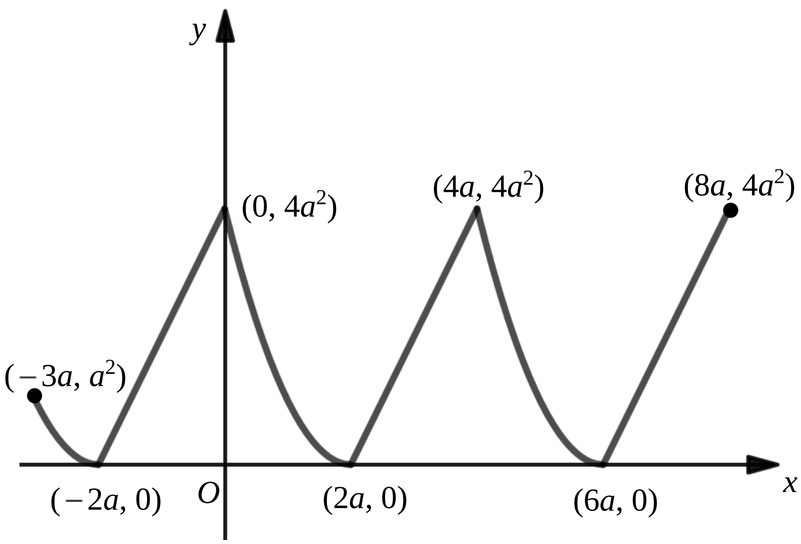
Yishun Innova Junior College ♦ Mathematics Department
2019 JC 2 Mathematics H2 9758
Prelim Examination P1
Solutions

Qn	Solution	Remarks
1(i)	$\sin\left(\frac{\pi}{4}-2x\right)=\sin\frac{\pi}{4}\cos 2x-\cos\frac{\pi}{4}\sin 2x------*$ $=\frac{1}{\sqrt{2}}(\cos 2x-\sin 2x)$ $=\frac{1}{\sqrt{2}}\left[\left(1-\frac{(2x)^2}{2}+...\right)-\left(2x-\frac{(2x)^3}{3!}+...\right)\right]$ $=\frac{1}{\sqrt{2}}\left(1-2x-2x^2+\frac{4x^3}{3}+...\right)$	You cannot substitute $\frac{\pi}{4}-2x$ into the standard expansion formula directly. In general, we can apply the standard expansions when x is replaced by g(x), provided g(0) = 0. For instance, $\mathrm{g}(x)=x+x^2$.
	<u>Alternative Method</u> Let $y=\sin\left(\frac{\pi}{4}-2x\right)$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-2\cos\left(\frac{\pi}{4}-2x\right)$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-4\sin\left(\frac{\pi}{4}-2x\right)$ $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=8\cos\left(\frac{\pi}{4}-2x\right)$ When x = 0, $y=\frac{1}{\sqrt{2}}$, $\frac{\mathrm{d}y}{\mathrm{d}x}=-\sqrt{2}$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-2\sqrt{2}$, $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=4\sqrt{2}$ $\sin\left(\frac{\pi}{4}-2x\right)=\frac{1}{\sqrt{2}}-\sqrt{2}x-\frac{2\sqrt{2}}{2}x^2+\frac{4\sqrt{2}}{3!}x^3+\ldots$ $=\frac{1}{\sqrt{2}}-\sqrt{2}x-\sqrt{2}x^2+\frac{2\sqrt{2}}{3}x^3+\ldots$ $=\frac{1}{\sqrt{2}}\left(1-2x-2x^2+\frac{4x^3}{3}+...\right)$	Be careful with the signs when you doing the higher derivatives.

