Qn

1

$y^2 = x^2 + 9$

Scale 2 parallel to x

$$y^2 = \left(\frac{x}{2}\right)^2 + 9$$

Translate by -4 units parallel to x

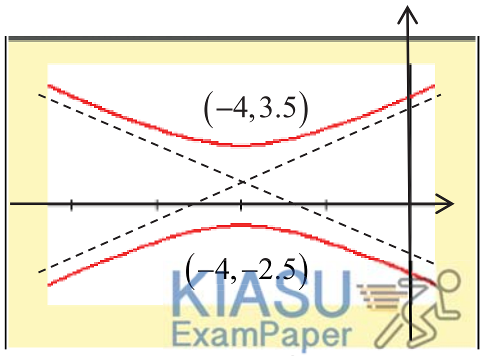
$$y^2 = \left(\frac{(x+4)}{2}\right)^2 + 9$$

Translate by 1/2 units parallel to y

$$\left(y - \frac{1}{2}\right)^2 = \left(\frac{(x+4)}{2}\right)^2 + 9$$

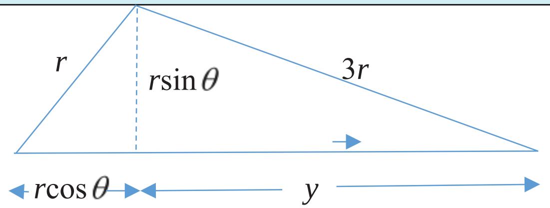
Equations of asymptotes

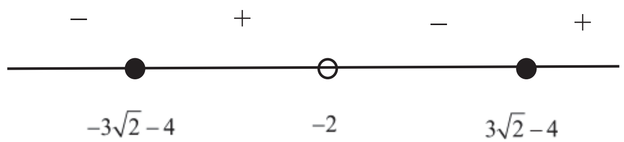
$$y = \frac{1}{2} \pm \frac{x+4}{2} = \frac{x}{2} + \frac{5}{2}, -\frac{x}{2} - \frac{3}{2}$$



$$y = \frac{x}{2} + \frac{5}{2}$$

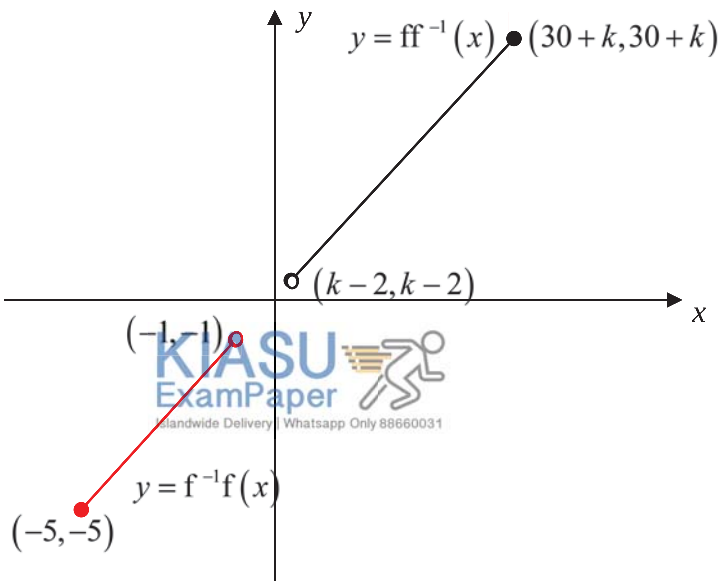
$$y = -\frac{x}{2} - \frac{3}{2}$$

Qn	
2(i)	 Using Pythagoras' Theorem, $y=\sqrt{9r^2-r^2\sin^2\theta}$ $\therefore x=r\cos\theta+r\sqrt{9-\sin^2\theta}\ =x=r\left[\cos\theta+\sqrt{\left(9-\sin^2\theta\right)}\right]$
2(ii)	Max $x=4r$
2(iii)	$x\approx r\left[\left(1-\frac{\theta^2}{2}\right)+\left(9-\theta^2\right)^{\frac{1}{2}}\right]$ $\approx r\left[1-\frac{\theta^2}{2}+3\left(1-\frac{\theta^2}{9}\right)^{\frac{1}{2}}\right]$ $\approx r\left[1-\frac{\theta^2}{2}+3\left(1-\frac{1}{2}\left(\frac{\theta^2}{9}\right)\right)\right]$ $=r\left(4-\frac{2}{3}\theta^2\right)$

Qn	
3	$\frac{x^2-3x+4}{x+2} \ge 2x+1$ $\frac{x^2-3x+4-(2x+1)(x+2)}{x+2} \ge 0$ $\frac{-x^2-8x+2}{x+2} \ge 0$ $\frac{x^2+8x-2}{x+2} \le 0$ $\frac{(x+4)^2-18}{x+2} \le 0$ $\frac{\left(x+4-3\sqrt{2}\right)\left(x+4+3\sqrt{2}\right)}{x+2} \le 0$  $x \le -3\sqrt{2}-4$ or $-2 < x \le 3\sqrt{2}-4$ Replacing x by $-a^x$ $\frac{a^{2x}+3a^x+4}{-a^x+2} \ge -2a^x+1$ $\frac{a^{2x}+3a^x+4}{a^x-2} \le 2a^x-1$ Since $-a^x < 0$ $-a^x \le -3\sqrt{2}-4$ or $-2 < -a^x \le 3\sqrt{2}-4$ $x \ge \frac{\ln\left(3\sqrt{2}+4\right)}{\ln a}$ or $x < \frac{\ln 2}{\ln a}$

Qn	
4(i)	$\sin^4\theta = \frac{1}{4}\left(2\sin^2\theta\right)^2$ $= \frac{1}{4}(1-\cos 2\theta)^2$ $= \frac{1}{4}\left(1-2\cos 2\theta+\cos^2 2\theta\right)$ $= \frac{1}{4}\left(1-2\cos 2\theta+\frac{1+\cos 4\theta}{2}\right)$ $= \frac{1}{8}(3-4\cos 2\theta+\cos 4\theta)$
4(ii)	Let $x=2\cos\theta$. Thus $\frac{\mathrm{d}x}{\mathrm{d}\theta}=-2\sin\theta$. When $x=0$, $\theta=\frac{\pi}{2}$; when $x=2$, $\theta=0$. $\int_0^2 \left(4-x^2\right)^{\frac{3}{2}}\,\mathrm{d}x = \int_{\frac{\pi}{2}}^{0}\left(4-4\cos^2\theta\right)^{\frac{3}{2}}(-2\sin\theta)\,\mathrm{d}\theta$ $= 2\int_0^{\frac{\pi}{2}} \sin\theta\left(4\sin^2\theta\right)^{\frac{3}{2}}\,\mathrm{d}\theta$ $= 16\int_0^{\frac{\pi}{2}} \sin^4\theta\,\mathrm{d}\theta$ $= 2\int_0^{\frac{\pi}{2}} (3-4\cos 2\theta+\cos 4\theta)\,\mathrm{d}\theta$ $= 2\left[3\theta-2\sin 2\theta+\frac{1}{4}\sin 4\theta\right]_0^{\frac{\pi}{2}}$ $= 3\pi$

