ANNEX

H2 MA 2019 JC2 Prelim (Paper 1 and Paper 2)

***Filename**: Change* SCHOOL *to your school name, e.g. NYJC*

Paper 1

Select topic from the dropdown list. If the question consists of multiple topics, choose 1 topic.

QN	TOPIC (H2) Paper 1	ANSWERS (Exclude graphs and text answers)
1	Equations & Inequalities	72 units
2	Sigma Notation & MOD	(i) $\ln\dfrac{1+x}{1+x^{N+1}}$ (ii) $\ln(1+x)$
3	Maclaurin & Binomial Series	$p=\sqrt{3}, q=1, r=\dfrac{1}{2\sqrt{3}}$
4	Graphs & Transformations	(i) $D=(-\infty,-11]\cup[1,\infty)$ (ii) $y=\pm\dfrac{x+5}{6}$
5	Vectors	(i) $\overrightarrow{OM}=\dfrac{1}{3}(3\mathbf{a}+\lambda\mathbf{b}), \overrightarrow{ON}=\dfrac{4}{7}\lambda\mathbf{b}$ (ii) $k=\dfrac{1}{3}$
6	Differentiation & Applications	(i) $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{-1}{25\cos t}$ (ii) $\dfrac{225}{4\sqrt{2}}$ (iii) $y=0$
7	Integration & Applications	(a)(i) $-\sin\dfrac{x}{2}\left(\mathrm{e}^{\cos\frac{x}{2}}\right)$ (ii) $-2\cos\dfrac{x}{2}\left(\mathrm{e}^{\cos\frac{x}{2}}\right)+2\mathrm{e}^{\cos\frac{x}{2}}+C$ (b) $-\ln\left\|1-\mathrm{e}^{x}\right\|+x+C$
8	Functions	(ii) greatest value of k is a. (iii) $\mathrm{f}^{-1}: x\mapsto a-2\sqrt{a^2-x^2},\ x\in\mathbb{R},\ 0\le x\le a$ (iv) $\mathrm{R}_{\mathrm{gf}}=\left[\dfrac{\sqrt{3}}{2}a, a\right]$
9	APGP	(i) $d=268.0$ (ii) 1339.9 cm (iii) red

		(iv) yellow
10	Complex Numbers	(a) $u=\dfrac{a}{4+a^2}-\dfrac{6+2a^2}{4+a^2}\mathrm{i}, v=\dfrac{2}{4+a^2}-\dfrac{a}{4+a^2}\mathrm{i}$ (b)(i) $k=12$ (iii) $b(x+12)(x+\dfrac{1+\sqrt{1+4b^2}}{2b}\mathrm{i})(x+\dfrac{1-\sqrt{1+4b^2}}{2b}\mathrm{i})$
11	Differentiation & Applications	(a) -0.916 cm/min (b) -2.29 cm/min (c) 4293.510 cm^3
12	Vectors	(ii) 11.1° (iv) $(0.547,-0.237,0.00325)$ (v) 117 m
13	H2 Prelim P1 Q13 Topic	
14	H2 Prelim P1 Q14 Topic	

Paper 2

Select topic from the dropdown list. If the question consists of multiple topics, choose 1 topic.

QN	TOPIC (H2) Paper 2	ANSWERS (Exclude graphs and text answers)
1	Equations & Inequalities	(i) $x\le -\dfrac{a}{2}$ or $x=a$ or $x\ge \dfrac{5a}{2}$ (ii) $x\le -a$ or $x=\dfrac{a}{2}$ or $x\ge 2a$
2	Maclaurin & Binomial Series	(ii) $y=1+x^2-\dfrac{3}{2}x^3+\ldots$ (iii) 0.985 (iv) $\dfrac{1}{\mathrm{e}^{\sin x}\sqrt{1-2x}}\approx 1+x^2+\dfrac{3}{2}x^3$
3	Complex Numbers	(i) $\dfrac{a}{2}$; $-\dfrac{\pi}{3}$ (ii) $-\dfrac{5\pi}{12}$; $a\cos\dfrac{\pi}{12}$ (iii) 6
4	Differential Equations	(a) (ii) $P=2\sqrt{1+3\mathrm{e}^{-2t}}$ (iii) The population of the bugs will decrease and approach 2000 in the long run. (b) $N=4t\ln t+t$
5	Normal Distribution	(ii) 7.6
6	Correlation & Regression	(ii) $(69,53)$ (iii) Since $\lvert r\rvert=0.96785$ for Model B is nearest to 1, Model B is the most accurate model (iv) 62 marks (v) unreasonable as 120 is not within data range

		where $48 \le x \le 84$
7	Normal Distribution	(i) a "6-inch" loaf more likely to be less than 6 inches (ii) $\frac{1}{2}$ (iii) 0.855 (iv) 0.446
8	PnC & Probability	(a) (i) 3360 (ii) 1200 (iii) 240 (b) (i) $\frac{13}{14}$ (ii) 0.644
9	Hypothesis Testing	(a) (i) 949.84 ; 1,73 (ii) H_0: $\mu = 950$ H_1: $\mu < 950$ (iii) 0.135 (iv) $\{\alpha \in \mathbb{R} : 13.5 \le \alpha \le 100\}$ (b) $12.5 \le k \le 104.8$ (c) There is no need for the floor supervisor to assume the volume of shower gel follow a normal distribution as the sample sizes in both part (a) and (b) are large, the sample mean volume of shower gel can be approximated to follow a normal distribution by Central Limit Theorem.
10	DRV	(i) \$2.50; \$3.75 (ii) Option A is a "sure win" option where the player would definitely gain a positive amount in all cases, whereas option B has a risk of losing money in some cases. (iv) $A \sim \mathrm{N}\left(125, \frac{125}{2}\right)$; $B \sim \mathrm{N}\left(\frac{375}{2}, \frac{22175}{8}\right)$
11	H2 Prelim P2 Q11 Topic	
12	H2 Prelim P2 Q12 Topic	
13	H2 Prelim P2 Q13 Topic	
14	H2 Prelim P2 Q14 Topic	

Qn	Solutions	C
1	$D(x)=\dfrac{40320}{g(x)}$ $\Rightarrow g(x)=\dfrac{40320}{D(x)}$ $ax^2+bx+c=\dfrac{40320}{D(x)}$ Given $5^2a+5b+c=\dfrac{40320}{384}=105$ -- (1) $8^2a+8b+c=\dfrac{40320}{224}=180$ -- (2) $10^2a+10b+c=\dfrac{40320}{168}=240$ -- (3) Using GC, $a=1,\ b=12,\ c=20$ When $x=18$, $D(18)=\dfrac{40320}{18^2+12(18)+20}=72$ units	G w A g(5^2 of 5^2 A de in pc
2i	<u>Method 1</u>: (method of differences) $\displaystyle\sum_{n=1}^{N}p_n=\sum_{n=1}^{N}\ln\frac{1+x^n}{1+x^{n+1}}$ $\displaystyle=\sum_{n=1}^{N}\left(\ln(1+x^n)-\ln(1+x^{n+1})\right)$ $=\ln(1+x)-\ln(1+x^2)$ $+\ln(1+x^2)-\ln(1+x^3)$ $+\ln(1+x^3)-\ln(1+x^4)$ $\vdots$ $+\ln(1+x^N)-\ln(1+x^{N+1})$ $=\ln(1+x)-\ln(1+x^{N+1})$ $=\ln\dfrac{1+x}{1+x^{N+1}}$	Tl dc dc su di cl th Sc en lir co se A fi ln of

Method 2: (using property of logarithm)

$$\begin{aligned}\sum_{n=1}^{N} p_n &= \sum_{n=1}^{N} \ln \frac{1+x^n}{1+x^{n+1}} \\ &= \ln\frac{1+x}{1+x^2} + \ln\frac{1+x^2}{1+x^3} + \ln\frac{1+x^3}{1+x^4} + \ldots + \ln\frac{1+x^N}{1+x^{N+1}} \\ &= \ln \frac{1+x}{\cancel{1+x^2}} \cdot \frac{\cancel{1+x^2}}{\cancel{1+x^3}} \cdot \frac{\cancel{1+x^3}}{\cancel{1+x^4}} \cdot \ldots \cdot \frac{\cancel{1+x^N}}{1+x^{N+1}} \\ &= \ln \frac{1+x}{1+x^{N+1}}\end{aligned}$$

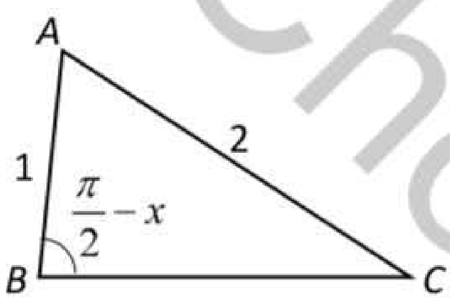
2ii

Since $-1 < x < 1$, as $N \to \infty$, $x^{N+1} \to 0$.

