 12\lambda + 21\lambda^2}$$

$$= \sqrt{\frac{23}{7} + 21(\lambda - \frac{6}{21})^2}$$

$\therefore$ shortest dist $= \sqrt{\frac{23}{7}}$

(iii)	$F_2 : \mathbf{r} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 5 \\ -7 \end{pmatrix}$ $F_3 : \mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ $\mathbf{n} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 5 \\ -7 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \\ 5 \end{pmatrix}$ $\mathbf{a.n} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} . \begin{pmatrix} 5 \\ 6 \\ 5 \end{pmatrix} = 11$ $\therefore \mathbf{r.} \begin{pmatrix} 5 \\ 6 \\ 5 \end{pmatrix} = 11 \Rightarrow 5x + 6y + 5z = 11$	Many students did not know what **cartesian** form "$ax+by+cz=p$" of equation of plane is and left answer in parametric form: $\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 5 \\ -7 \end{pmatrix}$.
(iv)	When F_2 and F_3 intersect, $\begin{pmatrix} -2+\mu \\ 1+5\mu \\ 3-7\mu \end{pmatrix} = \begin{pmatrix} 1+\alpha \\ 1 \\ -\alpha \end{pmatrix} \quad \therefore \mu = 0, \alpha = -3$ ie. $\begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$ Let B be another point on F_2 ($\mu = 1$). $\overrightarrow{OB} = \begin{pmatrix} -2+1 \\ 1+5(1) \\ 3-7(1) \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \\ -4 \end{pmatrix}$ Find p.v. of foot of perpendicular to F_3: <u>Method 1:</u> Let $\overrightarrow{ON} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ for some α. $\overrightarrow{BN} = \begin{pmatrix} 1+\alpha \\ 1 \\ -\alpha \end{pmatrix} - \begin{pmatrix} -1 \\ 6 \\ -4 \end{pmatrix} = \begin{pmatrix} 2+\alpha \\ -5 \\ 4-\alpha \end{pmatrix}$ $\overrightarrow{BN}.\mathbf{b} = 0$ $\begin{pmatrix} 2+\alpha \\ -5 \\ 4-\alpha \end{pmatrix} . \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = 0 \Rightarrow (2+\alpha)-(4-\alpha) = 0 \therefore \alpha = 1$ $\therefore \overrightarrow{ON} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ $\overrightarrow{ON} = \dfrac{\overrightarrow{OB} + \overrightarrow{OB'}}{2} \Rightarrow \overrightarrow{OB'} = 2\overrightarrow{ON} - \overrightarrow{OB}$ $\overrightarrow{OB'} = 2\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} - \begin{pmatrix} -1 \\ 6 \\ -4 \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \\ 2 \end{pmatrix}$ $\overrightarrow{PB'} = \begin{pmatrix} 5 \\ -4 \\ 2 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 7 \\ -5 \\ -1 \end{pmatrix} \quad \therefore F_4 : \mathbf{r} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 7 \\ -5 \\ -1 \end{pmatrix}$ <u>Method 2:</u>	Most students just gave up. Out of those who did not, many used the intersection of F_2 and F_3 (−2, 1, 3) to find foot of perpendicular, which failed.

	$\begin{pmatrix}-1\\6\\-4\end{pmatrix}-\begin{pmatrix}-2\\1\\3\end{pmatrix}=\begin{pmatrix}1\\5\\-7\end{pmatrix}$ $\overrightarrow{PN}=\left(\overrightarrow{PB}.\hat{\mathbf{b}}\right).\hat{\mathbf{b}}=\left(\begin{pmatrix}1\\5\\-7\end{pmatrix}.\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\-1\end{pmatrix}\right).\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\-1\end{pmatrix}=4\begin{pmatrix}1\\0\\-1\end{pmatrix}$ $\overrightarrow{PN}=\frac{\overrightarrow{PB}+\overrightarrow{PB'}}{2}\Rightarrow\overrightarrow{PB'}=2\overrightarrow{PN}-\overrightarrow{PB}$ $\overrightarrow{PB'}=8\begin{pmatrix}1\\0\\-1\end{pmatrix}-\begin{pmatrix}1\\5\\-7\end{pmatrix}=\begin{pmatrix}7\\-5\\-1\end{pmatrix}$ $\therefore F_4:\mathbf{r}=\begin{pmatrix}-2\\1\\3\end{pmatrix}+\mu\begin{pmatrix}7\\-5\\-1\end{pmatrix}$	