(ii)	$(a+bx)^{-1} = a^{-1}\left(1+\frac{b}{a}x\right)^{-1}$ $= a^{-1}\left[1+(-1)\left(\frac{b}{a}x\right)+\frac{(-1)(-2)}{2}\left(\frac{b}{a}x\right)^2+\ldots\right]$ $=\frac{1}{a}\left(1-\frac{b}{a}x+\left(\frac{b}{a}\right)^2x^2+\ldots\right)$ $\therefore a=\sqrt{2}$ and $\frac{b}{a}=2 \Rightarrow b=2\sqrt{2}$ Third non-zero term: $\frac{1}{a}\left(\frac{b}{a}\right)^2 x^2 = \frac{1}{\sqrt{2}}(2)^2x^2 = \frac{4}{\sqrt{2}}x^2 = 2\sqrt{2}x^2$	Note that power is -1, not a positive integer so we need to use the series expansion of $(1+x)^n$ found in MF26 Don't forget to apply the power -1 to a after factorize a out. Third non-zero term and the coefficient of third non-zero term are different.
2(a)	$\lvert\mathbf{a}+\mathbf{b}\rvert=\lvert\mathbf{a}-\mathbf{b}\rvert$ $\lvert\mathbf{a}+\mathbf{b}\rvert^2=\lvert\mathbf{a}-\mathbf{b}\rvert^2$ $(\mathbf{a}+\mathbf{b})\bullet(\mathbf{a}+\mathbf{b})=(\mathbf{a}-\mathbf{b})\bullet(\mathbf{a}-\mathbf{b})$ $\lvert\mathbf{a}\rvert^2+2\mathbf{a}\bullet\mathbf{b}+\lvert\mathbf{b}\rvert^2=\lvert\mathbf{a}\rvert^2-2\mathbf{a}\bullet\mathbf{b}+\lvert\mathbf{b}\rvert^2$ $4\mathbf{a}\bullet\mathbf{b}=0$ $\mathbf{a}\bullet\mathbf{b}=0$ Hence **a** and **b** are perpendicular. <u>Alternative Method:</u> Since $\lvert\mathbf{a}+\mathbf{b}\rvert=\lvert\mathbf{a}-\mathbf{b}\rvert$, the diagonals of the parallelogram (with sides $\mathbf{a}$ and $\mathbf{b}$) are equal in length and thus the parallelogram must be a rectangle. Therefore, **a** and **b** are perpendicular.	There are two forms of vector product: dot (scalar) and cross (product). Hence expressions such as $\mathbf{a}^2$ will not make sense, as it will be ambiguous whether it means $\mathbf{a}\cdot\mathbf{a}$ or $\mathbf{a}\times\mathbf{a}$. However $\lvert\mathbf{a}\rvert^2$ is meaningful as $\lvert\mathbf{a}\rvert$ means the length of a vector, so it is a number. Next, as $\lvert\mathbf{a}+\mathbf{b}\rvert$ is the length of the vector $\mathbf{a}+\mathbf{b}$, so $\lvert\mathbf{a}+\mathbf{b}\rvert\neq\lvert\mathbf{a}\rvert+\lvert\mathbf{b}\rvert$, and therefore $\lvert\mathbf{a}+\mathbf{b}\rvert^2\neq\lvert\mathbf{a}\rvert^2+2\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert+\lvert\mathbf{b}\rvert^2$. It is wrong to say that $\lvert\mathbf{a}+\mathbf{b}\rvert=\lvert\mathbf{a}-\mathbf{b}\rvert$ implies $\mathbf{a}+\mathbf{b}=\mathbf{a}-\mathbf{b}$ or $\mathbf{a}+\mathbf{b}=-(\mathbf{a}-\mathbf{b})$. Take for example the vectors $\mathbf{i}$ and $\mathbf{j}$, they are obviously pointing in different directions, so neither $\mathbf{i}=\mathbf{j}$ nor $\mathbf{i}=-\mathbf{j}$, but $\lvert\mathbf{i}\rvert=\lvert\mathbf{j}\rvert=1$.

(b)	$\overrightarrow{OP} = \lambda \mathbf{c}$ $\overrightarrow{OM} = \dfrac{2\overrightarrow{OP} + 3\overrightarrow{OD}}{5} = \dfrac{2\lambda\mathbf{c} + 3\mathbf{d}}{5}$ **Area of triangle** $OPM = \dfrac{1}{2}\left\|\overrightarrow{OP} \times \overrightarrow{OM}\right\|$ $= \dfrac{1}{2}\left\|\lambda\mathbf{c} \times \left(\dfrac{2\lambda\mathbf{c}+3\mathbf{d}}{5}\right)\right\|$ $= \dfrac{1}{2}\left\|\lambda\mathbf{c} \times \dfrac{2\lambda\mathbf{c}}{5} + \lambda\mathbf{c} \times \dfrac{3\mathbf{d}}{5}\right\|$ $= \dfrac{1}{2}\left\|\lambda\mathbf{c} \times \dfrac{3\mathbf{d}}{5}\right\| = \dfrac{3}{10}\lambda\left\|\mathbf{c} \times \mathbf{d}\right\|$	Read carefully: *P* lies on *OC* produced. So *P* does not lie in between *O* and *C*. Ratio theorem is a very useful and quick way to get $\overrightarrow{OM}$. $\mathbf{c} \times \mathbf{c}$ is a vector product, so the result should be a vector. So $\mathbf{c} \times \mathbf{c} \neq 0$, it should be $\mathbf{0}$, the zero vector. Because of the definition of vector product, $\mathbf{c} \times \mathbf{c} \neq \|\mathbf{c}\|^2$, and $\mathbf{c} \times \mathbf{d} \neq \mathbf{d} \times \mathbf{c}$.
3(i)	 	-label all vertices and end points -Quadratic curve shape (part) for $0 \le x \le 2a$ and straight line segment for $2a \le x \le 4a$
(ii)	$\int_{-2a}^{8a} \mathrm{f}\left(\|x\|\right)\,\mathrm{d}x = 3\int_0^{2a}(x-2a)^2\,\mathrm{d}x + 2\left(\dfrac{1}{2}(2a)(4a^2)\right)$ $= 3\left[\dfrac{1}{3}(x-2a)^3\right]_0^{2a} + 8a^3$ $= 8a^3 + 8a^3$ $= 16a^3$	To draw graph of f(|*x*|), keep RHS and reflect in the y-axis. Note that $(x-2a)^2$ is only defined for $0 \le x \le 2a$ while $2ax - 4a^2$ is only defined for $2a \le x \le 4a$. It is **wrong** to take for instance: $\int_{4a}^{6a}(x-2a)^2\,\mathrm{d}x$ as the area of the region from $4a \le x \le 6a$.