$$\therefore \sum_{n=1}^{\infty} p_n = \ln \frac{1+x}{1+0} = \ln(1+x)$$

So
as

3



By Cosine Rule,

$\cos\left(\frac{\pi}{2} - x\right)$

$$2^2 = 1^2 + (BC)^2 - 2(BC)\cos\left(\frac{\pi}{2} - x\right)$$

$$4 = 1 + (BC)^2 - 2(BC)\sin x$$

$$(BC)^2 - 2(BC)\sin x - 3 = 0$$

$$\begin{aligned}BC &= \frac{2\sin x \pm \sqrt{4\sin^2 x + 12}}{2} \\ &= \sin x \pm \sqrt{\sin^2 x + 3} \\ &\approx x \pm \sqrt{x^2 + 3} \\ &= x + \sqrt{x^2 + 3} \text{ , since } BC > 0 \\ &= x + \sqrt{3}\left(1 + \frac{x^2}{3}\right)^{1/2} \\ &= x + \sqrt{3}\left[1 + \frac{1}{2}\frac{x^2}{3} + \ldots\right] \\ &\approx \sqrt{3} + x + \frac{x^2}{2\sqrt{3}}\end{aligned}$$

$$p = \sqrt{3}, q = 1, r = \frac{1}{2\sqrt{3}}$$

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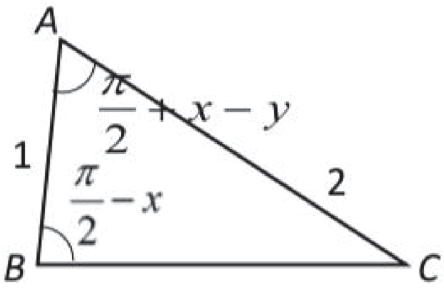
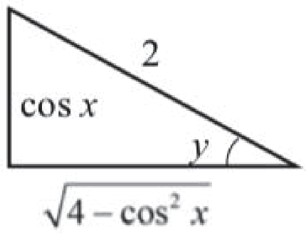
Alternative Solution



By sine rule,

$$\frac{BC}{\sin\left(\frac{\pi}{2}+x-y\right)}=\frac{2}{\sin\left(\frac{\pi}{2}-x\right)}=\frac{1}{\sin y}$$

$$\Rightarrow \frac{BC}{\cos(y-x)}=\frac{2}{\cos x}=\frac{1}{\sin y}$$



$$\Rightarrow BC=\frac{2\cos(y-x)}{\cos x}$$

$$\Rightarrow BC=\frac{2(\cos y\cos x+\sin y\sin x)}{\cos x}$$

$$\Rightarrow BC=\frac{2(\cos y\cos x+\sin y\sin x)}{\cos x}$$

$$\Rightarrow BC=\frac{2\left(\frac{\sqrt{4-\cos^2 x}}{2}\cos x+\frac{\cos x}{2}\sin x\right)}{\cos x}$$

$$\Rightarrow BC=\frac{\sqrt{4-\cos^2 x}\cos x+\cos x\sin x}{\cos x}$$

$$\Rightarrow BC=\sqrt{4-\cos^2 x}+\sin x$$

$$\Rightarrow BC=\sqrt{3+\sin^2 x}+\sin x$$

4i Sketch the graph using GC, remember to set window appropriately.

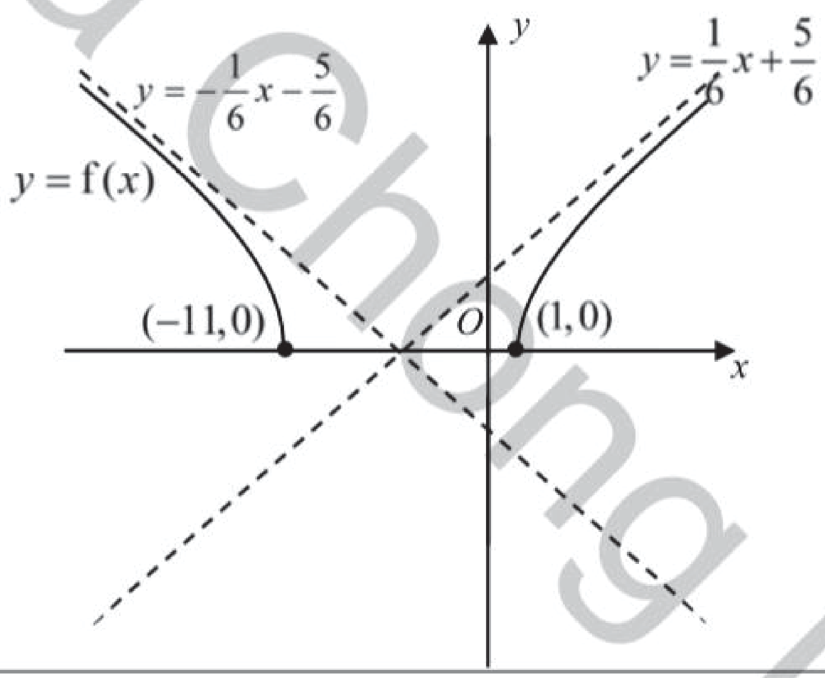
OR

Notice that it is the upper half of a hyperbola.

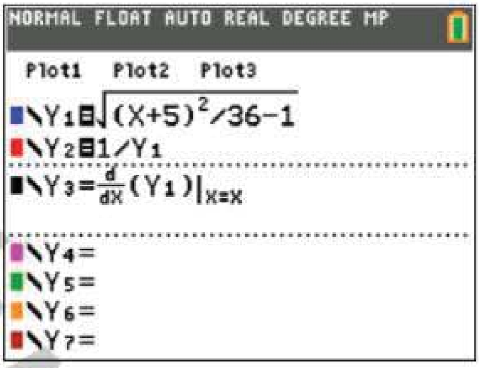
$$y=\sqrt{\frac{(x+5)^2}{36}-1}$$

$$\frac{(x+5)^2}{6^2}-y^2=1$$

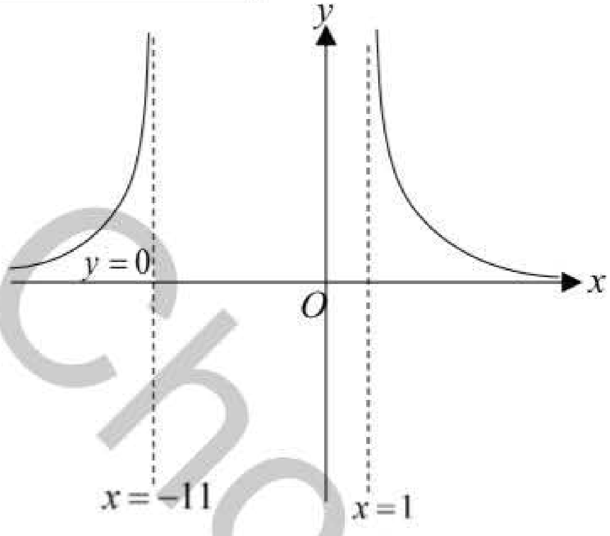
We have $D=(-\infty,-11]\cup[1,\infty)$.

ii	Let $y=\sqrt{\frac{(x+5)^2}{36}-1}$ $\frac{(x+5)^2}{6^2}-y^2=1$ To find asymptotes: $y^2=\frac{(x+5)^2}{6^2}$ $y=\pm\frac{x+5}{6}$
iii	

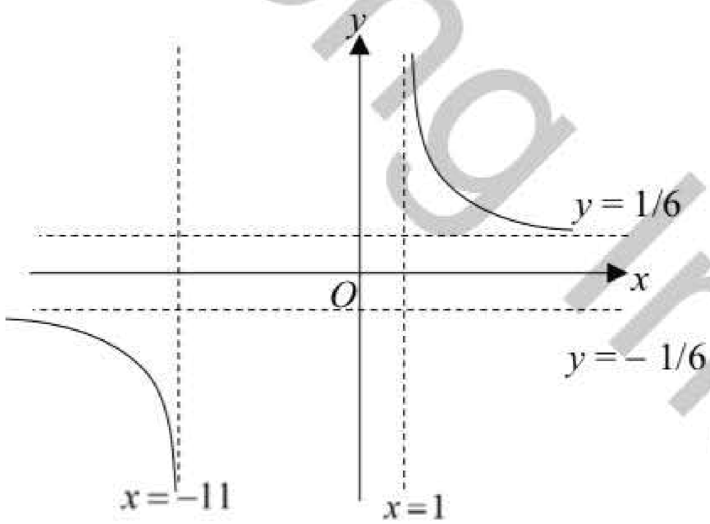
iv	Using GC to help if students is not sure



