



Name: _____ () Date: _____
 Class: Secondary 4 / _____

REVISION WORKSHEET : MATRICES

Prior Knowledge - Matrices

✓ Types of Matrices

- Row Matrix - Matrix which has only one row
 Eg. (2 3), (4 3 7 13)

- Column Matrix - Matrix which has only one column

$$\text{Eg. } \begin{pmatrix} 1 \\ 20 \\ 5 \\ 3 \\ 9 \end{pmatrix}$$

- Square Matrix - Matrix which has the same number of rows and columns

$$\text{Eg. } \begin{pmatrix} 3 & 4 \\ 5 & 7 \end{pmatrix}, \begin{pmatrix} -2 & 3 & 10 \\ -5 & 0 & 1 \\ 8 & -5 & 6 \end{pmatrix}$$

- Zero Matrix - Denoted by 0, also known as null matrix, has all its elements as zero
 Eg. $0 = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}, 0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

- Identity Matrix - Denoted by I, has its leading diagonal elements as ones while all other elements as zeros

$$\text{Eg. } I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

✓ Addition and Subtraction of Matrices

Matrices can be added or subtracted if they are of the same order.

$$\text{Given } A = \begin{pmatrix} 2 & 4 & 3 \\ 3 & 6 & 5 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 8 & 2 \\ 4 & 7 & 1 \end{pmatrix}$$

$$\text{Then } A+B = \begin{pmatrix} 2+1 & 4+8 & 3+2 \\ 3+4 & 6+7 & 5+1 \end{pmatrix} \quad \text{and} \quad A-B = \begin{pmatrix} 2-1 & 4-8 & 3-2 \\ 3-4 & 6-7 & 5-1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 12 & 5 \\ 7 & 13 & 6 \end{pmatrix} \quad = \begin{pmatrix} 1 & -4 & 1 \\ -1 & -1 & 4 \end{pmatrix}$$

- Matrix addition is commutative: $A+B = B+A$
- Matrix addition is associative: $(A+B)+C = A+(B+C)$

✓ Scalar Multiplication of a Matrix

When a matrix is multiplied by a real number (known as scalar), each element of the matrix is multiplied by the real number.

$$\text{Given } A = \begin{pmatrix} 1 & 4 \\ 2 & 6 \end{pmatrix}$$

$$\text{Then } 3A = 3 \begin{pmatrix} 1 & 4 \\ 2 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \times 1 & 3 \times 4 \\ 3 \times 2 & 3 \times 6 \end{pmatrix} = \begin{pmatrix} 3 & 12 \\ 6 & 18 \end{pmatrix}$$

✓ Multiplication of Matrices

Only matrices that are compatible can be multiplied, i.e. number of columns in first matrix = number of rows in second matrix.

$$\text{Given } A = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 6 & 5 \end{pmatrix}$$

$$B = \begin{pmatrix} 5 & 6 \\ 2 & 7 \\ 3 & 9 \end{pmatrix}$$

$$\text{Then } AB = \begin{pmatrix} (4)(5) + (2)(2) + (1)(3) & (4)(6) + (4)(7) + (5)(9) \\ (3)(5) + (6)(2) + (5)(3) & (3)(6) + (6)(7) + (5)(9) \end{pmatrix} = \begin{pmatrix} 27 & 97 \\ 42 & 105 \end{pmatrix}$$

- Unlike real numbers, matrices are not commutative under matrix multiplication
 i.e. $AB \neq BA$.

Skills

- ✓ Able to display information in the form of matrix of any order
- ✓ Able to interpret data in a given matrix
- ✓ Able to multiply a scalar quantity to a matrix
- ✓ Able to add, subtract and multiply two matrices

Objectives

- ✓ Solve real-life problems by representing data using matrices

1. ABC Supermarket sells two types of baby diapers, Superdry Diapers and Happy Diapers. Each type of diapers is sold in packets of three different sizes, small (S), medium (M) and large (L). The sales in two successive weeks are given in the table below.

	First Week			Second Week		
	S	M	L	S	M	L
Superdry Diapers	10	16	15	12	20	19
Happy Diapers	14	20	17	15	18	14

The information for the first week's sales can be represented by the matrix

$$W = \begin{pmatrix} 10 & 16 & 15 \\ 14 & 20 & 17 \end{pmatrix}$$

The information for the second week is represented by a 2×3 matrix X .

- Write down the matrix X .
- Calculate $W + X$.
- Describe what is represented by the elements in $W + X$.
- Given that $Y = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, calculate $Z = WY$.
- Describe what is represented by the elements of Z .