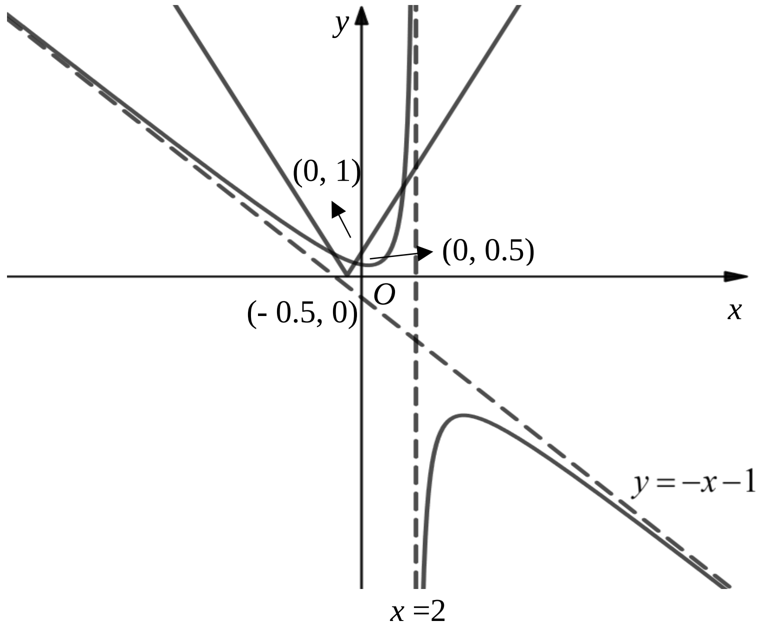
4(i)	Substitute $x=t^2$, $y=\frac{1}{\sqrt{t}}$ into $y=\frac{8}{x}$, $\frac{1}{\sqrt{t}}=\frac{8}{t^2}$ $t^{\frac{3}{2}}=8$ $t=4$ When $t=4$, $x=4^2=16$ $y=\frac{1}{\sqrt{4}}=\frac{1}{2}$ Coordinates of A is $\left(16,\frac{1}{2}\right)$.	Substitute the parametric equation of C into the Cartesian equation Avoid changing the parametric equation to Cartesian form
(ii)	$\frac{dx}{dt}=2t$, $\frac{dy}{dt}=-\frac{1}{2}t^{-\frac{3}{2}}$ $\frac{dy}{dx}=-\frac{1}{2}t^{-\frac{3}{2}}\times\frac{1}{2t}=-\frac{1}{4}t^{-\frac{5}{2}}$ Equation of tangent at point P on curve C, $y-\frac{1}{\sqrt{p}}=-\frac{1}{4}p^{-\frac{5}{2}}\left(x-p^2\right)$ At D , $y=0$ $\therefore 0-\frac{1}{\sqrt{p}}=-\frac{1}{4}p^{-\frac{5}{2}}\left(x-p^2\right)\Rightarrow x=5p^2$ At E , $x=0$ $\therefore y-\frac{1}{\sqrt{p}}=-\frac{1}{4}p^{-\frac{5}{2}}\left(0-p^2\right)\Rightarrow y=\frac{5}{4}p^{-\frac{1}{2}}$ Coordinates of F : $\left(\frac{5p^2}{2},\frac{\frac{5}{4}p^{-\frac{1}{2}}}{2}\right)\Rightarrow\left(\frac{5p^2}{2},\frac{5}{8\sqrt{p}}\right)$ $y=\frac{5}{8\sqrt{p}}$ $\Rightarrow\sqrt{p}=\frac{5}{8y}$, substitute into $x=\frac{5p^2}{2}$ $x=\frac{5}{2}\left(\frac{5}{8y}\right)^4=\frac{3125}{8192y^4}$ Cartesian equation of the curve traced by F is $x=\frac{3125}{8192y^4}$.	Take note that we are finding the gradient of tangent $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$ Write down as header what you are trying to find, for example equation of tangent at point P. When $t=p$, Gradient of tangent at point P is $-\frac{1}{4}p^{-\frac{5}{2}}$ instead of $-\frac{1}{4}t^{-\frac{5}{2}}$ Find the midpoint F which follows the formula $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ From F, let $x=\frac{5p^2}{2}$, $y=\frac{5}{8\sqrt{p}}$ which is in the parametric form. Convert to Cartesian form so that we can trace how the path that point F moves as p varies.