$y = \dfrac{1}{\mathrm{f}(x)}$



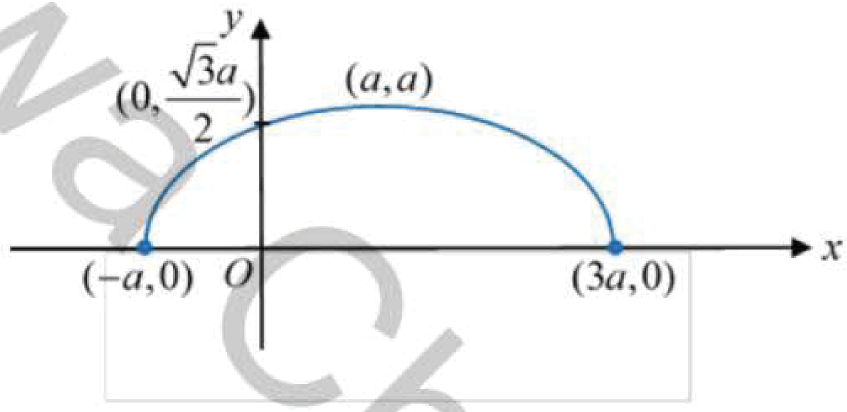
$y = \mathrm{f}'(x)$

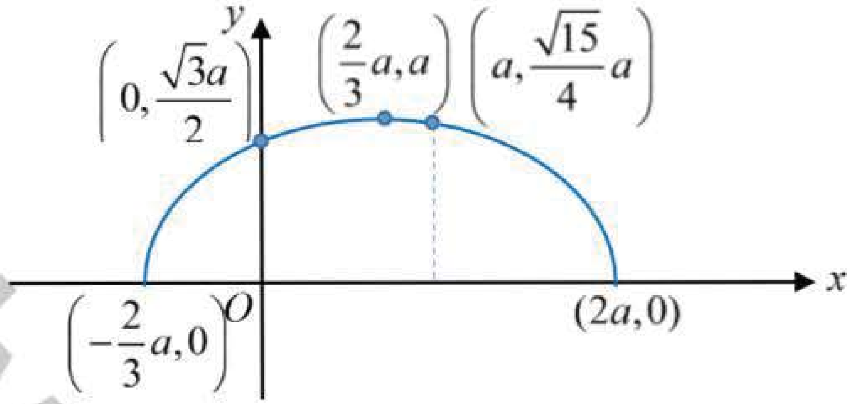


5i	$\overrightarrow{OC} = \lambda\mathbf{b}$ $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{OC} = a + \lambda\mathbf{b}$ $\overrightarrow{ON} = \frac{4}{7}\lambda\mathbf{b}$ $\overrightarrow{OM} = \frac{\overrightarrow{OD} + 2\overrightarrow{OA}}{3}$ $= \frac{\mathbf{a} + \lambda\mathbf{b} + 2\mathbf{a}}{3}$ $= \frac{1}{3}(3\mathbf{a} + \lambda\mathbf{b})$
ii	Area of triangle $OMD = \frac{1}{2}\left\|\overrightarrow{OM} \times \overrightarrow{OD}\right\|$ $= \frac{1}{2}\left\|\frac{1}{3}(\lambda\mathbf{b} + 3\mathbf{a}) \times (\lambda\mathbf{b} + \mathbf{a})\right\|$ $= \frac{1}{6}\left\|\lambda^2\mathbf{b} \times \mathbf{b} + \lambda\mathbf{b} \times \mathbf{a} + 3\lambda\mathbf{a} \times \mathbf{b} + 3\mathbf{a} \times \mathbf{a}\right\|$ $= \frac{1}{6}\left\|-\lambda\mathbf{a} \times \mathbf{b} + 3\lambda\mathbf{a} \times \mathbf{b}\right\| \quad (\because \mathbf{b} \times \mathbf{b} = \mathbf{0} = \mathbf{a} \times \mathbf{a})$ $= \frac{1}{3}\lambda\left\|\mathbf{a} \times \mathbf{b}\right\|$
iii	$\left\|\mathbf{p}.\mathbf{a}\right\|$ is the length of projection of $\overrightarrow{OA}$ on $\overrightarrow{OD}$

6i	$\dfrac{dx}{dt} = 50\sin t\cos t$, $\dfrac{dy}{dt} = -2\sin t$ $\dfrac{dy}{dx} = \dfrac{-1}{25\cos t}$	
ii	When $t = \dfrac{\pi}{4}$, $\dfrac{dy}{dx} = \dfrac{-\sqrt{2}}{25}$, $x = \dfrac{25}{2}$, $y = \sqrt{2}$ $y = -\dfrac{\sqrt{2}}{25}x + c$ $c = \sqrt{2} + \dfrac{1}{\sqrt{2}} = \dfrac{3}{\sqrt{2}}$ Equation of tangent: $y = -\dfrac{\sqrt{2}}{25}x + \dfrac{3}{\sqrt{2}}$ When $x = 0$, $y = \dfrac{3}{\sqrt{2}}$ When $y = 0$, $x = \dfrac{75}{2}$ Area of triangle $OPQ = \dfrac{1}{2}\left(\dfrac{75}{2}\right)\left(\dfrac{3}{\sqrt{2}}\right) = \dfrac{75}{4}\left(\dfrac{3}{\sqrt{2}}\right) = \dfrac{225}{4\sqrt{2}}$	

iii	$y=0$
7ai	Let $y=2\mathrm{e}^{\cos\frac{x}{2}}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{1}{2}\sin\dfrac{x}{2}\left(2\mathrm{e}^{\cos\frac{x}{2}}\right)$ $=-\sin\dfrac{x}{2}\left(\mathrm{e}^{\cos\frac{x}{2}}\right)$
aii	$\displaystyle\int\frac{1}{2}\sin x\left(\mathrm{e}^{\cos\frac{x}{2}}\right)\mathrm{d}x$ $\displaystyle=\int\frac{1}{2}\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)\left(\mathrm{e}^{\cos\frac{x}{2}}\right)\mathrm{d}x$ $\displaystyle=-\int\left(\cos\frac{x}{2}\right)\left(-\sin\frac{x}{2}\mathrm{e}^{\cos\frac{x}{2}}\right)\mathrm{d}x$ $\displaystyle=\left(-\cos\frac{x}{2}\right)\left(2\mathrm{e}^{\cos\frac{x}{2}}\right)-\int\left(\frac{1}{2}\sin\frac{x}{2}\right)\left(2\mathrm{e}^{\cos\frac{x}{2}}\right)\mathrm{d}x$ $\displaystyle=-2\cos\frac{x}{2}\left(\mathrm{e}^{\cos\frac{x}{2}}\right)+2\mathrm{e}^{\cos\frac{x}{2}}+C$
b	$\displaystyle\int\frac{1}{1-\mathrm{e}^{x}}\,\mathrm{d}x$ Let $u=1-\mathrm{e}^{x}\Rightarrow\dfrac{\mathrm{d}u}{\mathrm{d}x}=-\mathrm{e}^{x}=u-1\Rightarrow\dfrac{\mathrm{d}x}{\mathrm{d}u}=\dfrac{1}{u-1}$ $\displaystyle=\int\frac{1}{u(u-1)}\,\mathrm{d}u$ $\displaystyle=\int-\frac{1}{u}+\frac{1}{u-1}\,\mathrm{d}u$ $=-\ln\lvert u\rvert+\ln\lvert u-1\rvert+C$

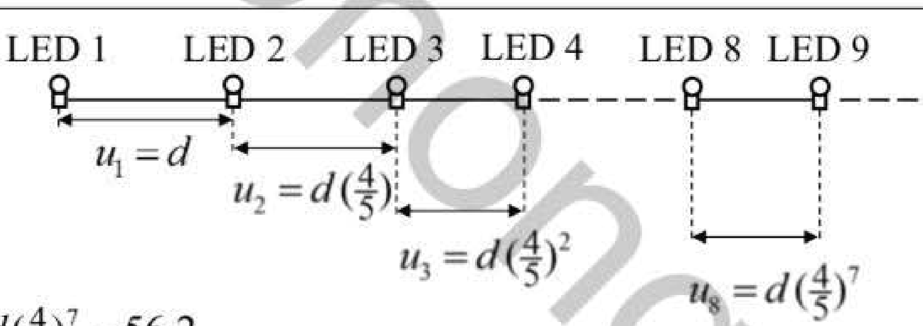
	$= -\ln\lvert 1-e^{x}\rvert + \ln\lvert -e^{x}\rvert + C$ $= -\ln\lvert 1-e^{x}\rvert + x + C$
8i	
ii	For f^{-1} to exist, f must be one-one. i.e. every horizontal line $y=h$, $h \in \mathbb{R}$ can only cut the graph of $y = f(x)$ at most once. Hence, the greatest value of k is a.
iii	Let $y = \sqrt{a^2 - \frac{(x-a)^2}{4}}$ $\Rightarrow y^2 = a^2 - \frac{(x-a)^2}{4}$ $\Rightarrow (x-a)^2 = 4(a^2 - y^2)$ $\Rightarrow x = a \pm 2\sqrt{a^2 - y^2}$ $\Rightarrow x = a - 2\sqrt{a^2 - y^2}$ $(\because -a \le x \le a)$ $f^{-1} : x \mapsto a - 2\sqrt{a^2 - x^2}$, $x \in \mathbb{R}$, $0 \le x \le a$
iv	$g(x) = f\left(\frac{3}{2}x\right)$ for $x \in \mathbb{R}$, $-\frac{2}{3}a \le x \le 2a$. Since $R_f = [0,a] \subseteq \left[-\frac{2}{3}a, 2a\right] = D_g$, the composite function gf exists.



$$D_{gf} = D_f \xrightarrow{f} R_f = [0, a] \xrightarrow{g} \left[\frac{\sqrt{3}}{2}a, a\right] = R_{gf}$$

$$R_{gf} = \left[\frac{\sqrt{3}}{2}a, a\right]$$

9i



$d(\frac{4}{5})^7 = 56.2$

$\therefore \ d = 267.9824829 = 268.0$ (1 d.p.)

ii

Since $|r| = \left|\frac{4}{5}\right| < 1$, sum to infinity exists.