Qn	
5(i)	$\overrightarrow{OD} = \lambda b + (1-\lambda)a, \quad \lambda \in \mathbb{R}$ $l_{OD} : r = s(\lambda b + (1-\lambda)a), \qquad s \in \mathbb{R}$
5(ii)	$\overrightarrow{OE} = s(\lambda \mathbf{b} + (1-\lambda)\mathbf{a}),$ for some $s, \lambda \in \mathbb{R}$. $\overrightarrow{OE} = \frac{1}{2}(\mathbf{b} + 3\mathbf{a})$ $s(\lambda \mathbf{b} + (1-\lambda)\mathbf{a}) = \frac{1}{2}(\mathbf{b} + 3\mathbf{a})$ Since $\mathbf{a}$ and $\mathbf{b}$ are non-zero and non-parallel ($\lambda > 0$), $s\lambda = \frac{1}{2}$ $s(1-\lambda) = \frac{3}{2}$ Solving, $\lambda = \frac{1}{4}$. **Area of** $\Delta BED = \frac{1}{2}\left\|\overrightarrow{BE} \times \overrightarrow{BD}\right\|$ $= \frac{1}{2}\|(-\frac{1}{2}\mathbf{b} + \frac{3}{2}\mathbf{a}) \times (-\frac{3}{4}\mathbf{b} + \frac{3}{4}\mathbf{a})\|$ $= \frac{1}{2}\|\frac{3}{8}\mathbf{b}\times\mathbf{b} - \frac{3}{8}\mathbf{b}\times\mathbf{a} - \frac{9}{8}\mathbf{a}\times\mathbf{b} + \frac{9}{8}\mathbf{a}\times\mathbf{a}\|$ $= \frac{1}{2}\left\|\frac{3}{8}\mathbf{a}\times\mathbf{b} - \frac{9}{8}\mathbf{a}\times\mathbf{b}\right\|$ $= \frac{3}{8}\|\mathbf{a}\times\mathbf{b}\|$ $k = \frac{3}{8}$

Qn	
6(i)	$\mathrm{f}(x)=2x^2+4x+k=2(x+1)^2+k-2$ For f^{-1} to exist, f must be one-one. Largest value of $a=-1$
6(ii)	Let $y=\mathrm{f}(x)$ $y=2(x+1)^2+k-2$ $x=-1\pm\sqrt{\frac{1}{2}(y-k+2)}$ Since $x\in[-5,-1)$, $x<-1$ Hence $x=-1-\sqrt{\frac{1}{2}(y-k+2)}$ For $-5\le x<a$, $k-2<\mathrm{f}(x)\le 30+k$, $\mathrm{f}(x)=-1-\sqrt{\frac{1}{2}(x-k+2)}$, $D_{\mathrm{f}^{-1}}=(k-2,30+k]$
6(iii)	 Number of solutions to $\mathrm{ff}^{-1}(x)=\mathrm{f}^{-1}\mathrm{f}(x)$ is 0.

Qn	
7(i)	$V=\frac{1}{3}\pi x^2(3a-x) \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}t}=(2\pi ax-\pi x^2)\frac{\mathrm{d}x}{\mathrm{d}t}$ Since $\frac{\mathrm{d}V}{\mathrm{d}t}=-\pi k\sqrt{x}$, $(2\pi ax-\pi x^2)\frac{\mathrm{d}x}{\mathrm{d}t}=-\pi k\sqrt{x}$ $(2ax-x^2)\frac{\mathrm{d}x}{\mathrm{d}t}=-k\sqrt{x}$
7(ii)	$\int \frac{2ax-x^2}{\sqrt{x}}\,\mathrm{d}x=\int -k\,\mathrm{d}t$ $\Rightarrow \int 2a\sqrt{x}-x^{\frac{3}{2}}\,\mathrm{d}x=-kt+c$ $\Rightarrow \frac{4}{3}ax^{\frac{3}{2}}-\frac{2}{5}x^{\frac{5}{2}}=-kt+c$ $\Rightarrow t=\frac{1}{k}\left[c-\frac{4}{3}ax^{\frac{3}{2}}+\frac{2}{5}x^{\frac{5}{2}}\right]$
7(iii)	If the tank is initially full, $x=2a$, thus $c=\frac{4}{3}a(2a)^{\frac{3}{2}}-\frac{2}{5}(2a)^{\frac{5}{2}}=\frac{16}{15}a^2\sqrt{2a}$ Thus $T_1=\frac{c}{k}=\frac{16}{5k}a^2\sqrt{2a}$ If the tank is initially half full, $x=a$, thus $c=\frac{4}{3}a(a)^{\frac{3}{2}}-\frac{2}{5}(a)^{\frac{5}{2}}=\frac{14}{15}a^2\sqrt{a}$ Thus $T_2=\frac{c}{k}=\frac{14}{5k}a^2\sqrt{a}$ Thus $\frac{T_1}{T_2}=\frac{16a^2\sqrt{2a}}{14a^2\sqrt{a}}=\frac{8\sqrt{2}}{7}$ Required ratio is $8\sqrt{2}:7$

Qn	
8(i)	$y^2+xy=4 \qquad$ ___(1) Differentiate w.r.t. x, $2y\dfrac{dy}{dx}+y+x\dfrac{dy}{dx}=0$ $(2y+x)\dfrac{dy}{dx}=-y \qquad$ ___(2) $\dfrac{dy}{dx}=-\dfrac{y}{2y+x}$ When $\dfrac{dy}{dx}=-\dfrac{y}{2y+x}=-\dfrac{1}{5}$ $5y=2y+x$ $x=3y$ Substitute $x=3y$ in (1), $y^2+3y^2=4$ $y^2=1$ Hence $\quad y=1$ ($\because y>0$) Coordinates of the point are $(3,1)$
8(ii)	$y^2+z^2=10y \qquad$ ___(3) Differentiate (3) with respect to y, $2y+2z\dfrac{dz}{dy}=10$ $y+z\dfrac{dz}{dy}=5$ $\dfrac{dz}{dy}=\dfrac{5-y}{z}$ $\dfrac{dz}{dt}=\dfrac{dz}{dy}\cdot\dfrac{dy}{dx}\cdot\dfrac{dx}{dt}$ $=\dfrac{5-y}{z}\left(-\dfrac{y}{2y+x}\right)\dfrac{dx}{dt}$