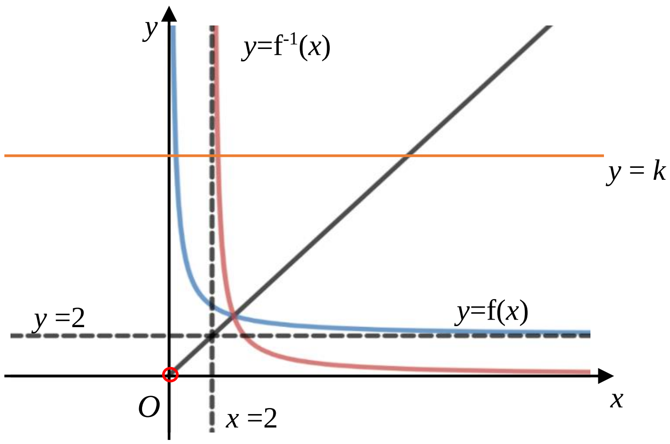
5(i)	$2xy+(1+y)^2=x$ Differentiating wrt x, $2y+2x\frac{\mathrm{d}y}{\mathrm{d}x}+2(1+y)\frac{\mathrm{d}y}{\mathrm{d}x}=1$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1-2y}{2(x+y+1)}$ When the tangent is parallel to the y-axis, $\frac{\mathrm{d}y}{\mathrm{d}x}$ is undefined. $\therefore 2(x+y+1)=0$ $y=-1-x$, substitute this into the equation of the curve, $2x(-1-x)+(1-1-x)^2=x$ $x^2+3x=0$ $x=0$ or $x=-3$	Do not confuse "tangent is parallel to y-axis", with x-axis: "parallel with y-axis": $\frac{\mathrm{d}y}{\mathrm{d}x}$ is undefined. "parallel with x-axis": $\frac{\mathrm{d}y}{\mathrm{d}x}=0$. Vertical lines in the Cartesian grid are of the form "x=…." instead of "y=…"
(ii)	Gradient of normal at $A=\frac{-1}{\frac{1-2(0)}{2(1+0+1)}}=-4$ Equation of normal at A: $y-0=-4(x-1)$ At B, $x=0\Rightarrow y=4$ => Coordinates of B are $(0,4)$ Area of triangle $OAB=\frac{1}{2}\times4\times1=2$ units^2	
6(a)(i)	$S_n=an^3+bn^2+cn+d$ $S_1=a(1)^3+b(1)^2+c(1)+d=2$ $\Rightarrow a+b+c+d=2$ ---- (1) $S_2=a(2)^3+b(2)^2+c(2)+d=6$ $\Rightarrow 8a+4b+2c+d=6$ ----(2) $S_5=a(5)^3+b(5)^2+c(5)+d=90$ $\Rightarrow 125a+25b+5c+d=90$ ----(3)	Don't assume that sequence is an AP/GP. It's neither. It's pointless to just write $S_5=T_1+T_2+...+T_5$ The sum is polynomial in n i.e. dependent on n and generally includes a constant term. Read question carefully. '4' is not S_2.