2. A confectionery sells small, median and large strawberry cakes and chocolate cakes. The number of each type of cake that was sold on a particular week are summarised in the following table.

	Chocolate		Strawberry	
	Small	Medium	Large	Small
Small	10	18	15	12
Medium		17		
Large			24	

- The prices of each small, medium and large cakes are \$8, \$15 and \$20 respectively and the matrix $P = \begin{pmatrix} 8 & 15 & 20 \end{pmatrix}$ can represent the information.
- The information on the above table can be represented by a matrix Q . Write down the matrix Q .
 - $N = PQ$, evaluate N and describe the elements in N .
 - Write down a matrix M such that the multiplication of matrices M and N gives the total amount of money collected by the confectionery shop on the particular week. Find this amount of money by matrix multiplication.

3. Bandung, Ice Milo and Soya Bean are drinks sold by 4 local fast food outlets. The number of cups of each type of drink is shown in the table below.

Name of Fast Food Outlet	Number of cups sold				
	Ali's	MachHusseins	Pereira's	Ho's Fast Food	
Types of Drink	Bandung	70	240	30	200
	Ice Milo	130	100	160	0
	Soya Bean	50	0	50	140

- Represent the above information as a matrix P .
- During a local drink promotion week where the proceeds are donated to the SPCA, the four outlets charge a standard price for any cup of their drinks. Ali sells his drinks at \$1.40 per cup, MachHusseins at \$1.80 per cup while Pereira's and Ho's Fast Food charge \$1.50 per cup.
- Evaluate $(1 \ 1 \ 1 \ 1)P$ and explain what the elements of the matrix product represent.
 - Using the above matrix product and another matrix, find the total amount of money that will be collected from the four Fast Food outlets from the sale of all the three drinks and hence given to the SPCA.

4. The table below shows the exchange rates of foreign currencies against one Singapore dollar.

Singapore dollar (S\$)	United States (US\$)	Hong Kong (HK\$)	Malaysia (RM)
1	0.65	5.00	2.20

- Represent the exchange rate by a 3×1 matrix E .
- Let $S = 1000I$ where I is a 3×3 identity matrix. Evaluate and explain the possible meanings of the non-zero elements in SE .
- Find, by matrix multiplication, the total amount of S\$ that can be exchanged for a combined sum of US\$390, HK\$1200 and RM2200.

Answers:

1. (a) $X = \begin{pmatrix} 12 & 20 & 19 \\ 15 & 18 & 14 \end{pmatrix}$ (b) $W + X = \begin{pmatrix} 22 & 36 & 34 \\ 29 & 38 & 31 \end{pmatrix}$

(c) Total no. of small, medium and large packets of Superdry and Happy diapers sold respectively for 2 weeks.

(d) $Z = \begin{pmatrix} 41 \\ 51 \end{pmatrix}$

(e) No. of packets of Superdry and Happy diapers sold respectively in the first week.

2.

(a) $Q = \begin{pmatrix} 10 & 12 \\ 18 & 17 \\ 15 & 24 \end{pmatrix}$

(b) $N = \begin{pmatrix} 650 & 831 \end{pmatrix}$

The total amount of money collected for chocolate and strawberry cakes respectively.

(c) $M = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, \$1481

3. (a) $\begin{pmatrix} 70 & 240 & 30 & 200 \\ 130 & 100 & 160 & 0 \\ 50 & 0 & 5 & 14 \end{pmatrix}$

(b) (i) (250 340 240 340): Each element reflects the total number of cups sold of all three drinks altogether by each outlet.

(ii) \$1832

4. (a) $E = \begin{pmatrix} 0.65 \\ 5 \\ 2.2 \end{pmatrix}$

(b) $S = 1000I$ $SE = \begin{pmatrix} 650 \\ 5000 \\ 2200 \end{pmatrix}$

\$S1000 can be exchanged for US\$650, HK\$5000 and RM2200.

(c) $\begin{pmatrix} 1840 \end{pmatrix}$

The total amount of \$S that can be exchanged for a combined sum of US\$390, HK\$1200 and RM2200 is \$S1840.