Qn	
	At $x=3$, $y=1$ and $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{5}$ from (i). From (3), $1^2+z^2=10$ $z=3$ ($\because z>0$) Hence $\frac{\mathrm{d}z}{\mathrm{d}t}=\frac{5-1}{3}\left(-\frac{1}{5}\right)\frac{1}{2}=-\frac{2}{15}$ Alternatively, $y^2+z^2=10y$ ___(3) Differentiate (3) with respect to y, $2y+2z\frac{\mathrm{d}z}{\mathrm{d}y}=10$ $y+z\frac{\mathrm{d}z}{\mathrm{d}y}=5$ $\frac{\mathrm{d}z}{\mathrm{d}y}=\frac{5-y}{z}$ From (2), $\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{d}y}{\mathrm{d}x}\cdot\frac{\mathrm{d}x}{\mathrm{d}t}$ $=-\frac{y}{2y+x}\frac{\mathrm{d}x}{\mathrm{d}t}$ At $x=3$, $y=1$ and $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{5}$ from (i). $\frac{\mathrm{d}y}{\mathrm{d}t}=-\frac{1}{5}\cdot\frac{1}{2}=-\frac{1}{10}$ $\frac{\mathrm{d}z}{\mathrm{d}t}=\frac{\mathrm{d}z}{\mathrm{d}y}\cdot\frac{\mathrm{d}y}{\mathrm{d}t}$ $=\frac{5-y}{z}\frac{\mathrm{d}y}{\mathrm{d}t}$ From (3), $1^2+z^2=10$ $z=3$ ($\because z>0$) $\frac{\mathrm{d}z}{\mathrm{d}t}=\frac{5-1}{3}\left(-\frac{1}{10}\right)=-\frac{2}{15}$

Qn	
9(a)	$\lvert z\rvert - w^{*} = -3-\sqrt{2}\mathrm{i}$ $\Rightarrow w^{*} = \lvert z\rvert + 3+\sqrt{2}\mathrm{i}$ and $w = \lvert z\rvert + 3-\sqrt{2}\mathrm{i}$ Sub into $w^{*}+w+5z = 1+20\mathrm{i}$, Let $z = x+y\mathrm{i}$, where x and y are real. $2\sqrt{x^2+y^2}+5x+5y\mathrm{i} = -5+20\mathrm{i}$ Comparing real and imaginary components, $2\sqrt{x^2+y^2}+5x = -5$, $5y = 20 \Rightarrow y = 4$ $2\sqrt{x^2+16}+5x = -5$ $2\sqrt{x^2+16} = -5x-5$ $4\left(x^2+16\right) = 25x^2+50x+25$ $21x^2+50x-39 = 0$ $x = \frac{13}{21}$ or $x = -3$ (reject $x = \frac{13}{21} \because 2\sqrt{x^2+y^2}+5x = -5$) $z = -3+4\mathrm{i}$, $w = 8-\sqrt{2}\mathrm{i}$
9(b) (i)	$\mathrm{i}(8\mathrm{i})^3+(8-2\mathrm{i})(8\mathrm{i})^2+a(8\mathrm{i})+40 = 0$ $512-64(8-2\mathrm{i})+8a\mathrm{i}+40 = 0$ $a\mathrm{i} = -5-16\mathrm{i} \Rightarrow a = -16+5\mathrm{i}$
9(b) (ii)	$(z-8\mathrm{i})\left(Az^2+Bz+C\right) = 0$ Comparing coefficient for z^3, $A = \mathrm{i}$ Comparing coefficient for constant, $C = 5\mathrm{i}$ Comparing coefficient for z^2, $B-8\mathrm{i}A = 8-2\mathrm{i}$ $B = -2\mathrm{i}$

Qn	
	$(z-8\text{i})(\text{i}z^2-2\text{i}z+5\text{i})=0$ $(z-8\text{i})(z^2-2z+5)=0$ The other roots are $z=\dfrac{2\pm\sqrt{4-20}}{2}$ $=1\pm 2\text{i}$
9(b) **(iii)**	Replacing z with iz, $\text{i}z=8\text{i}$ or $\text{i}z=1\pm 2\text{i}$ $z=8$ $\quad$ $z=\pm 2-\text{i}$ Therefore 1 real root.

Qn	
10(i)	$\sum_{r=1}^{n} r^2(2r-1)=\sum_{r=1}^{n}(2r^3-r^2)$ $=\frac{2}{4}n^2(n+1)^2-\frac{n}{6}(n+1)(2n+1)$ $=\frac{1}{6}n(n+1)\left[3n(n+1)-(2n+1)\right]$ $=\frac{1}{6}n(n+1)(3n^2+n-1)$
10(ii)	$\sum_{r=1}^{n} r^2(r-1)=\sum_{r=1}^{n}(r^3-r^2)$ $=\frac{1}{4}n^2(n+1)^2-\frac{n}{6}(n+1)(2n+1)$ $=\frac{1}{12}n(n+1)\left[3n(n+1)-2(2n+1)\right]$ $=\frac{1}{12}n(n+1)(3n^2-n-2)$ $=\frac{1}{12}n(n+1)(3n+2)(n-1)$ $\sum_{r=2}^{n-1} r(r+1)^2=\sum_{k-1=2}^{k-1=n-1}(k-1)k^2$ $=\sum_{k=3}^{k=n}k^2(k-1)$ $=\sum_{k=1}^{k=n}k^2(k-1)-\sum_{k=1}^{k=2}k^2(k-1)$ $=\frac{1}{12}n(n+1)(n-1)(3n+2)-\frac{1}{12}(2)(3)(1)(8)$ $=\frac{1}{12}n(n+1)(n-1)(3n+2)-4$

Qn	
10(iii)	**(iii)** $4(25)-5(36)-\cdots-59(3600)$ $=4(25)+5(36)+\cdots+59(3600)-2\left[5(36)+7(64)\cdots+59(3600)\right]$ $=\sum_{r=5}^{60} r^2(r-1)-2\sum_{r=3}^{30}(2r)^2(2r-1)$ $=\sum_{r=1}^{60} r^2(r-1)-\sum_{r=1}^{4} r^2(r-1)-2\sum_{r=1}^{30}(2r)^2(2r-1)+2\sum_{r=1}^{2}(2r)^2(2r-1)$ $=\frac{1}{12}(60)(61)(59)(182)-\frac{1}{12}(4)(5)(3)(14)$ $-\frac{4}{3}(30)(31)(2729)+\frac{4}{3}(2)(3)(13)$ $=-108836$

Qn	
11(i)	Denote the position of the boy by X.

Let $\angle OXA = \alpha$ and $\angle OXB = \beta$. Then $\theta = \beta - \alpha$ and

$$\tan\theta = \tan(\beta - \alpha) = \frac{\tan\beta - \tan\alpha}{1 + \tan\beta\tan\alpha}$$

$$= \frac{\frac{b}{x} - \frac{a}{x}}{1 + \frac{b}{x}\cdot\frac{a}{x}}$$

$$= \frac{\left(\frac{b-a}{x}\right)x^2}{\left(1 + \frac{ab}{x^2}\right)x^2} = (b-a)\frac{x}{x^2 + ab}$$

Alternatively:

Applying sine rule,

$$\frac{\sin\theta}{b-a} = \frac{\sin B}{\sqrt{x^2 + a^2}} \quad \Rightarrow \quad \sin\theta = \frac{(b-a)}{\sqrt{x^2+a^2}}\sin B$$

$$= \frac{(b-a)}{\sqrt{x^2+a^2}}\cdot\frac{x}{\sqrt{x^2+b^2}}$$

Applying cosine rule,

$$(b-a)^2 = (x^2 + a^2) + (x^2 + b^2) - 2\sqrt{x^2+a^2}\sqrt{x^2+b^2}\cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{(x^2+a^2) + (x^2+b^2) - (b-a)^2}{2\sqrt{x^2+a^2}\sqrt{x^2+b^2}}$$

$$= \frac{x^2 + ab}{\sqrt{x^2+a^2}\sqrt{x^2+b^2}}$$

Hence $\tan\theta = \dfrac{(b-a)}{\sqrt{x^2+a^2}}\cdot\dfrac{x}{\sqrt{x^2+b^2}} \Big/ \dfrac{x^2+ab}{\sqrt{x^2+a^2}\sqrt{x^2+b^2}}$

$$= (b-a)\frac{x}{x^2+ab}$$

Qn	
(ii)	Differentiate $\tan\theta = \dfrac{5x}{x^2+300}$ with respect to x: $\sec^2\theta \dfrac{\mathrm{d}\theta}{\mathrm{d}x} = \dfrac{5\left(x^2+300\right)-5x\cdot 2x}{\left(x^2+300\right)^2}$ $= \dfrac{5\left(-x^2+300\right)}{\left(x^2+300\right)^2}$ $\dfrac{\mathrm{d}\theta}{\mathrm{d}x} = 0 \Rightarrow x^2 = 300$ $x = \sqrt{300}$ or 17.3 (3s.f.) Alternatively, Differentiate $\tan\theta\left(x^2+300\right) = 5x$ with respect to x: $\sec^2\theta \dfrac{\mathrm{d}\theta}{\mathrm{d}x}\left(x^2+300\right) + \tan\theta(2x) = 5$ $\dfrac{\mathrm{d}\theta}{\mathrm{d}x} = 0 \Rightarrow \tan\theta(2x) = 5$ Substitute $\tan\theta = \dfrac{5}{2x}$ into $\tan\theta = \dfrac{5x}{x^2+300}$: $\dfrac{5}{2x} = \dfrac{5x}{x^2+300}$ $x^2+300 = 2x^2$ $x^2 = 300$ $x = \sqrt{300}$ $\dfrac{\mathrm{d}\theta}{\mathrm{d}x} = \dfrac{5\left(-x^2+300\right)}{\left(x^2+300\right)^2\sec^2\theta}$.

Qn	

Since $\left(x^2+300\right)^2 \sec^2\theta > 0$ for any x and θ, it suffices to test the sign of $-x^2+300$.

x	$\left(\sqrt{300}\right)^-$	$\sqrt{300}$	$\left(\sqrt{300}\right)^+$
$\dfrac{d\theta}{dx}=\dfrac{5\left(-x^2+300\right)}{\left(x^2+300\right)^2\sec^2\theta}$	>0	=0	<0

Hence θ is maximum

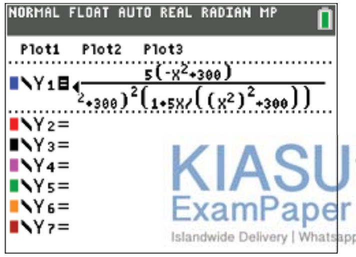
Alternatively, apply second derivative test:

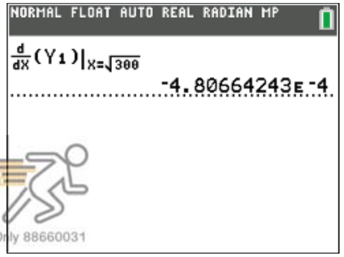
Using GC:

$$\frac{d\theta}{dx}=\frac{5\left(-x^2+300\right)}{\left(x^2+300\right)^2\sec^2\theta}$$

$$=\frac{5\left(-x^2+300\right)}{\left(x^2+300\right)^2\left(1+\tan^2\theta\right)}$$

$$=\frac{5\left(-x^2+300\right)}{\left(x^2+300\right)^2\left(1+\frac{5x}{x^2+300}\right)}$$





$$\left.\frac{d^2\theta}{dx^2}\right|_{x=\sqrt{300}}=-4.81\times10^{-4}<0.$$ Hence θ is maximum.

Or:

Qn	

Differentiate $\sec^2\theta\dfrac{\mathrm{d}\theta}{\mathrm{d}x}=\dfrac{5\left(-x^2+300\right)}{\left(x^2+300\right)^2}$ with respect to *x*:

$$2\sec\theta\left(\sec\theta\tan\theta\frac{\mathrm{d}\theta}{\mathrm{d}x}\right)\frac{\mathrm{d}\theta}{\mathrm{d}x}+\sec^2\theta\frac{\mathrm{d}^2\theta}{\mathrm{d}x^2}$$

$$=\frac{5(-2x)\left(x^2+300\right)^2-5\left(-x^2+300\right)\cdot 2\cdot 2x\left(x^2+300\right)}{\left(x^2+300\right)^4}$$

$$=\frac{-10x\left(x^2+300\right)\left[\left(x^2+300\right)+2\left(-x^2+300\right)\right]}{\left(x^2+300\right)^4}$$

$$=\frac{-10x\left(x^2+300\right)\left[-x^2+900\right]}{\left(x^2+300\right)^4}$$

At $x=\sqrt{300}$, $\dfrac{\mathrm{d}\theta}{\mathrm{d}x}=0$, $-x^2+900=-300+900=600$

$\dfrac{-10x\left(x^2+300\right)(600)}{\left(x^2+300\right)^4}<0$, and $\sec^2\theta>0$,

Hence $\dfrac{\mathrm{d}^2\theta}{\mathrm{d}x^2}<0$ and θ is maximum.

(iii)

Since $b=2a$, $\tan\theta=(2a-a)\dfrac{x}{x^2+a(2a)}$

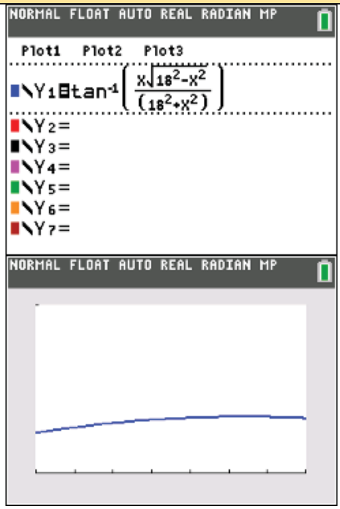
$$\tan\theta=\frac{ax}{x^2+2a^2}$$

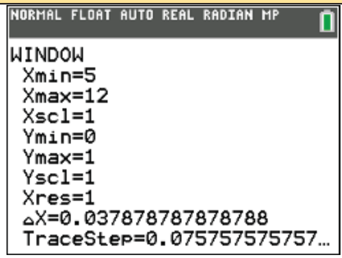
$$x^2+a^2=18^2$$
$$x^2=18^2-a^2$$

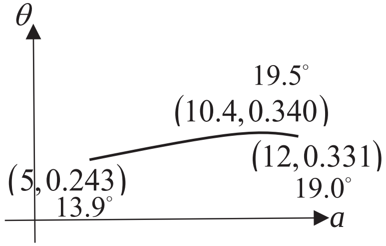
Hence $$\tan\theta=\frac{a\sqrt{18^2-a^2}}{18^2-a^2+2a^2}$$

$$\theta=\tan^{-1}\left(\frac{a\sqrt{18^2-a^2}}{18^2+a^2}\right)$$

Qn







Largest possible $\theta = 0.340$ rad (3 s.f.) or $\theta = 19.5^\circ$ (1 d.p.)