	$S_{10}=a(10)^3+b(10)^2+c(10)+d=830$ $\Rightarrow\quad 1000a+100b+10c+d=830\quad$ ----(4) Using GC: $a=1,\quad b=-2,\quad c=3,\quad d=0$ $S_n=n^3-2n^2+3n,\ \ n\ge 1,\ n\in\mathbb{Z}^+$	
(ii)	54th term of the sequence $u_{54}=S_{54}-S_{53}$ $=(54)^3-2(54)^2+3(54)-(53)^3+2(53)^2-3(53)$ $=8376$	
(b)(i)	Given that $\cos(2n-1)\alpha-\cos(2n+1)\alpha=2\sin\alpha\sin 2n\alpha$ $\sin(2n\alpha)=\dfrac{\cos(2n-1)\alpha-\cos(2n+1)\alpha}{2\sin\alpha}$ $\therefore\sum_{n=1}^{N}\sin(2n\alpha)=\sum_{n=1}^{N}\dfrac{\cos(2n-1)\alpha-\cos(2n+1)\alpha}{2\sin\alpha}$ $=\dfrac{1}{2\sin\alpha}\left[\begin{array}{l} \cos\alpha-\cos 3\alpha \\ +\ \cos 3\alpha-\cos 5\alpha \\ +\ \cos 5\alpha-\cos 7\alpha \\ +\ \cdots \\ +\ \cos(2N-3)\alpha-\cos(2N-1)\alpha \\ +\ \cos(2N-1)\alpha-\cos(2N+1)\alpha \end{array}\right]$ $=\dfrac{\cos\alpha-\cos(2N+1)\alpha}{2\sin\alpha}\quad$ ------(*) $=\dfrac{\cos\alpha}{2\sin\alpha}-\dfrac{\cos(2N+1)\alpha}{2\sin\alpha}$ $=\dfrac{\cot\alpha}{2}-\dfrac{\operatorname{cosec}\alpha\cos(2N+1)\alpha}{2}\quad$ (shown)	Use what was given. Don't waste time to derive it when it's already given. Note that $2\sin\alpha$ (independent of *n*) is a constant in this case Remember to cancel sufficient number of rows at the beginning and at the end.
(ii)	Let $\alpha=\dfrac{\pi}{3}$, $\quad\sum_{n=1}^{N}\sin\left(\dfrac{2n\pi}{3}\right)$ $=\dfrac{\cot\frac{\pi}{3}}{2}-\dfrac{\operatorname{cosec}\frac{\pi}{3}\cos(2N+1)\frac{\pi}{3}}{2}$ As $N\to\infty$, $\cos(2N+1)\dfrac{\pi}{3}$ takes values $\dfrac{1}{2}$ or -1. OR cannot converge to a constant number. $\therefore\sum_{n=1}^{\infty}\sin\dfrac{2n\pi}{3}$ does not converge.	Note that it's the letter *N* that tends to infinity and not the letter *n*.

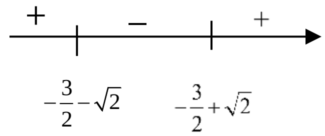
7(a)(i)	$\int \cos(\ln x)\ \mathrm{d}x = x\cos(\ln x) - \int x\left(-\frac{1}{x}\sin(\ln x)\right)\mathrm{d}x$ $= x\cos(\ln x) + \int \sin(\ln x)\ \mathrm{d}x$ $= x\cos(\ln x) + x\sin(\ln x) - \int x\left(\frac{1}{x}\cos(\ln x)\right)\ \mathrm{d}x$ $= x\cos(\ln x) + x\sin(\ln x) - \int \cos(\ln x)\ \mathrm{d}x$ $2\int \cos(\ln x))\ \mathrm{d}x = x\cos(\ln x) + x\sin(\ln x)$ $\int \cos(\ln x)\ \mathrm{d}x = \frac{1}{2}x\left(\cos(\ln x) + \sin(\ln x)\right) + C$	Use integration by parts directly. $\cos(\ln x)$ is a composite function. It is not a product of $(\cos x)(\ln x)$. Let u = $\cos(\ln x)$ and $\frac{\mathrm{d}v}{\mathrm{d}x} = 1$ apply integration by parts $uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\mathrm{d}x$ twice Bring $-\int \cos(\ln x)\ \mathrm{d}x$ to the LHS to stop the loop. Put + C in the final step
(ii)	Area = $\int_{e^{-\frac{\pi}{2}}}^{e^{\frac{\pi}{2}}} \cos(\ln x))\ \mathrm{d}x \quad = \frac{1}{2}\left[\ x\cos(\ln x) + x\sin(\ln x)\ \right]_{e^{-\frac{\pi}{2}}}^{e^{\frac{\pi}{2}}}$ $= \frac{1}{2}e^{\frac{\pi}{2}}\left(\cos\left(\ln e^{\frac{\pi}{2}}\right) + \sin\left(\ln e^{\frac{\pi}{2}}\right)\right) -$ $\frac{1}{2}e^{-\frac{\pi}{2}}\left(\cos\left(\ln e^{-\frac{\pi}{2}}\right) + \sin\left(\ln e^{-\frac{\pi}{2}}\right)\right)$ $= \frac{1}{2}e^{\frac{\pi}{2}}\left(\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\right) - \frac{1}{2}e^{-\frac{\pi}{2}}\left(\cos\left(-\frac{\pi}{2}\right) + \sin\left(-\frac{\pi}{2}\right)\right)$ $= \frac{1}{2}\left(e^{\frac{\pi}{2}} + e^{-\frac{\pi}{2}}\right)$	Read question carefully. Integrate from $e^{-\frac{\pi}{2}}$ to $e^{\frac{\pi}{2}}$. Use (i) answer. F(Upper limit)– F(lower limit) Note that $\ln e^{\frac{\pi}{2}} = \frac{\pi}{2}$, $\sin\left(-\frac{\pi}{2}\right) = -1$
(b)	Using $u = \cot x$, $\frac{\mathrm{d}u}{\mathrm{d}x} = -\mathrm{cosec}^2 x = -\frac{1}{\sin^2 x}$ $\Rightarrow ...\mathrm{d}u = ... - \frac{1}{\sin^2 x}\ \mathrm{d}x$ When $x = \frac{\pi}{6} \Rightarrow u = \frac{1}{\tan x} = \sqrt{3}$ When $x = \frac{2\pi}{3} \Rightarrow u = -\frac{1}{\sqrt{3}}$	Differentiate the given substitution. Memorise the differentiation of the 6 trigo functions. Only differentiation of sec x and cosec x formula are in MF26 Find $\frac{\mathrm{d}u}{\mathrm{d}x}$ and hence dx = .. Need to change limit to u value. Write down the expression of the volume first. Then do substitution. $V = \int_{x_1}^{x_2} y^2\ \mathrm{d}x$