Hence maximum theoretical length of wire

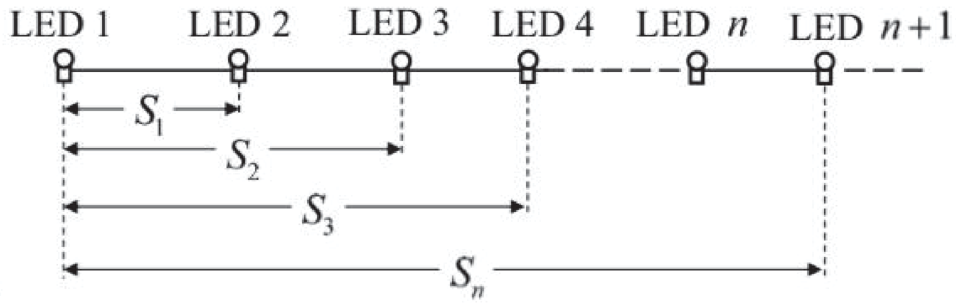
$= \dfrac{d}{1-r}$

$= \dfrac{267.9824829}{1-\frac{4}{5}}$

$= 1339.912415$

$= 1339.9$ cm (1 d.p.)

iii



Let S_n be distance from LED 1 to LED $n+1$.

<u>Method 1</u>: (using GC table)

Using sum of GP, $S_n = \dfrac{267.9824829[1-(\frac{4}{5})^n]}{1-\frac{4}{5}}$

Using GC,

n	S_n	$\lvert S_n - 1290\rvert$
14	1281.0	9.0
15	1292.8	2.8

$\therefore$ closest LED to 1290 is when $n = 15$.

$\therefore$ required LED is LED $(15+1)$ = LED 16.

Hence colour of LED 16 is red.

<u>Method 2</u>: (using algebraic manipulation)

Let $S_n = \dfrac{267.9824829[1-(\frac{4}{5})^n]}{1-\frac{4}{5}} = 1290$

$1-(\frac{4}{5})^n = 0.9627494943$

$n\ln(\frac{4}{5}) = \ln 0.0372505057$

$\therefore\ n = 14.74427443$

Using GC,

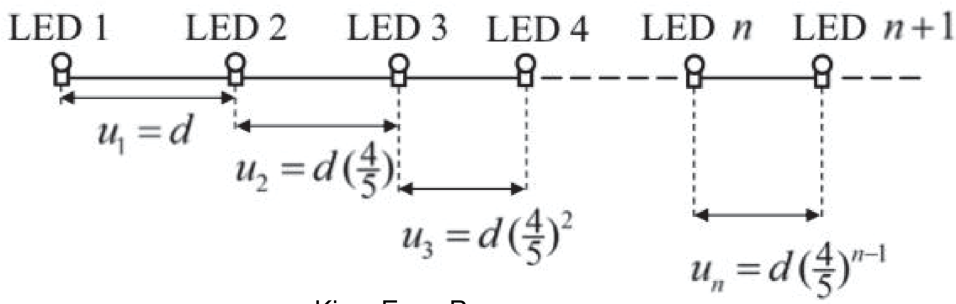
n	S_n	$\lvert S_n - 1290\rvert$
14	1281.0	9.0
15	1292.8	2.8

$\therefore$ closest LED to 1290 is when $n = 15$.

$\therefore$ required LED is LED $(15+1)$ = LED 16.

Hence colour of LED 16 is red.

iv



	Let $u_n = d\,(\frac{4}{5})^{n-1} = 267.9824829(\frac{4}{5})^{n-1} \geq 1$ $(\frac{4}{5})^{n-1} \geq \frac{1}{267.9824829}$ $\ln(\frac{4}{5})^{n-1} \geq \ln\frac{1}{267.9824829}$ $(n-1)\ln(\frac{4}{5}) \geq \ln\frac{1}{267.9824829}$ $n-1 \leq 25.05526859$ $\therefore \quad n \leq 26.05526859$ Since largest integer $n = 26$, last LED is LED $(26+1) =$ LED 27 Hence colour of last LED 27 is yellow.	
10a	$v + \mathrm{i}u = 2$ ----(1) $av - 2u = 3\mathrm{i}$ ----(2) $(1)\times a - (2)$: $\mathrm{i}au + 2u = 2a - 3\mathrm{i}$ $u = \frac{2a-3\mathrm{i}}{2+\mathrm{i}a} \times \frac{2-\mathrm{i}a}{2-\mathrm{i}a}$ $u = \frac{4a-6\mathrm{i}-2a^2\mathrm{i}-3a}{4+a^2}$ $u = \frac{a-(6+2a^2)\mathrm{i}}{4+a^2}$ $= \frac{a}{4+a^2} - \frac{6+2a^2}{4+a^2}\mathrm{i}$ Sub u into (1) $v = 2 - \mathrm{i}u$ $= 2 - \mathrm{i}(\frac{a}{4+a^2} - \frac{6+2a^2}{4+a^2}\mathrm{i})$ $= 2 - \frac{6+2a^2}{4+a^2} - \frac{a}{4+a^2}\mathrm{i}$ $= \frac{2}{4+a^2} - \frac{a}{4+a^2}\mathrm{i}$	

bi	Sub $x=-k$ into $bx^3+(12b+\mathrm{i})x^2+(12\mathrm{i}+b)x+12b=0$ We have: $-bk^3+(12b+\mathrm{i})k^2-(12\mathrm{i}+b)k+12b=0$ $-bk^3+12bk^2-bk+12b=0 \quad ---(1)$ and $k^2-12k=0----(2)$ (2) implies $k(k-12)=0$ $k=0,12$ Reject $k=0$ as $b\neq 0$.	• • • •
bii	$bx^2+\mathrm{i}x+b=0$ $x=\dfrac{-\mathrm{i}\pm\sqrt{-1-4b^2}}{2b}$ $x=\dfrac{-1\pm\sqrt{1+4b^2}}{2b}\mathrm{i}$ Hence roots are purely imaginary	• • •
biii	Hence $bx^3+(12b+\mathrm{i})x^2+(12\mathrm{i}+b)x+12b=(x+12)(bx^2+cx+b)$ By compare coeff of x, $c=\mathrm{i}$, using (i):	•

	$bx^3+(12b+\mathrm{i})x^2+(12\mathrm{i}+b)x+12b=(x+12)(bx^2+\mathrm{i}x+b)$ $=b(x+12)(x+\frac{1+\sqrt{1+4b^2}}{2b}\mathrm{i})(x+\frac{1-\sqrt{1+4b^2}}{2b}\mathrm{i})$	• •
11 a	Height of liquid in the cylinder $= x-4$ $V=\pi(2.5)^2(x-4)+\frac{1}{3}\pi(2.5)^2(4)$ $=6.25\pi(x-4)+\frac{25}{3}\pi$ $\frac{dV}{dx}=6.25\pi$ and $\frac{dV}{dt}=-18$ $\frac{dx}{dt}=\frac{dx}{dV}\times\frac{dV}{dt}$ $=\frac{-18}{6.25\pi}$ $=-0.916$ cm/min	• •
b	$\frac{r}{x}=\frac{2.5}{4}\Rightarrow x=1.6r$ When $x=2$, $r=\frac{5}{4}$ $V_c=\frac{1}{3}\pi r^2 x$ $=\frac{1}{3}\pi r^2(1.6r)$ $=\frac{1.6}{3}\pi r^3$ $\frac{dV}{dr}=1.6\pi r^2$	• •

	$\frac{dr}{dt}=\frac{dr}{dV}\times\frac{dV}{dt}$ $=\frac{1}{1.6\pi\left(\frac{5}{4}\right)^2}\times(-18)$ $=-2.29$ cm/min	
c	Vol of bowl $=\pi\int_{-10}^{0}225\left(1-\frac{y^2}{100}\right)\mathrm{d}y$ Vol. of empty space $=\pi\int_{-10}^{0}225\left(1-\frac{y^2}{100}\right)\mathrm{d}y-\pi\underbrace{(2.5)^2(20)}_{\boxed{A}}-\frac{1}{3}\pi\underbrace{(2.5)^2(4)}_{\boxed{B}}$ $=4293.510$ cm^3 (3 d.p.)	• • •
12i	Equation of line through MD: $\mathbf{r}=\begin{pmatrix}0.8\\0.6\\0\end{pmatrix}+\lambda\begin{pmatrix}2\\7\\-1\end{pmatrix},\ \lambda\in\mathbb{R}$ $\begin{pmatrix}0.8\\0.75\\0.3\end{pmatrix}-\begin{pmatrix}0.4\\-0.9\\0\end{pmatrix}=\begin{pmatrix}0.4\\1.65\\0.3\end{pmatrix}$ Equation of line through ND: $\mathbf{r}=\begin{pmatrix}0.4\\-0.9\\0\end{pmatrix}+\mu\begin{pmatrix}0.4\\1.65\\0.3\end{pmatrix},\ \lambda\in\mathbb{R}$. Since lines through MD and ND intersect at D,	St th lir r $\overline{M}$ W In