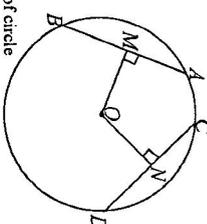
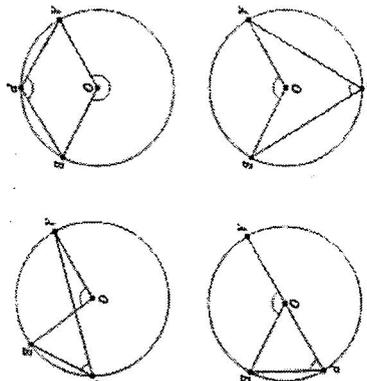
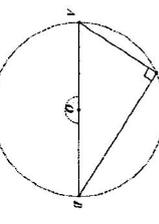
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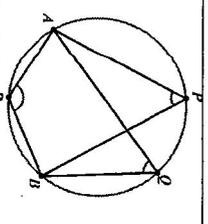
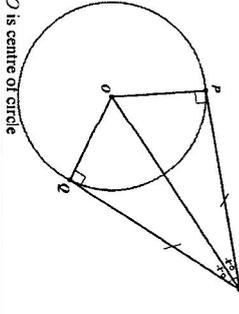


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REVISION WORKSHEET : PROPERTIES OF CIRCLES AND RADIAN MEASURE

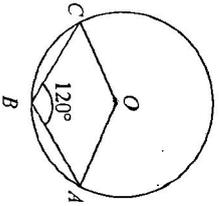
Prior Knowledge / Essential Concepts

<p>• Circle Properties</p>  <p>O is centre of circle</p> <p>Tip: Two of the three conditions (line from centre; midpoint of chord; line is perpendicular to chord) imply the third. For this property, you can state the reason as "\perp from centre bisects chord"</p>	<p>If $AB = CD$, $OM \perp AB$ and $ON \perp CD$, then $OM = ON$ (equal chords, equidistant from centre)</p> <p>If $OM = ON$, $OM \perp AB$ and $ON \perp CD$, then $AB = CD$ (chords equidistant from centre are equal)</p> <p>If $\angle OMA = 90^\circ$, $AM = BM$ (\perp from centre bisects chord)</p> <p>If $AM = BM$ and O is centre of circle, then $\angle OMA = 90^\circ$ (line through midpoint of chord and centre of circle \perp chord)</p> <p>If $\angle OMA = 90^\circ$ and $AM = BM$, then OM passes through centre of circle (\perp bisector of chord)</p>
 <p>O is centre of circle</p>	<p>If $\angle AOB$ and $\angle APB$ are subtended by the same arc, then $\angle AOB = 2 \times \angle APB$ (\angle at centre = $2 \times \angle$ at circumference)</p> <p>Note: Question must state that O is centre of circle.</p>
 <p>O is centre of circle and AB is diameter</p>	<p>If AOB is the diameter, then $\angle ADB = 90^\circ$ (right \angle in semi-circle)</p> <p>Note: Question must state that O is centre of circle and AOB is a straight line or that AB is the diameter.</p>

	<p>If $\angle APB$ and $\angle AOB$ are in the same segment, then $\angle APB = \angle AOB$ ($\angle s$ in same segment).</p> <p>If $\angle APB$ and $\angle ANB$ are in the opposite segments, then $\angle APB + \angle ANB = 180^\circ$ ($\angle s$ in opposite segments).</p>
 <p>O is centre of circle</p>	<p>If TP is a tangent to a circle and OP is radius of the circle, then $OP \perp TP$ (tangent \perp radius)</p> <p>If TP and TQ are tangents to a circle and OP and OQ are radii of circle, then $TP = TQ$ and $\angle PTO = \angle QTO$ (tangents from external point)</p> <p>Note: Converse of the above are true as well.</p> <p>Question must state that O is centre of circle and TP and TQ are tangents before you can apply the property.</p>
<p>• Radian Measure (θ is an angle in radians)</p> <ul style="list-style-type: none"> ○ 2π radians = 360° ○ Arc length = $r\theta$ ○ Area of sector = $\frac{1}{2}r^2\theta = \frac{1}{2}rs$ (s = arc length) ○ Area of segment = $\frac{1}{2}r^2\theta - \frac{1}{2}ab \sin C = \frac{1}{2}r^2\theta - \frac{1}{2} \times \text{base} \times \text{height}$ <p>Exam tips:</p> <ul style="list-style-type: none"> • Read the question carefully and transfer the necessary information into the diagram. Highlight any information you are unable to transfer (you will need it later). • Recall properties of circles and deduce all the angles you are able to find. • Tips: <ul style="list-style-type: none"> • If there is a circle, is there a centre mark? If yes, there'll be a radius → Look for isosceles triangle → Look for angle at centre = $2 \times$ angle at circumference • If diameter is marked, there might be a semicircle → angle in semicircle = 90° ○ If center of circle not marked <ul style="list-style-type: none"> → Look for angles in the same segment → Look for angles in the opposite segment • Is there a tangent? And a center? Then, tangent perpendicular to radius • If there are external points, then look for equal lengths • If need to prove for midpoint, then likely from similar triangles or isosceles triangles. • Avoid converting radians to degrees as this will affect your accuracy. 	