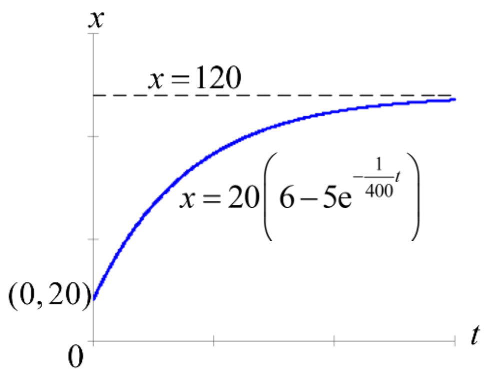
	Required volume is $=\pi\int_{\frac{\pi}{6}}^{\frac{2\pi}{3}}\left(\frac{e^{\frac{1}{2}\cot x}}{\sin x}\right)^2\ dx=\pi\int_{\frac{\pi}{6}}^{\frac{2\pi}{3}}\frac{e^{\cot x}}{\sin^2 x}\ dx$ $=\pi\int_{\frac{\pi}{6}}^{\frac{2\pi}{3}}e^{\cot x}\frac{1}{\sin^2 x}\ dx$ $=-\pi\int_{\sqrt{3}}^{-\frac{1}{\sqrt{3}}}e^{u}\ du$ or $\pi\int_{-\frac{1}{\sqrt{3}}}^{\sqrt{3}}e^{u}\ du$ $=\pi\left(e^{\sqrt{3}}-e^{-\frac{1}{\sqrt{3}}}\right)$	Change to $-\pi\int_{\sqrt{3}}^{-\frac{1}{\sqrt{3}}}e^{u}\ du$ Integrate e^u w.r.t u is e^u
8(i)	$y=\frac{x-x^2-1}{x-2}=\frac{-3}{x-2}-(x+1)$ $y=\frac{1}{3-x}-\frac{x}{3}$ $=\frac{-1}{x-3}-\frac{1}{3}x$ 1) A scaling parallel to the y-axis by a factor of 3. 2) A translation of 1 unit in the negative x-direction.	Be careful in your algebraic manipulation when trying to express $y=\frac{x-x^2-1}{x-2}$ in the form $y=\frac{A}{x-2}+B(x+1)$. Note that you are required to **describe** the sequence of transformations that will transform the curve with equation $y=\frac{1}{3-x}-\frac{x}{3}$ to $y=\frac{x-x^2-1}{x-2}=\frac{-3}{x-2}-(x+1)$, NOT the reverse. It is **INCORRECT to use** the word **"shift"** to describe translation and the word **"flip/rotate"** to describe reflection. One of the transformations involved is scaling parallel to the y-axis with a factor of 3, it is **<u>NOT with a factor of -3 or 3 units.</u>**