	$\begin{cases} 0.8+2\lambda = 0.4+0.4\mu \\ 0.6+7\lambda = -0.9+1.65\mu \\ \quad -\lambda = 0.3\mu \end{cases}$ Solving using GC, $\lambda = -\frac{3}{25}$, $\mu = \frac{2}{5}$. Substitute $\lambda = -1.2$ into equation of line through MD, $\begin{pmatrix} 0.8 \\ 0.6 \\ 0 \end{pmatrix} - \frac{3}{25}\begin{pmatrix} 2 \\ 7 \\ -1 \end{pmatrix} = \begin{pmatrix} 0.56 \\ -0.24 \\ 0.12 \end{pmatrix}$ $\therefore$ coordinates of D is $(0.56, -0.24, 0.12)$. (shown)	Ma ans or sho cur
ii	Equation of horizontal ground: $\mathbf{r} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0$ Required angle $= 90° - \cos^{-1}\left\lvert \dfrac{\begin{pmatrix} 0.56 \\ -0.24 \\ 0.12 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}{\sqrt{0.56^2 + (-0.24)^2 + 0.12^2}\sqrt{1}} \right\rvert$ $= 11.14233567°$ $= 11.1°$ (1 d.p.)	Ma for for Ma no hor So equ wr is r
iii	Given $T(0,0,0.07)$. $\overrightarrow{NM} = \begin{pmatrix} 0.8 \\ 0.6 \\ 0 \end{pmatrix} - \begin{pmatrix} 0.4 \\ -0.9 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.4 \\ 1.5 \\ 0 \end{pmatrix}$ $\overrightarrow{TM} = \begin{pmatrix} 0.8 \\ 0.6 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 0.07 \end{pmatrix} = \begin{pmatrix} 0.8 \\ 0.6 \\ -0.07 \end{pmatrix}$ $\begin{pmatrix} 0.4 \\ 1.5 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0.8 \\ 0.6 \\ -0.07 \end{pmatrix} = \begin{pmatrix} -0.105 \\ 0.028 \\ -0.96 \end{pmatrix} = 0.01\begin{pmatrix} -10.5 \\ 2.8 \\ -96 \end{pmatrix}$ $\therefore$ equation of p: $\mathbf{r} \cdot \begin{pmatrix} -10.5 \\ 2.8 \\ -96 \end{pmatrix} = \begin{pmatrix} 0.8 \\ 0.6 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -10.5 \\ 2.8 \\ -96 \end{pmatrix} = -6.72$ (Shown)	Ma 0.0 Stu equ wr pre

iv

Equation of line perpendicular to p passing through D is

$$\mathbf{r}=\begin{pmatrix}0.56\\-0.24\\0.12\end{pmatrix}+t\begin{pmatrix}-10.5\\2.8\\-96\end{pmatrix},\ t\in\mathbb{R}.$$

Substitute equation of line into p,

$$\begin{pmatrix}0.56-10.5t\\-0.24+2.8t\\0.12-96t\end{pmatrix}\cdot\begin{pmatrix}-10.5\\2.8\\-96\end{pmatrix}=-6.72$$

$\therefore\ t=0.00121618712$

Hence

$$\begin{pmatrix}0.56\\-0.24\\0.12\end{pmatrix}+0.00121618712\begin{pmatrix}-10.5\\2.8\\-96\end{pmatrix}=\begin{pmatrix}0.5472300352\\-0.2365946761\\0.00324603648\end{pmatrix}$$

$$=\begin{pmatrix}0.547\\-0.237\\0.00325\end{pmatrix}$$

$\therefore$ coordinates of G is $(0.547,-0.237,0.00325)$.

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Method 1: [Hence using $(0.547,-0.237,0.00325)$]

Required distance DG

$$=\sqrt{(0.56-0.54723)^2+(-0.24-(-0.23659))^2+(0.12-0.003246)^2}$$

$$=0.1174998$$

$$=0.117\quad(3\text{ s.f.})$$

$\therefore$ required distance $=117$ m

Method 2: [Otherwise using dot product]

$$\overrightarrow{MD}=\begin{pmatrix}0.56\\-0.24\\0.12\end{pmatrix}-\begin{pmatrix}0.8\\0.6\\0\end{pmatrix}=\begin{pmatrix}-0.24\\-0.84\\0.12\end{pmatrix}$$

Required distance

$$=\left|\begin{pmatrix}-0.24\\-0.84\\0.12\end{pmatrix}\cdot\frac{\begin{pmatrix}-10.5\\2.8\\-96\end{pmatrix}}{\sqrt{(-10.5)^2+2.8^2+(-96)^2}}\right|$$

$$=0.1174996006$$

$$=0.117\quad(3\text{ s.f.})$$

$\therefore$ required distance $=117$ m

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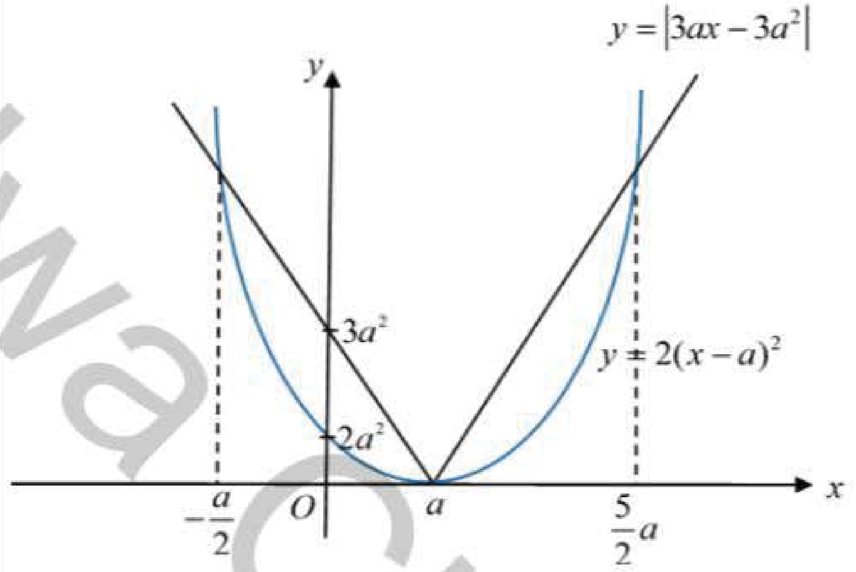
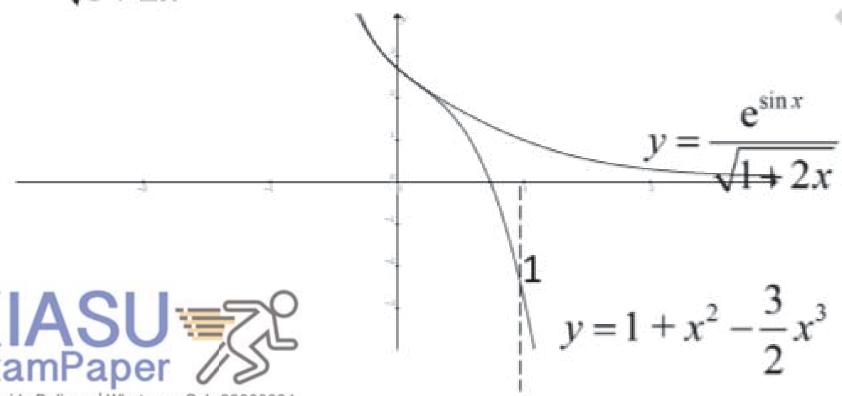
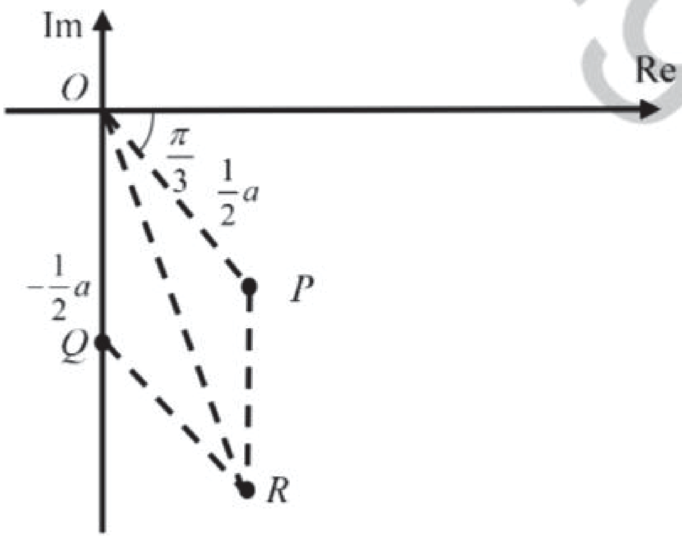
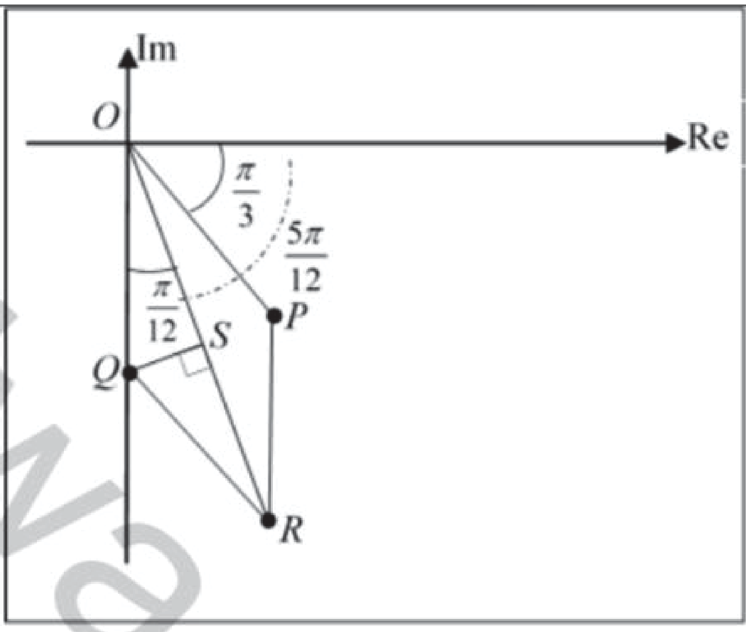
	Method 3 $\left\|\overrightarrow{DG}\right\| = \beta\left\|\begin{pmatrix}-10.5\\2.8\\-96\end{pmatrix}\right\|$ $= 0.00121618712\sqrt{9334.09}$ $= 0.1174996006$ $= 0.117 \ \ (3 \text{ s.f.})$ $\therefore$ required distance $= 117$ m	