(iv)

Differentiate $h = -\left(\frac{1}{10}k+2\right)^2 + 6$ with respect to k:

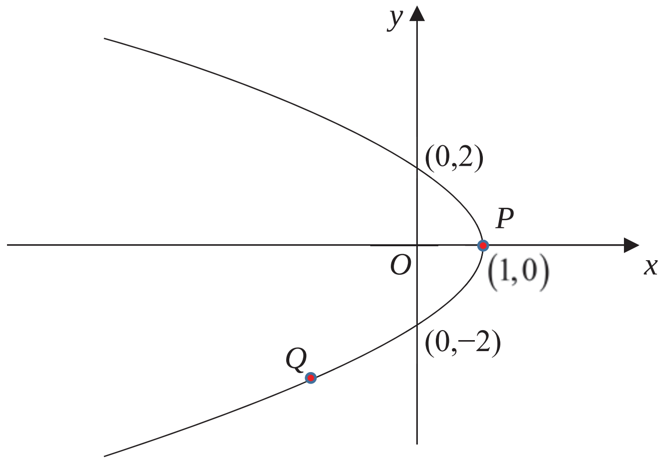
$$\frac{dh}{dk} = -\frac{2}{10}\left(\frac{1}{10}k+2\right)$$

At the instant when the ball crosses the goal line, $k = 0$

$$\frac{dh}{dk} = -\frac{2}{5}$$

$$\tan\phi = -\frac{2}{5}$$

$\phi = -0.381$ (3 s.f.) or $\theta \approx -21.8^\circ$ (1 d.p.)

Qn	
12(i)	
12(ii)	$y^2 = 4(1-x)$ Differentiate wrt x: $2y\dfrac{\mathrm{d}y}{\mathrm{d}x} = -4$ $\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{2}{y}$
12(iii)	At P, $x=1$, $y=0$; at Q, $x=-3$, $y=-4$. Thus equation of line PQ is $\dfrac{y}{x-1} = \dfrac{0-(-4)}{1-(-3)} \quad \Rightarrow \quad y = x-1$

<table>
<tr><th>Qn</th><th></th></tr>
<tr><td>12(iv)</td><td>

Along arc QP, $y=-2\sqrt{1-x}$.

$$W_C=\int_{-3}^{1}\left(x^2+xy^2\cdot\left(\frac{-2}{y}\right)\right)\mathrm{d}x$$
$$=\int_{-3}^{1}\left(x^2-2xy\right)\mathrm{d}x$$
$$=\int_{-3}^{1}\left(x^2+4x(1-x)^{\frac{1}{2}}\right)\mathrm{d}x$$

<table>
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Method 1

$$W_C=\left[\frac{x^3}{3}\right]_{-3}^{1}-\left[\frac{8}{3}x(1-x)^{\frac{3}{2}}\right]_{-3}^{1}+\int_{-3}^{1}\frac{8}{3}(1-x)^{\frac{3}{2}}\,\mathrm{d}x$$
$$=\frac{28}{3}-64-\left[\frac{16}{15}(1-x)^{\frac{5}{2}}\right]_{-3}^{1}$$
$$=-\frac{164}{3}+\frac{512}{15}=-\frac{308}{15}$$

</td><td>

Method 2

$$W_C=\int_{-3}^{1}\left(x^2+4x(1-x)^{\frac{1}{2}}\right)\mathrm{d}x$$
$$=\int_{-3}^{1}\left(x^2-4(1-x)(1-x)^{\frac{1}{2}}+4(1-x)^{\frac{1}{2}}\right)\mathrm{d}x$$
$$=\left[\frac{x^3}{3}+\frac{8}{5}(1-x)^{\frac{5}{2}}-\frac{8}{3}(1-x)^{\frac{3}{2}}\right]_{-3}^{1}$$
$$=\frac{28}{3}-\frac{256}{5}+\frac{64}{3}=-\frac{308}{15}$$

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<tr><td>12(v)</td><td>

$$W_L=\int_{-3}^{1}\left(x^2+xy^2\right)\mathrm{d}x=\int_{-3}^{1}\left(x^2+x(x-1)^2\right)\mathrm{d}x$$
$$=-33.33$$

</td></tr>
<tr><td>12(vi)</td><td>Since the work done for the two paths are different, the force field F is not conservative.</td></tr>
</table>

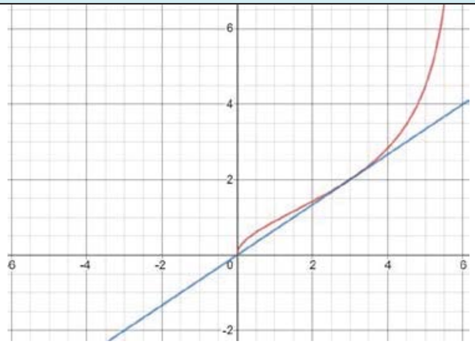
Qn	
1(i)	$\frac{1}{(n-1)!}-\frac{3}{n!}+\frac{2}{(n+1)!}=\frac{n(n+1)-3(n+1)+2}{(n+1)!}$ $=\frac{n^2+n-3n-3+2}{(n+1)!}=\frac{n^2-2n-1}{(n+1)!}$ Hence $A=1, B=-2, C=-1$
1(ii)	$\sum_{n=1}^{N}\frac{n^2-2n-1}{5(n+1)!}=\frac{1}{5}\sum_{n=1}^{N}\frac{n^2-2n-1}{(n+1)!}=\frac{1}{5}\sum_{n=1}^{N}\left[\frac{1}{(n-1)!}-\frac{3}{n!}+\frac{2}{(n+1)!}\right]$ $=\frac{1}{5}\left[\begin{array}{c}\frac{1}{0!}-\frac{3}{1!}+\cancel{\frac{2}{2!}} \\ +\frac{1}{1!}-\cancel{\frac{3}{2!}}+\cancel{\frac{2}{3!}} \\ +\cancel{\frac{1}{2!}}-\cancel{\frac{3}{3!}}+\cancel{\frac{2}{4!}} \\ +\vdots \\ +\cancel{\frac{1}{(N-3)!}}-\cancel{\frac{3}{(N-2)!}}+\cancel{\frac{2}{(N-1)!}} \\ +\cancel{\frac{1}{(N-2)!}}-\cancel{\frac{3}{(N-1)!}}+\frac{2}{N!} \\ +\cancel{\frac{1}{(N-1)!}}-\frac{3}{N!}+\frac{2}{(N+1)!}\end{array}\right]$ $=\frac{1}{5}\left[\frac{1}{0!}-\frac{3}{1!}+\frac{1}{1!}+\frac{2}{N!}+\frac{2}{(N+1)!}-\frac{3}{N!}\right]=\frac{1}{5}\left(\frac{2}{(N+1)!}-\frac{1}{N!}-1\right)$
1(iii)	$\sum_{n=1}^{\infty}\frac{n^2-2n-1}{5(n+1)!}=\lim_{N\to\infty}\sum_{n=1}^{N}\frac{n^2-2n-1}{5(n+1)!}=\lim_{N\to\infty}\left[\frac{1}{5}\left(\frac{2}{(N+1)!}-\frac{1}{N!}-1\right)\right]$ Since $\frac{1}{(N+1)!}\to 0$ & $\frac{1}{N!}\to 0$ when $N\to\infty$, $\sum_{n=1}^{\infty}\frac{n^2-2n-1}{5(n+1)!}$ converges to $-\frac{1}{5}$