(ii)		Note that the curve $y=\frac{x-x^2-1}{x-2}$ has asymptotes $y=-x-1$ and $x=2$. You are expected to draw properly and clearly, including the behaviour of the curve $y=\frac{x-x^2-1}{x-2}$ near its asymptotes. Note that the oblique asymptote $y=-x-1$ meets the x-axis and y-axis at $(-1,0)$ and $(0,-1)$ respectively and the curve cuts the y-axis at the point $\left(0,\frac{1}{2}\right)$ which is NOT the minimum point. You are expected to get the <u>correct shape</u> of the graph from your GC. You are required to state the **coordinates of the points where the curves cross the axes.** For the modulus graph $y=\lvert 2x+1\rvert$, the coordinates are $\left(-\frac{1}{2},0\right)$ and $(0,1)$. It has a line of symmetry $x=-\frac{1}{2}$. You should have the sense of the relative positions for the points of intersection with the axes when sketching the two graphs, including the asymptotes on the same axes.
	$\frac{x-x^2-1}{x-2}=-2x-1$ From GC, the first point of intersection has x-coordinate -1 . $\frac{x-x^2-1}{x-2}=2x+1$ $x-x^2-1=(2x+1)(x-2)$ $3x^2-4x-1=0$ $x=\frac{4\pm\sqrt{16-4(3)(-1)}}{2(3)}$ $x=\frac{2\pm\sqrt{7}}{3}$	The question states that "**Hence**, find the **exact range** of …..", so you are required to use graphical method.

	Thus, the range of values of x is $x < -1,\ \frac{2-\sqrt{7}}{3} < x < \frac{2+\sqrt{7}}{3}$ or $x > 2$	
9(a)(i)	 Since any horizontal line $y = k,\ k \in \mathbb{R}$ intersects the graph of f at most once, f is one-one and it has an inverse.	Must sketch the graph of f for the given domain only ($x > 0$) with the asymptotes. Never use a particular line to explain one-one function and you must state the range of k. Note the 2 possible explanations. If you use any line $y = k,\ k \in \mathbb{R}$, then the line cuts the graph of f **at most once.** If you use any line $y = k,\ k \in R_f$, then the line cuts the graph of f **exactly once.** It is important to mention that f is one-one and not just f has an inverse.
(ii)	Let $y = 2 + \frac{3}{x}$ $\frac{3}{x} = y - 2$ $x = \frac{3}{y-2}$ $\therefore \mathrm{f}^{-1}(x) = \frac{3}{x-2}$ $D_{\mathrm{f}^{-1}} = R_{\mathrm{f}} = (2, \infty)$	To find the rule of f^{-1} : Let y = f (x) and then make x the subject
(iii)	See diagram in (i)	Must use the same scale for both axes when sketching the graphs of f and f^{-1} on the same diagram. The graph of f^{-1} is a reflection of the graph of f in the line $y=x$

		The graph of $f^{-1}f$ is not simply the line $y = x$. Must sketch for the correct domain and passing through (2, 2). The graphs of f, f^{-1} and $f^{-1}f$ must intersect at the same point.
(iv)	$D_f = (0, \infty)$ and $R_f = (2, \infty)$ Since $R_f \subseteq D_f$, f^2 exists. $f^2(x) = f(f(x))$ $= 2 + \dfrac{3}{2 + \frac{3}{x}}$ $= 2 + \dfrac{3x}{2x+3}$ $= \dfrac{2(2x+3)+3x}{2x+3}$ $= \dfrac{7x+6}{2x+3}$	It is not sufficient to state $R_f \subseteq D_f$. $D_f = (0, \infty)$, $R_f = (2, \infty)$ must be stated to justify the subset.
(b)	Given $h(x) = \dfrac{3-x}{x^2-1}$ To find the range of g, the graph must intersect the horizontal line $y = k$, therefore, $D \geq 0$ Let $k = \dfrac{3-x}{x^2-1}$ $kx^2 - k = 3 - x$ $kx^2 + x - (k+3) = 0$ $(1)^2 + 4k(k+3) \geq 0$ $1 + 12k + 4k^2 \geq 0$ $4k^2 + 12k + 1 \geq 0$ Consider $4k^2 + 12k + 1 = 0$, $k = \dfrac{-12 \pm \sqrt{12^2 - 4(4)(1)}}{2(4)}$ $= \dfrac{-12 \pm \sqrt{128}}{8}$ $= \dfrac{-3 \pm 2\sqrt{2}}{2} = -\dfrac{3}{2} \pm \sqrt{2}$	This question required an algebraic approach so the GC cannot be used to obtain the graph. It is very tedious to sketch the graph without using a GC as the stationary points would have to be found by differentiation. Students would also need to show the nature of the stationary points before they can sketch the graph on their own. Moreover, it would take some time to find the exact y coordinates of the stationary points in order to obtain the range of h. Thus, students are strongly encouraged to use the discriminant instead (see solution)