Qn	Solutions	Commen
1		In genera part. Common 1. Fo in 2. Th stu of 3. Sc nu gr
1i	To find points of intersection, we solve $y=\|3ax-3a^2\|$ $y=2(x-a)^2$ i.e. $2(x-a)^2=\|3ax-3a^2\|$ $2(x-a)^2=3ax-3a^2$ ------(1) $\Rightarrow 2x^2-7ax+5a^2=0$ $\Rightarrow (2x-5a)(x-a)=0$ $\Rightarrow x=\frac{5}{2}a$ or $x=a$ Or $2(x-a)^2=-(3ax-3a^2)$ -------(2) $\Rightarrow 2x^2-ax-a^2=0$ $\Rightarrow (2x+a)(x-a)=0$ $\Rightarrow x=-\frac{1}{2}a$ or $x=a$ For $2(x-a)^2\geq\|3ax-3a^2\|$, $x\leq-\frac{a}{2}$ or $x=a$ or $x\geq\frac{5a}{2}$	A lot of st previous g Majority that $x=a$ The easie intercepts based on t final conc A lot of st method to directly w manipulat Common 1. $\|a$ m M lo 2. Sc wa in fa Th (

1ii	Replace x with $x+\frac{a}{2}$, $2(x+\frac{a}{2}-a)^2 \geq \left\lvert 3a\left(x+\frac{a}{2}\right)-3a^2\right\rvert$ $2\left(x-\frac{a}{2}\right)^2 \geq \left\lvert 3ax-\frac{3a^2}{2}\right\rvert$ $\therefore\ x+\frac{a}{2}\leq-\frac{a}{2}$ or $x+\frac{a}{2}=a$ or $x+\frac{a}{2}\geq\frac{5a}{2}$ $\Rightarrow x\leq -a$ or $x=\frac{a}{2}$ or $x\geq 2a$	A lot replac
2i	$y=\frac{e^{\sin x}}{\sqrt{1+2x}}$ $y\sqrt{1+2x}=e^{\sin x}$ $\sqrt{1+2x}\frac{dy}{dx}+\frac{1}{2}\frac{2}{\sqrt{1+2x}}y=\cos x\, e^{\sin x}$ $\frac{dy}{dx}+\frac{y}{1+2x}=\frac{e^{\sin x}}{\sqrt{1+2x}}\cos x$ $=y\cos x$ $\frac{1}{y}\frac{dy}{dx}+\frac{1}{1+2x}=\cos x$ Alternatively, $\ln\left[y\sqrt{1+2x}\right]=\sin x$ $\frac{\sqrt{1+2x}\left(\frac{dy}{dx}\right)+(1+2x)^{-\frac{1}{2}}}{y\sqrt{1+2x}}=\cos x$ $\frac{1}{y}\frac{dy}{dx}+\frac{1}{1+2x}=\cos x$ (Shown)	This Stude detail quest credit

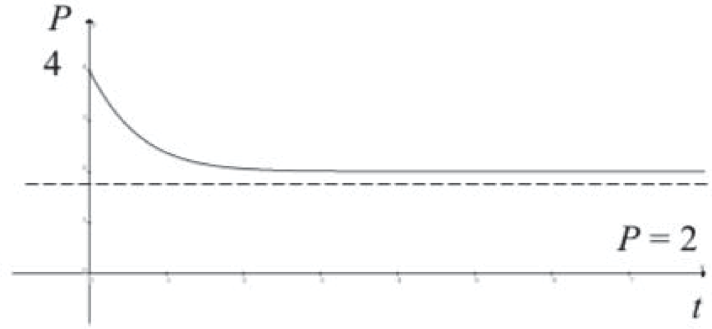
ii	Differentiating again w.r.t. x, $\frac{1}{y}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-\frac{1}{y^2}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2-\frac{2}{(1+2x)^2}=-\sin x$ $\frac{1}{y}\frac{\mathrm{d}^3y}{\mathrm{d}x^3}-\frac{1}{y^2}\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-\frac{2}{y^2}\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\frac{2}{y^3}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3+\frac{8}{(1+2x)^3}=-\cos x$ $\frac{1}{y}\frac{\mathrm{d}^3y}{\mathrm{d}x^3}-\frac{3}{y^2}\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\frac{2}{y^3}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3+\frac{8}{(1+2x)^3}=-\cos x$ When $x=0, y=1, \frac{\mathrm{d}y}{\mathrm{d}x}=0, \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2, \frac{\mathrm{d}^3y}{\mathrm{d}x^3}=-9$. The Maclaurin's series for y is $y=1+\frac{2}{2!}x^2-\frac{9}{3!}x^3+\ldots$ $=1+x^2-\frac{3}{2}x^3+\ldots$	The n questi differ $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}$ $\frac{\mathrm{d}y}{\mathrm{d}x}$ a expre differ $\frac{\mathrm{d}}{\mathrm{d}x}\Big($
iii	$\int_0^1 \frac{e^{\sin x}}{\sqrt{1+2x}}\,\mathrm{d}x \approx \int_0^1 1+x^2-\frac{3}{2}x^3\,\mathrm{d}x$ $=\frac{23}{24}$ $=0.985$ (3 s.f.)	A lot they c integr evalu
iii	$\int_0^1 \frac{e^{\sin x}}{\sqrt{1+2x}}\,\mathrm{d}x=0.058$  The graphs of $y=\frac{e^{\sin x}}{\sqrt{1+2x}}$ and $y=1+x^2-\frac{3}{2}x^3$ differ significantly near to $x=1$. Hence the approximation is not good.	1. Ma the M powe powe appro answe expla good. 2. Ma that x witho affect 3. So range range for ap valid

		not sr less tl good
iv	Replace x with $-x$ $y=\dfrac{\mathrm{e}^{\sin(-x)}}{\sqrt{1-2x}}=\dfrac{\mathrm{e}^{-\sin x}}{\sqrt{1-2x}}=\dfrac{1}{\mathrm{e}^{\sin x}\sqrt{1-2x}}$ $\dfrac{1}{\mathrm{e}^{\sin x}\sqrt{1-2x}}\approx 1+x^2+\dfrac{3}{2}x^3$	Many series conne (ii).
3i	$\lvert p\rvert=\left\lvert\dfrac{a}{1+\sqrt{3}\mathrm{i}}\right\rvert=\dfrac{a}{2}$ $\arg(p)=\arg\left(\dfrac{a}{1+\sqrt{3}\mathrm{i}}\right)$ $=\arg(a)-\arg\left(1+\sqrt{3}\mathrm{i}\right)$ $=0-\dfrac{\pi}{3}$ $=-\dfrac{\pi}{3}$	• Ins ar mu p. mi • A 4^{th} • A
ii	 The shape is a rhombus.	• Al fo ve p sh • Sc di • A qu tri
ii	$\angle QOR=\dfrac{\frac{\pi}{2}-\frac{\pi}{3}}{2}=\dfrac{\pi}{12}$ $\arg(p+q)=-(\dfrac{\pi}{3}+\dfrac{\pi}{12})=-\dfrac{5\pi}{12}$	• Sc ar or ro • Sc ar an

	 $\lvert p+q \rvert = 2\times OS = 2\times\frac{a}{2}\cos\frac{\pi}{12} = a\cos\frac{\pi}{12}$	• M fo • Sc ar
iii	For $(p+q)^n$ to be purely imaginary, $(p+q)^n = (a\cos\frac{\pi}{12})^n[\cos(-\frac{5n\pi}{12}) + \mathrm{i}\sin(-\frac{5n\pi}{12})]$ $\cos(-\frac{5n\pi}{12}) = 0$ Hence the smallest positive integer n is 6, as $\cos(-\frac{5\pi}{2}) = 0$.	• Sc su • a f

4ai	$\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{a}{P}-bP$, where a, b are constants When $P=2$, $\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{a}{2}-2b=0\Rightarrow a=4b$ $\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{4b}{P}-bP$ $=b\left(\frac{4}{P}-P\right)$ $=k\left(\frac{4}{P}-P\right)$, where $k=b$
4aii	When $P=4$, $\frac{\mathrm{d}P}{\mathrm{d}t}=-3$. $\frac{\mathrm{d}P}{\mathrm{d}t}=k\left(\frac{4}{4}-4\right)=-3$ $\Rightarrow k=1$ $\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{4}{P}-P$ $=\frac{4-P^2}{P}$ $\frac{\mathrm{d}t}{\mathrm{d}P}=\frac{P}{4-P^2}$ $=-\frac{1}{2}\frac{-2P}{4-P^2}$ $t=-\frac{1}{2}\ln\left\|4-P^2\right\|+C$ $-2t=\ln\left\|4-P^2\right\|-2C$ $\ln\left\|4-P^2\right\|=2C-2t$ $4-P^2=\pm e^{2C}e^{-2t}$ $=Ae^{-2t}$, where $A=\pm e^{2C}$ When $t=0$, $P=4$. $A=-12$ $4-P^2=-12e^{-2t}$ $P^2=4+12e^{-2t}$ $P=2\sqrt{1+3e^{-2t}}$

4a iii



The population of the bugs will decrease and approach 2000 in the long run.