Qn	
2(i)	$x = 6t^2 \qquad y = \dfrac{2t}{\sqrt{1-t^2}}$ $\dfrac{\mathrm{d}x}{\mathrm{d}t} = 12t \qquad \dfrac{\mathrm{d}y}{\mathrm{d}t} = \dfrac{2\sqrt{1-t^2} - 2t\left(\frac{-2t}{2\sqrt{1-t^2}}\right)}{1-t^2}$ $= \dfrac{2\left(1-t^2\right)+2t^2}{\left(1-t^2\right)\sqrt{1-t^2}}$ $= \dfrac{2}{\left(1-t^2\right)^{3/2}}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{6t\left(1-t^2\right)^{3/2}}$ The tangent to the curve C has equation $y = \dfrac{1}{6t\left(1-t^2\right)^{3/2}}x$ for some t. $\dfrac{2t}{\sqrt{1-t^2}} = \dfrac{1}{6t\left(1-t^2\right)^{3/2}} \cdot 6t^2 \quad (t \neq 0)$ $2\left(1-t^2\right) = 1$ $t^2 = \dfrac{1}{2}$ $t = \dfrac{1}{\sqrt{2}}$ since $0 < t < 1$ Hence the tangent line has equation $y = \dfrac{1}{6\frac{1}{\sqrt{2}}\left(1-\frac{1}{2}\right)^{3/2}}x$ $y = \dfrac{2}{3}x$ Alternative method:

Qn	
	Cartesian equation of curve C: Sub $t=\sqrt{\frac{x}{6}}$ into $y=\frac{2t}{\sqrt{1-t^2}}$ to get $y=\frac{2\sqrt{\frac{x}{6}}}{\sqrt{1-\frac{x}{6}}}=\frac{2\sqrt{x}}{\sqrt{6-x}}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2\cdot\frac{1}{2\sqrt{x}}\cdot\sqrt{6-x}-2\sqrt{x}\cdot\frac{-1}{2\sqrt{6-x}}}{6-x}$ $=\frac{6}{(6-x)^{3/2}\sqrt{x}}$ The required tangent line passes through the point $\left(6t^2,\frac{2t}{\sqrt{1-t^2}}\right)$ for some x. $y=\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=6t^2}x$ $\frac{2t}{\sqrt{1-t^2}}=\frac{6}{(6-6t^2)^{3/2}\sqrt{6t^2}}\cdot 6t^2 \quad (t\neq 0)$ $2=\frac{1}{(1-t^2)}$ $2(1-t^2)=1$ $t^2=\frac{1}{2}$ $t=\frac{1}{\sqrt{2}}$ since $0\le t<1$ Hence the tangent line has equation $y=\frac{1}{6\frac{1}{\sqrt{2}}\left(1-\frac{1}{2}\right)^{3/2}}x$ $y=\frac{2}{3}x$

Qn

2(ii)



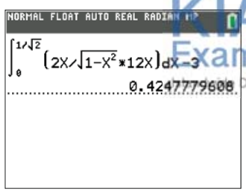
The point of intersection, A, of the tangent line and the curve corresponds to $t=\frac{1}{\sqrt{2}}$.

Coordinates of point A are $(3,2)$.

$$\text{Area of } R=\int_0^3 y\ \mathrm{d}x-\frac{1}{2}(3)(2)$$
$$=\int_0^{\frac{1}{\sqrt{2}}}\frac{2t}{\sqrt{1-t^2}}\cdot 12t\mathrm{d}t-\frac{1}{2}(3)(2)$$
$$=0.425 \text{ (3 s.f.)}$$

Alternatively,

$$\text{Area of } R=\int_0^3 y-\frac{2}{3}x\ \mathrm{d}x=\int_0^{\frac{1}{\sqrt{2}}}\frac{2t}{\sqrt{1-t^2}}\cdot 12t\mathrm{d}t-\int_0^3\frac{2}{3}x\ \mathrm{d}x$$
$$=0.425 \text{ (3 s.f.)}$$



Qn	
2(iii)	Sub $t=\sqrt{\frac{x}{6}}$ into $y=\frac{2t}{\sqrt{1-t^2}}$ to get $y=\frac{2\sqrt{\frac{x}{6}}}{\sqrt{1-\frac{x}{6}}}=\frac{2\sqrt{x}}{\sqrt{6-x}}$
2(iv)	Volume, $V=\pi\int_0^3 y^2\ \mathrm{d}x-\frac{1}{3}\pi\left(2^2\right)(3)$ $=\pi\int_0^3 \frac{4x}{6-x}\ \mathrm{d}x-4\pi$ $=\pi\int_0^3 -4+\frac{24}{6-x}\ \mathrm{d}x-4\pi$ $=\pi\left[-4x-24\ln\lvert 6-x\rvert\right]_0^3-4\pi$ $=\pi\left[-12-24\ln 3+24\ln 6\right]-4\pi$ $=\pi\left[24\ln 2\right]-16\pi$ $=\left(24\ln 2-16\right)\pi$ <u>Alternatively</u>, Volume, $V=\pi\int_0^3 \left(\frac{2\sqrt{x}}{\sqrt{6-x}}\right)^2-\left(\frac{2}{3}x\right)^2\ \mathrm{d}x$ $=\pi\int_0^3 \frac{4x}{6-x}-\frac{4}{9}x^2\ \mathrm{d}x$ $=\pi\int_0^3 -4+\frac{24}{6-x}-\frac{4}{9}x^2\ \mathrm{d}x$ $=\pi\left[-4x-24\ln\lvert 6-x\rvert-\frac{4}{27}x^3\right]_0^3$ $=\pi\left[\left(-12-24\ln 3-4\right)+24\ln 6\right]$ $=\left(24\ln 2-16\right)\pi$