What's wrong?

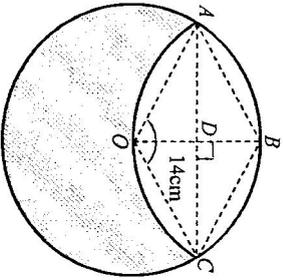
Given that $\angle ABC = 120^\circ$, find $\angle AOC$.



Solution:

$$\begin{aligned} \angle AOC &= 120 \times 2 \quad (\text{at centre} = 2 \times \angle \text{at circumference}) \\ &= 240^\circ \end{aligned}$$

Practice Questions



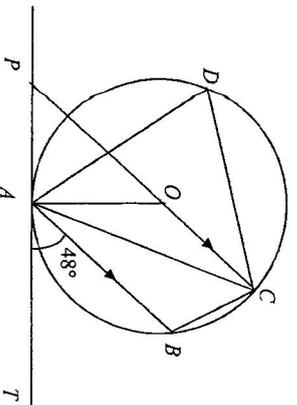
The diagram shows three points A , B and C on the circumference of a circle, centre O and radius 14 cm. ABC is a segment such that when it is folded along AC , B meets O . AC and OB meet at the point D .

- State the shape of the quadrilateral $OABC$.
- Show that the length AC is 24.2 cm

Find,

- $\angle AOC$ in radians,
- the area of triangle ABC ,
- the area of segment $ADCB$,
- the area of the shaded region

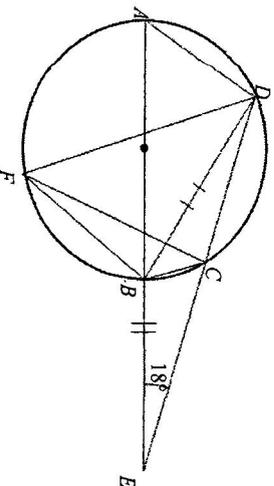
2.



In the diagram, O is the centre of the circle of radius 7 cm, PAT is the tangent to the circle at A , AB is parallel to OC , $\angle BAT = 48^\circ$ and AC bisects the angle OAB .

- Calculate, stating the reasons clearly
 - $\angle OAC$,
 - $\angle ADC$,
 - $\angle CPA$.
- Calculate the area of the sector OCA .

3.



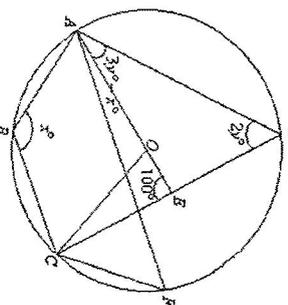
AB is the diameter of the circle $AFBCD$ as shown in the diagram. E is the point on AB produced where $DB = BE$ and $\angle AED = 18^\circ$. The straight line ED cuts the circle at C .

- Explain why $\angle CFB = 18^\circ$.
- Find $\angle ABC$.
- Show that BD bisects $\angle ABC$.
- Given also that $\angle BDF = 51^\circ$, calculate $\angle FBC$.

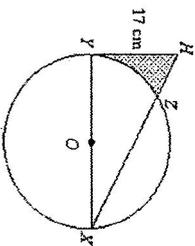
4.

In the diagram, the points A , B , C , F and D lie on the circle with centre O . CED is a straight line, $\angle AEC = 100^\circ$, $\angle ADC = 2y^\circ$, $\angle ABC = x^\circ$ and $\angle DAE = 3y^\circ - x^\circ$.

- Give a reason why
 - $x + 2y = 180$,
 - $3y - x = 100$
- Find the value of x and of y .
- Hence, calculate, giving the reasons,
 - $\angle AOC$
 - $\angle AFC$
 - $\angle ACD$.



5. The diagram below shows a circle with centre O and radius of 12 cm. XY is a diameter of the circle and YH is a tangent to the circle at Y with $YH = 17$ cm.

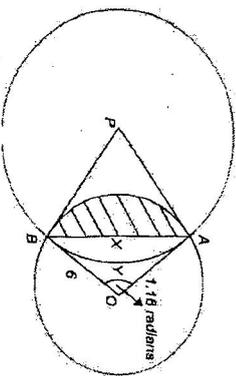


- (a) Show that $\angle ZOY = 1.23$ radians correct to 3 significant figures.

Calculate

- (b) (i) the length of minor arc ZY ,
 (ii) the area of ΔXOZ ,
 (iii) the area of minor sector OYZ ,
 (iv) the area of the shaded region.