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Method 1:

$$u = \frac{N}{t}$$

$$\frac{du}{dt} = \frac{t\frac{dN}{dt} - N}{t^2}$$

$$t\frac{du}{dt} = \frac{dN}{dt} - \frac{N}{t}$$

Substitute into $\frac{dN}{dt} = 4 + \frac{N}{t}$:

$$t\frac{du}{dt} = 4$$

$$\frac{du}{dt} = \frac{4}{t}$$

$$u = 4\ln t + C$$

$$\frac{N}{t} = 4\ln t + C$$

When $t = 1$, $N = 1 \Rightarrow C = 1$.

$N = 4t\ln t + t$

Method 2:

$$u = \frac{N}{t}$$

$$N = ut$$

$$\frac{dN}{dt} = \frac{du}{dt}t + u$$

Substitute into $\frac{dN}{dt} = 4 + \frac{N}{t}$:

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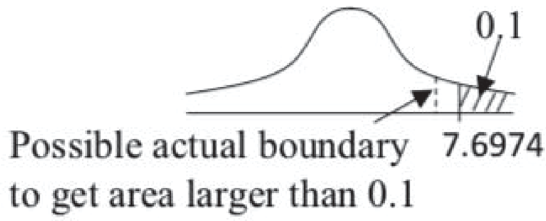
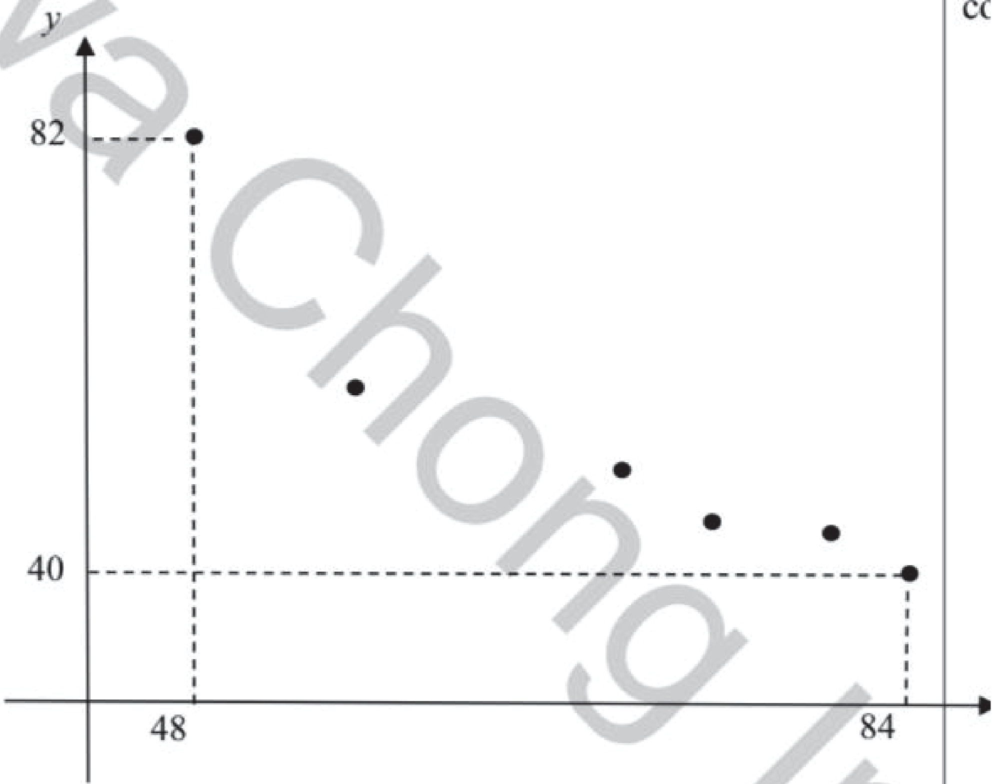
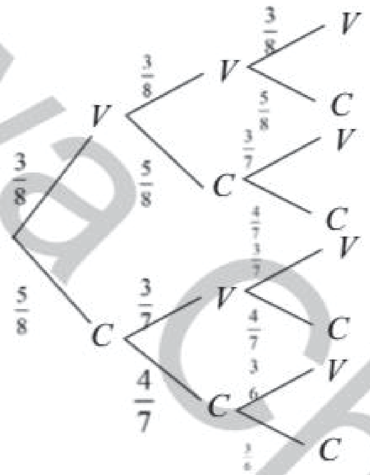
	$\frac{\mathrm{d}u}{\mathrm{d}t}t+u=4+u$ $\frac{\mathrm{d}u}{\mathrm{d}t}t=4$ $\frac{\mathrm{d}u}{\mathrm{d}t}=\frac{4}{t}$ $u=4\ln t+C$ $\frac{N}{t}=4\ln t+C$ When $t=1$, $N=1 \Rightarrow C=1$. $N=4t\ln t+t$	
5 **(i)**	Let X denote the daily rainfall in mm. $X\sim \mathrm{N}\left(\mu,\sigma^2\right)$ In 7 days, $\bar{X}\sim \mathrm{N}\left(\mu,\frac{\sigma^2}{7}\right)$ $\mathrm{P}\left(X>9.8\right)=0.1$ $\mathrm{P}\left(Z>\frac{9.8-\mu}{\sigma}\right)=0.1$ $\frac{9.8-\mu}{\sigma}=1.2815516$ $9.8=\mu+1.2815516\sigma$ --- (1) $\mathrm{P}\left(\bar{X}>8.2\right)=0.1$ $\mathrm{P}\left(Z>\frac{8.2-\mu}{\sigma/\sqrt{7}}\right)=0.1$ $\frac{8.2-\mu}{\sigma/\sqrt{7}}=1.2815516$ $8.2=\mu+0.4843810\sigma$ --- (2) Solving (1) and (2), $\mu=7.22780,\sigma=2.00710$ $\therefore \sigma=2.01\left(2\mathrm{dp}\right)$ (shown)	S μ pa
(ii)	In 30 days, $\bar{X}\sim \mathrm{N}\left(7.22780,\frac{2.01^2}{30}\right)$ $\mathrm{P}\left(\bar{X}>k\right)\geq 0.1$ $k\leq 7.6981\left(5\mathrm{sf}\right)$ $\therefore$ max $k=7.6\left(1\mathrm{dp}\right)$	N al A th S ar

	Note: change in inequality sign because for area to be greater than or equal to 0.1, the value of k can be at 7.6974 or to its left, i.e. less than 7.6974 (see diagram below).  	
6 **(i)**	 	A cc
(ii)	(x_7, y_7) is $(\bar{x}, \bar{y})$ Using G.C, $(\bar{x}, \bar{y}) = (69, 53)$ OR $(x_7, y_7) =$ $\left(\frac{80+84+70+74+58+48}{6}, \frac{44+40+49+45+58+82}{6}\right)$ $= (69, 53)$	M
(iii)	Model A: $y = a + bx^2$ r-value $= -0.92386$ Model B: $y = \ln a + b \ln x$ r-value $= -0.96785$ Model C: $y = a + b\sqrt{x}$ r-value $= -0.95855$ Since $\lvert r \rvert$ for Model B is nearest to 1, Model B is the most accurate model	Sc di to bc C ar cc

(iv)	$y = \ln a + b \ln x$ By GC, $\ln a = 350.11$ $b = -70.469$ $y = 350.11 - 70.469 \ln x$ When $x = 60$, $y = 62$ marks. The student is estimated to score 62 marks.	N us of m A w A ec
(v)	$4 \times 30 = 120$ unreasonable as 120 is not within data range where $48 \le x \le 84$.	Sc hc Tl th ar th m
7 **(i)**	Let X and Y denote the length (in inches) of a “footlong” and a “6-inch” loaf respectively. $X \sim \mathrm{N}\left(12.2, 0.2^2\right)$, $Y \sim \mathrm{N}\left(6.1, \left(0.1\sqrt{2}\right)^2\right)$ $\mathrm{P}(Y < 6) = 0.239750 = 0.240(3\text{sf})$ $\mathrm{P}(X < 12) = 0.158655 = 0.159(3\text{sf}) < \mathrm{P}(Y < 6)$ $\therefore$ a “6-inch” loaf more likely to be less than 6 inches.	A cc va th 6, ga It pı St fu as qu
(ii)	$\mathrm{E}(Y_1 + Y_2 - X) = 2(6.1) - 12.2 = 0$ $\mathrm{Var}(Y_1 + Y_2 - X) = 2\left(0.1\sqrt{2}\right)^2 + 0.2^2 = 0.08$ $Y_1 + Y_2 - X \sim \mathrm{N}(0, 0.08)$ $\mathrm{P}(Y_1 + Y_2 > X) = \mathrm{P}(Y_1 + Y_2 - X > 0) = \dfrac{1}{2}$	It in M th th Sc w as ar