Qn	
3(i)	Let S_n be the total distance travelled by the ball just before the n-th bounce. Thus $S_n = 10 + 2(10e) + 2(10e^2) + \cdots + 2(10e^{n-1})$ $\quad = 20 + 20e + 20e^2 + \cdots + 20e^{n-1} - 10$ $\quad = \dfrac{20(1-e^n)}{1-e} - 10$ $\quad = \dfrac{10(1+e-2e^n)}{1-e}$
3(ii)	Let d_k be the maximum height of the ball after the k-th bounce. Thus $d_k = 10e^k$. Hence $t_k = 0.90305\sqrt{d_k}$. Thus for $k \in \mathbb{Z}^+$, $\dfrac{t_{k+1}}{t_k} = \dfrac{0.90305\sqrt{d_{k+1}}}{0.90305\sqrt{d_k}}$ $\quad = \dfrac{\sqrt{10e^{k+1}}}{\sqrt{10e^k}} = \sqrt{e}$ Hence t_n is a geometric sequence with common ratio $\sqrt{e}$.
3(iii)	As $n \to \infty$, the ball will come to rest. Thus total distance travelled is $S = \lim\limits_{n\to\infty}\left(\dfrac{10(1+e-2e^n)}{1-e}\right)$ $\quad = \dfrac{10(1+e)}{1-e}$ Total time taken $= 0.5(0.90305)\sqrt{10} + \sum\limits_{n=1}^{\infty} t_n$ $\quad = 1.4278 + \dfrac{0.90305\sqrt{10e}}{1-\sqrt{e}}$ $\quad = 1.43 + \dfrac{2.86\sqrt{e}}{1-\sqrt{e}}$

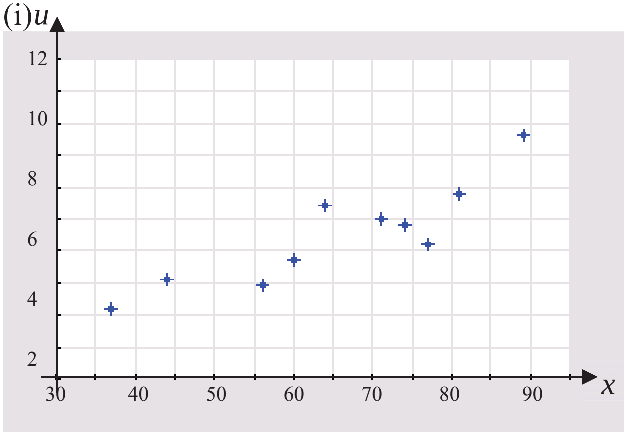
Qn	
4(i)	$\mathbf{n}_{\pi_1} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ a \end{pmatrix} = \begin{pmatrix} 2 \\ -(2a+1) \\ 4 \end{pmatrix}$ $\mathbf{d}_l \cdot \mathbf{n}_{\pi_1} = \begin{pmatrix} 4a \\ 4 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -2a-1 \\ 4 \end{pmatrix}$ $= 8a - 8a - 4 + 4 = 0$ Since $\mathbf{d}_l \perp \mathbf{n}_{\pi_1}$, then l is parallel to π_1
4(ii)	Equation of π_1: $\mathbf{r} \cdot \begin{pmatrix} 2 \\ -2a-1 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -2a-1 \\ 4 \end{pmatrix} = 10$ Since l is parallel to π_1, we want l to lie inside π_1. $\begin{pmatrix} 3 \\ 0 \\ a \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -2a-1 \\ 4 \end{pmatrix} = 10$ $6 + 4a = 10 \quad \Rightarrow \quad a = 1$
4(iii)	Since B lies on the line, required vector is the vector $\overrightarrow{FB}$, where F is the foot of perpendicular from A to π_1. $\overrightarrow{OF} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix} + k \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$ for some $k \in \mathbb{R}$. $\left[\begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix} + k \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \right] \cdot \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} = 10$ $-19 + 29k = 10 \quad \Rightarrow \quad k = 1$

Qn	
	$\overrightarrow{OF}=\begin{pmatrix}1\\0\\2\end{pmatrix}$ $\overrightarrow{FB}=\begin{pmatrix}3\\0\\1\end{pmatrix}-\begin{pmatrix}1\\0\\2\end{pmatrix}=\begin{pmatrix}2\\0\\-1\end{pmatrix}$
4(iv)	Let π_2 be the required plane. Point C is the reflection of A in π_1. $\overrightarrow{OC}=\begin{pmatrix}-1\\3\\-2\end{pmatrix}+2\begin{pmatrix}2\\-3\\4\end{pmatrix}=\begin{pmatrix}3\\-3\\6\end{pmatrix}$, $\overrightarrow{BC}=\begin{pmatrix}3\\-3\\6\end{pmatrix}-\begin{pmatrix}3\\0\\1\end{pmatrix}=\begin{pmatrix}0\\-3\\5\end{pmatrix}$ $\mathbf{n}_{\pi_2}=\begin{pmatrix}4\\4\\1\end{pmatrix}\times\begin{pmatrix}0\\-3\\5\end{pmatrix}=\begin{pmatrix}23\\-20\\-12\end{pmatrix}$ $\mathbf{r}\cdot\begin{pmatrix}23\\-20\\-12\end{pmatrix}=\begin{pmatrix}3\\0\\1\end{pmatrix}\cdot\begin{pmatrix}23\\-20\\-12\end{pmatrix}=57$. Thus $23x-20y-12z=57$
4(v)	Maximum value of $\angle ADC=2\times\angle CDF$ $=2\times\cos^{-1}\dfrac{\begin{pmatrix}2\\-3\\4\end{pmatrix}\cdot\begin{pmatrix}23\\-20\\-12\end{pmatrix}}{\left\|\begin{pmatrix}2\\-3\\4\end{pmatrix}\right\|\left\|\begin{pmatrix}23\\-20\\-12\end{pmatrix}\right\|}$ $=2\times\cos^{-1}\dfrac{58}{\sqrt{29}\sqrt{1073}}\approx 2(70.804)$ $=141.6°$ (1dp)

Qn	
5(i)	Number of ways = $\dfrac{9!}{2!2!3!} = 15120$
5(ii)	Number of ways = $\dfrac{7!}{3!} = 840$
5(iii)	Number of ways $= \dfrac{5!}{2!} \cdot {}^{6}C_{3}$ $= 1200$
5(iv)	Let the event D be such that the D's are together, the event E be such that the E's are together and S be such that the S's are together. $n(D \cup E \cup S) = n(D) + n(E) + n(S) - n(D \cap E)$ $- n(E \cap S) - n(D \cap S) + n(D \cap E \cap S)$ $= \dfrac{8!}{2!3!} + \dfrac{7!}{2!2!} + \dfrac{8!}{2!3!} - \dfrac{6!}{2!} - \dfrac{6!}{2!} - \dfrac{7!}{3!} + 5!$ $= 6540$ Number of ways $= n(D' \cap E' \cap S')$ $= n(S) - n(D \cup E \cup S)$ $= 15120 - 6540$ $= 8580$

Qn	
6(i)	Let X denotes the number of 1-year old flares that fail to fire successfully, out of the 100, $X \sim \mathrm{B}(100,\ 0.005)$ $\mathrm{P}(X \le 2) = 0.985897 \approx 0.986$
6(ii)	Let Y denotes the number of boxes with a hundred 1-year old flares with at most 2 that fail to fire, out of 50 boxes, ie $Y \sim \mathrm{B}(50,\ 0.985897)$ $\mathrm{P}(Y \le 48) = 0.156856 \approx 0.157$
6(iii)	Let T denotes the number of 10-year old flares that fire successfully, out of the 6, $T \sim \mathrm{B}(6,\ 0.75)$ (a) Required prob = $(1-0.970)\times \mathrm{P}(T \ge 4)$ $= 0.03\times\left(1-P(T \le 3)\right)$ $= 0.0249$ (b) P(at least 4 of the 7 flares fire successfully) $= 0.024917 + 0.970\times \mathrm{P}(T \ge 3)$ $= 0.024917 + 0.970\times\left(1-\mathrm{P}(T \le 2)\right)$ $= 0.958$