	$4k^2+12k+1\geq 0$ $\therefore\ k\leq -\frac{3}{2}-\sqrt{2}\ OR\ k\geq -\frac{3}{2}+\sqrt{2}$  Hence, the range of h is $\left(-\infty,-\frac{3}{2}-\sqrt{2}\right]\cup\left[-\frac{3}{2}+\sqrt{2},\infty\right)$	
10(i)	Concentration of the brine entering the tank is $\frac{C}{1000}$ kg/L Solution leaving the tank is $\frac{x}{2000}$ kg/L. The rate at which salt enters the tank is $\frac{5C}{1000}=\frac{C}{200}$ kg/min The rate at which leaves the tank is $\frac{5x}{2000}=\frac{x}{400}$ kg/min. $\frac{\mathrm{d}x}{\mathrm{d}t}=\text{inflow rate }-\text{outflow rate}$ $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{C}{200}-\frac{x}{400}=\frac{1}{400}(2C-x)$ (Shown) When $\frac{\mathrm{d}x}{\mathrm{d}t}=0$, $x=120$ kg and $2C-120=0\Rightarrow C=60$ kg	
(ii)	$\int\frac{1}{120-x}\ \mathrm{d}x=\int\frac{1}{400}\ \mathrm{d}t$ $-\ln\lvert 120-x\rvert=\frac{1}{400}t+B$ where $B=\text{const}$ $120-x=Ae^{-\frac{1}{400}t}$ where $A=\text{const}$ When $t=0$, $x=20$ kg, $Ae^0=A=120-20=100$ $120-x=100e^{-\frac{1}{400}t}$ $x=20\left(6-5e^{-\frac{1}{400}t}\right)$	Remember to include a negative sign and modulus sign after integrating $\frac{1}{120-x}$.

<table>
<tr><td>(iii)</td><td>


Using GC, $t = 204.33 = 204$ min.</td><td></td></tr>
<tr><td>(iv)</td><td>It is assumed that there is no evaporation in the system so that the concentrations of the solution remain as stated/unaffected.</td><td></td></tr>
<tr><td>11(i)</td><td>AP : first term = 1000, common difference = 250

$7500 = 1000 + 250(N-1)$
$N = 27$</td><td>It is an AP with first term 1000 and common difference 250.
$u_N = 7500$. Solve for N.</td></tr>
<tr><td>(ii)</td><td>Total number of power banks produced in 60 weeks
= $S_{27} + 33(7500)$
$= \frac{27}{2}\left(1000 + 7500\right) + 33(7500)$
=362250</td><td>For 1st to 27th week, it is an AP with first term 1000 and common difference 250.
For 28th to 60th week, the production is at 7500 per week, so $(60-27)\times 7500$.</td></tr>
<tr><td>(iii)</td><td>Number of power banks on demand on week 1= 50
Number of power banks on demand on week 2
= a+50b
Number of power banks on demand on week 3
= $a + b(a + 50b) = a + ba + 50b^2$</td><td>For 2nd week, the demand is
$a + b(50) = a + 50b$
For 3rd week, demand is
$a + b(\text{demand of 2nd week})$
$= a + b(a + 50b)$</td></tr>
<tr><td>(iv)</td><td>Number of power banks on demand on week 4
= $a + b(a + ba + 50b^2)$
= $a + ba + b^2a + 50b^3$
Number of power banks on demand on week n
$= a + ba + b^2a + b^3a + ... + b^{n-2}a + 50b^{n-1}$
$= \frac{a(b^{n-1}-1)}{b-1} + 50b^{n-1}$</td><td>Continue working out for week 4 and deduce the demand of the nth week.
Note that the last term of the expression is $b^{n-2}a$ for the GP.
The first part of the expression is a summation of GP with first term a, common ratio b and number of terms $n-1$, which can be written as $\frac{a(b^{n-1}-1)}{b-1}$.</td></tr>
<tr><td>(v)</td><td>Total number of power banks produced in 60 weeks
$= \frac{60}{2}\left(2(1000) + 59L\right)$
= 30(2000+59L)</td><td>Remember to take summation of the terms from the 1st to 60th week. The total demand can be found using GC (MATH, 0: Summation).</td></tr>
</table>

	Total number of power banks on demand in 60 weeks $=\sum_{r=1}^{60}\left(\frac{300(1-1.05^{r-1})}{1-1.05}\right)+\sum_{r=1}^{60}\left(1.05^{r-1}(50)\right)$ = 1779181.493 For $30\left(2000+59L\right)\geq 1779181.493$ $L\geq 971.29$ Least L = 972	For total demand to be met, total production $\geq$ total demand for the first 60 weeks, solve for L.