(iii)	Let A denote the number of "6-inch" sandwiches less than 6-inches in length in a week. $A \sim \mathrm{B}(3, 0.239750)$ $\mathrm{P}(A \le 1) = 0.855122 = 0.855\,(3\text{sf})$ <u>Alternative</u> $\text{required probability} = \mathrm{P}(Y<6)\left[\mathrm{P}(Y>6)\right]^2 \times 3 + \left[\mathrm{P}(Y>6)\right]^3$ $= 0.23975(1-0.23975)^2 \times 3 + (1-0.23975)^3$ $= 0.855\,(3\text{sf})$	St va w or di of sa "l S w m "x sa in
(iv)	Let B and C denote the number of "6-inch" sandwiches less than 6-inches in length in a 4-week and 3-week period respectively. $B \sim \mathrm{B}(12, 0.239750)$ and $C \sim \mathrm{B}(9, 0.239750)$ $\mathrm{P}(A=1 \mid B>4) = \dfrac{\mathrm{P}(A=1)\mathrm{P}(C>3)}{\mathrm{P}(B>4)}$ $= \dfrac{\mathrm{P}(A=1)\left[1-\mathrm{P}(C\le 3)\right]}{1-\mathrm{P}(B\le 4)}$ $= \dfrac{0.41571 \times 0.14710}{0.13721}$ $= 0.445674$ $= 0.446\,(3\text{sf})$	Tl ch fr Sc cc Sc de nu 4t P = Sc 2 up P — =
8 (a)(i)	Number of arrangements $= \dfrac{8!}{3!2!} = 3360$	Sc re
8 (a)(ii)	Number of arrangements $= \dfrac{5!}{2!} \times {}^6\mathrm{C}_3 = 1200$	M cc 2
8 (a)(iii)	Arrange 3 vowels (into a group) $\dfrac{3!}{2!} = 3$	M es er

	Arrange 5 consonants (for slotting) $\frac{5!}{3!}=20$ Since the ends must be consonants, only the middle 4 slots can be used for the group to slot. (ie. ${}^4C_1$) Number of arrangements$=3\times20\times4=240$ OR Case 1: First and Last 'L' $4!\times\frac{3!}{2!}=72$ Case 2: First and last 'L' and non-L $2\times2\times\frac{4!}{2!}\times\frac{3!}{2!}=144$ Case 3: First and last non- L $2\times\frac{4!}{3!}\times\frac{3!}{2!}=24$ Total= 240 ways	
(b)(i)	P(1V) + P(2V) + P(3V) $=\frac{{}^5C_3\,{}^3C_1+{}^5C_2\,{}^3C_2+{}^5C_1\,{}^3C_3}{{}^8C_4}$ $=\frac{65}{70}=\frac{13}{14}$ Probability = 1 – P(all consonants) $=1-\left(\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}\times\frac{2}{5}\right)=\frac{13}{14}$ OR 1 – P(no vowels) $=1-\frac{{}^5C_4}{{}^8C_4}$ $=1-\frac{5}{70}$ $=\frac{13}{14}$	G

	OR P(1 vowel) + P(2 vowels) + P(3 vowels) $=\frac{3\times5\times4\times3}{8\times7\times6\times5}\times\frac{4!}{3!}+\frac{3\times2\times5\times4}{8\times7\times6\times5}\times\frac{4!}{2!2!}+\frac{3\times2\times1\times5}{8\times7\times6\times5}\times\frac{4!}{3!}$ $=\frac{13}{14}$	
8 (b)(ii)	 Probability = P(VCC)+P(CVC)+P(CCV)+P(CCC) $=\frac{3}{8}\times\frac{5}{8}\times\frac{4}{7}+\frac{5}{8}\times\frac{3}{7}\times\frac{4}{7}+\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}+\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}$ $=\frac{505}{784}$ $=0.644$	M di
9(i)	Using GC, Unbiased estimate of population mean is $\bar{x}=949.84\ (2\text{ d.p.})$ Unbiased estimate of population variance is $s^2=1.31634^2=1.73\ (2\text{ d.p.})$ Note: If student write $\mu=949.84$ or $\sigma^2=1.31634^2=1.73$, annotate but do not deduct.	G St *ar* ar gi T m st 1. ge 2.

		N 1. σ sa U va (1 (1 fo **N** μ
9 (a) (ii)	Let X be the volume of shower gel dispensed by the machine. Let μ denote the population mean volume of shower gel dispensed by the machine. $\bar{X} \sim \mathrm{N}\left(\mu, \frac{s^2}{n}\right)$ approx. by Central Limit Theorem. $\bar{X} \sim \mathrm{N}\left(950, \frac{s^2}{n}\right)$ $\mathrm{H_0}$: $\mu = 950$ $\mathrm{H_1}$: $\mu < 950$	Pa ar M th de 's de of ex th as er
9 (a)(iii)	Under $\mathrm{H_0}$, using GC, $p-\text{value} = 0.13474 \approx 0.135$	R m fa st co

	The p - value is the probability of the sample mean volume of shower gel in a bottle is as extreme as 949.84 when the mean is actually 950.	Tl w 9 m M th cc S th w of hy st p- cc
9 (a)(iv)	Since we reject H_0 , $p-\text{value} < \alpha / 100$ $\alpha \geq 13.476$ $\{\alpha \in \mathbb{R} : 13.5 \leq \alpha \leq 100\}$	B di nc uj St us in ro th
9(b)	Let Y be the volume of shower gel dispensed by the machine after recalibration. Let μ denote the population mean volume of refill dispensed by the machine. $\bar{Y} \sim \mathrm{N}\left(\mu, \dfrac{s^2}{n}\right)$ approx. by Central Limit Theorem H_0: $\mu = 250$ H_1: $\mu \neq 250$ Test statistic: $Z = \dfrac{\bar{Y}-\mu}{s/\sqrt{n}}$ Level of significance: 1% Reject H_0 if $p-\text{value} \leq 0.01$	C th 1. 2. to 3. as cc 4.

$$\frac{249.5-250}{s/\sqrt{50}} \le -2.5758293 \quad \text{or} \quad \frac{249.5-250}{s/\sqrt{50}} \ge 2.5758293 \text{ (NA)}$$

$$0 < \frac{s}{\sqrt{50}} \le 0.19411$$

$$0 < s \le 1.3726$$

$$0 < \frac{1}{49}\left(k - \frac{(-25)^2}{50}\right) \le 1.3726^2$$

$$12.5 < k \le 104.8$$

9(c)

There is no need for the floor supervisor to assume the volume of shower gel follow a normal distribution as the sample sizes in both part (a) and (b) are large. The sample mean volume of shower gel can be approximated to follow a normal distribution by Central Limit Theorem.

10 (i)

Let $\$W$ denote the amount won by a player in one game.
Under option A,

$$\mathrm{E}(W_A) = 1\left(\frac{1}{4}\right) + 2\left(\frac{1}{4}\right) + 3\left(\frac{1}{4}\right) + 4\left(\frac{1}{4}\right)$$

$$= (1+2+3+4)\left(\frac{1}{4}\right)$$

$$= \frac{5}{2} = \$2.50$$

Under option B, the winning amount for each combination is listed below:

x \ y	1	2	3	4
1	0	4	6	8
2	-1	0	12	16
3	-2	-1	0	24
4	-3	-2	-1	0

	$\mathrm{E}(W_B)=\left(\frac{1}{16}\right)[4+6+12+8+16+24$ $-1-2-3-1-2-1]$ $=\left(\frac{1}{16}\right)[70-10]$ $=\frac{15}{4}=\$3.75$ Since $\mathrm{E}(W_B)>\mathrm{E}(W_A)$, option B is better. (shown)	ca ar It ex ar O
(ii)	Option A is a "sure win" option where the player would definitely gain a positive amount in all cases, whereas option B has a risk of losing money in some cases.	G
(iii)	$\mathrm{E}(W_A{}^2)=(1^2+2^2+3^2+4^2)\left(\frac{1}{4}\right)=\frac{15}{2}$ $\mathrm{Var}(W_A)=\mathrm{E}(W_A{}^2)-[\mathrm{E}(W_A)]^2=\frac{15}{2}-\left(\frac{5}{2}\right)^2=\frac{5}{4}=1.25(\text{shown})$	G Tl st cc
(iv)	Let A and B denote the total amount won by Abel and Benson respectively. $A=W_{A1}+W_{A2}+\cdots+W_{A50}$ Since $n=50$ is large, by Central Limit Theorem, $A\sim \mathrm{N}\left(50\left(\frac{5}{2}\right),50\left(\frac{5}{4}\right)\right)$ approximately i.e. $A\sim \mathrm{N}\left(125,\frac{125}{2}\right)$ $B=W_{B1}+W_{B2}+\cdots+W_{B50}$ $B\sim \mathrm{N}\left(50\left(\frac{15}{4}\right),50\left(\frac{887}{16}\right)\right)$ approximately by Central Limit Theorem since $n=50$ is large $B\sim \mathrm{N}\left(\frac{375}{2},\frac{22175}{8}\right)$	A w va ar w th Tl ap of ar O - - - - -

		-
(v)	$A-B \sim \mathrm{N}\left(125-\frac{375}{2}, \frac{125}{2}+\frac{22175}{8}\right)$ $A-B \sim \mathrm{N}\left(-\frac{125}{2}, \frac{22675}{8}\right)$ $\mathrm{P}(A>B)=\mathrm{P}(A-B>0)$ $=0.120207$ $=0.120(3\mathrm{sf})$	M at re pı A cc aı fo - -