Qn	
7(i)	Let X be the rv denoting the amount of time taken by a cashier to deal with a randomly chosen customer, ie $X \sim \mathrm{N}(150,\ 45^2)$. $\mathrm{P}(X > 180) = 0.25249 \approx 0.252$
7(ii)	Assume that the time taken to deal with each customer is independent of the other, ie $X_1 + X_2 \sim \mathrm{N}(2 \times 150, 2 \times 45^2)$ $\mathrm{P}(X_1 + X_2 < 200) = 0.058051 \approx 0.0581$
7(iii)	Let Y be the rv denoting the amount of time taken by a the second cashier to deal with a randomly chosen customer, ie $Y \sim \mathrm{N}(150,\ 45^2)$. $X_1 + X_2 + X_3 + X_4 \sim \mathrm{N}(4 \times 150, 4 \times 45^2)$ and $Y_1 + Y_2 + Y_3 \sim \mathrm{N}(3 \times 150, 3 \times 45^2)$ $\mathrm{P}(X_1 + X_2 + X_3 + X_4 < Y_1 + Y_2 + Y_3) = \mathrm{P}(X_1 + X_2 + X_3 + X_4 - (Y_1 + Y_2 + Y_3) < 0)$ Using $X_1 + X_2 + X_3 + X_4 - (Y_1 + Y_2 + Y_3) \sim N(150, 7 \times 45^2)$ $\mathrm{P}(X_1 + X_2 + X_3 + X_4 - (Y_1 + Y_2 + Y_3) < 0) = 0.10386 \approx 0.104$ KIASU ExamPaper Islandwide Delivery | Whatsapp Only 88660031

Qn	
8(i)	
8(ii)	Using GC, $r = 0.884$ for the model $u = ax + b$ $u = ae^{bx} \Rightarrow \ln u = bx + \ln a$ Using GC, $r =$ 0.906 for the model $u = ae^{bx}$ Since the value of r is closer to 1 for the 2nd model, $u = ae^{bx}$ is a better model. $\ln u = 0.013633x + 0.94964$ $u = e^{0.013633x+0.94964}$ $u = 2.58e^{0.0136x} = 2.6e^{0.014x}$
8(iii)	$7 = 2.58e^{0.0136x} \Rightarrow x = \dfrac{\ln\left(\dfrac{7}{2.58}\right)}{0.0136} = 73.391 \approx 73$ A patient with urea serum is 7 mmol per litre is approximately 73 years old. Since $r = 0.906$ is close to 1 and 7 is within the data range of urea serum, estimate is reliable.
8(iv) (a)	The product moment correlation coefficient in part **(ii)** will not be changed if the units for the urea serum is given in mmol per decilitre.
8(iv) (b)	$u = 0.258e^{0.0136x}$

Qn	
9(i)	$\mathrm{P}(X=2)=\frac{18}{18}\frac{2}{17}\frac{15}{16}\frac{3!}{2!}$ $=\frac{45}{136}$ $\mathrm{P}(X=0)=\frac{18}{18}\frac{15}{17}\frac{12}{16}$ $=\frac{45}{68}$ $\mathrm{P}(X=3)=\frac{18}{18}\frac{2}{17}\frac{1}{16}$ $=\frac{1}{136}$
9(ii)	$\mathrm{E}(X)=\frac{93}{136}$ $\mathrm{E}(X^2)=0\times\frac{45}{68}+2^2\times\frac{45}{136}+3^2\times\frac{1}{136}=\frac{189}{136}$ $\mathrm{Var}(X)=\frac{189}{136}-\left(\frac{93}{136}\right)^2$ ≈ 0.922
9(iii)	Since $n=40$ is large, by Central Limit Theorem, $\overline{X}\sim N\left(\frac{93}{136},\frac{0.922}{40}\right)$ approximately $P(\overline{X}>1)=0.0186$

Qn	
9(iv)	Expected winnings $= -\frac{45}{68}a + \frac{45}{136}(a+10) + \frac{1}{136}(a+10)$ $-\frac{11}{34}a + \frac{115}{34} > 0$ $a < \frac{115}{11}$ $a < 10.\dot{4}\dot{5}$ The possible amounts will be $1 \le a \le 10$ and $a \in \mathbb{Z}$.

Qn	
10(i)	Let X be the thickness of the coating on a randomly chosen computer device. Let μ be the mean thickness of the coating of a computer device. Assume that the standard deviation of the coating of a computer device remains unchanged. To test : $H_0 : \mu = 100$ $H_1 : \mu \neq 100$ Level of Significance: 5% Under H_0, since sample size $n = 50$ is large, by Central Limit Theorem, $Z = \dfrac{\overline{X} - 100}{10/\sqrt{50}} \sim N(0,1)$ approx. Reject H_0 if $p-value \leq 0.05$. Calculations: $\overline{x} = 103.4$ $p-value = 0.0162$ Conclusion: Since $p-value < 0.05$, we reject H_0 and conclude that there is significant evidence at 5% level of significance that the process is not in control.
10(ii)	Reject H_0 is $\lvert z_{calc} \rvert \geq 1.960$ For H_0 to be rejected, $\left\lvert \dfrac{\overline{x} - 100}{10/\sqrt{50}} \right\rvert \geq 1.95996$ $\Rightarrow \overline{x} \leq 100 - 1.95996\left(\dfrac{10}{\sqrt{50}}\right)$ or $\overline{x} \geq 100 + 1.95996\left(\dfrac{10}{\sqrt{50}}\right)$ $\Rightarrow \overline{x} \leq 97.228$ or $\overline{x} \geq 102.772$ Thus the required range of values of $\overline{x}$ is $0 < \overline{x} \leq 97.2$ or $\overline{x} \geq 102.8$.

Qn	
10(iii)	$\bar{y}=\frac{4164}{40}=104.1$ $\Sigma(y-100)=4164-4000=164$ $s^2=\frac{1}{39}\left[\Sigma(y-100)^2-\frac{(\Sigma(y-100))^2}{40}\right]$ $=\frac{1}{39}\left[9447-\frac{164^2}{40}\right]$ $=\frac{43873}{195}=224.9897$
10(iv)	The standard deviation may have changed due to the wear out of mechanical parts as well.
10(v)	To test : $H_0:\mu=100$ $H_1:\mu\neq 100$ Level of Significance: 4% Under H_0, since sample size $n=40$ is large, by Central Limit Theorem, $Z=\frac{\bar{Y}-100}{S/\sqrt{40}}\sim N(0,1)$ approx. Reject H_0 if $p-value\leq 0.04$. Calculations: $\bar{x}=104.1, s^2=224.9897$ $p-value=0.0839$ Conclusion: Since $p-value>0.04$, we do not reject H_0 and conclude that there is insignificant evidence at 4% level of significance that the process is not in control.