6. (a) The points A and B lie on a circle centre O . $AO = BO = 6$ cm and $\angle AOB = 1.16$ radians.



- (i) Find the length of minor arc AB .
 (ii) Show that the area of the sector OAB is 20.88 cm².
 (iii) Calculate the area of the shaded segment.

- (b) A second circle, centre P , cuts the first circle at A and B . Radius of the second circle is 12 cm. Calculate

- (i) $\angle PO_1$,
 (ii) angle APB , in radians,
 (iii) perimeter of the segment $AYBX$.

Answers:

- (a) Rhombus (b) $\angle \hat{O}C = 2.09$ rad (d) 84.9 cm² (e) 120 cm² (f) 375 cm²
- (a)(i) 21° (ii) 69° (iii) 48° (b) 59 cm²
- (b) 72° (d) 111°
- (b) $x = 100$ and $y = 40$ (c) (i) 160° (ii) 80° (iii) 70°
- (b) (i) 14.8 cm (ii) 67.9 cm² (iii) 88.7 cm² (iv) 47.3 cm²
- (a) (i) 6.96 cm (iii) 4.38 cm² (b) (i) 16.6 cm (3s.f.) (ii) 0.555 rad(3s.f.) (iii) 13.2 cm (3s.f.)



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REVISION WORKSHEET : SET LANGUAGE & NOTATION

Prior Knowledge

- Use of set language and the following notation:

Union of A and B	$A \cup B$
Intersection of A and B	$A \cap B$
Number of elements in set A	$n(A)$
"... is an element of ..."	\in
"... is not an element of ..."	\notin
Complement of set A	A'
The empty set	\emptyset
Universal set	\mathcal{E}
A is a subset of B	$A \subseteq B$
A is a proper subset of B	$A \subset B$
A is not a subset of B	$A \not\subseteq B$
A is not a proper subset of B	$A \not\subset B$

- Union and intersection of two sets
- Venn diagrams

Skills

- Able to use set language and notation to represent solutions
- Able to draw and use Venn diagrams to represent solutions

Objectives

- Apply set language and notation to represent solutions
- Draw and apply Venn diagrams in solving real-life problems

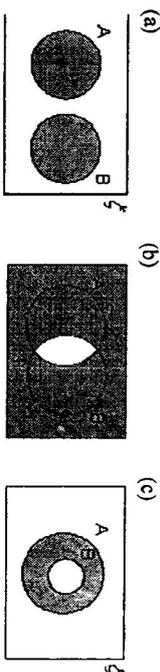
Key teaching points

- Is E equal to F if $n(E) = n(F)$?
 $E = \{\text{odd numbers less than 11}\}$, $F = \{\text{odd numbers less than 11}\}$
 Find $n(E)$ and $n(F)$.
- If $G = \{\text{even numbers less than 11}\}$, G a null set?

Practice Questions

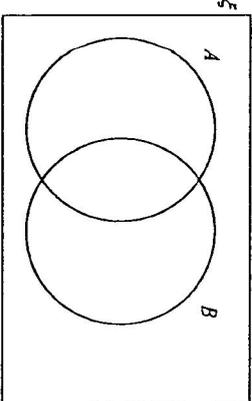
- Illustrate on separate Venn diagrams, the sets \mathcal{E} , A and B if,
 - $A \cap B = A$,
 - $A \cup B = A$,
 - $n(A \cap B) = 0$.

- Write the set notation for the sets shaded in the following Venn diagrams.



- Let $A = \{\text{points lying on the line } y = x + 2\}$ and $B = \{\text{points lying on the line } 3x + y = 4\}$
 - Show that $(1, 3) \in A$.
 - Find the element p such that $p \in (A \cap B)$.
 - If $(2, q) \in B$, find the value of q .
 - If $C = \{\text{points lying on the line } 2y - 2x = 8\}$, explain clearly why $A \cap C = \emptyset$.

- Shade the region $(A \cup B) \cap (A \cap B)$.
- Given that $\mathcal{E} = \{\text{bulbs}\}$, $B = \{\text{black bulbs}\}$, $W = \{\text{white bulbs}\}$, $S = \{\text{strong bulbs}\}$ and $E = \{\text{weak bulbs}\}$, express in set notation each of the following statements:
 - All strong bulbs are white.
 - No bulb can be both strong and weak.

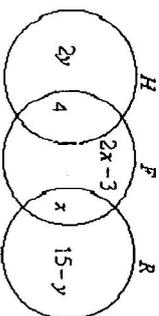


5. The findings of a survey involving 80 students on food they consume during their recess break are as follows:

59 students eat noodles/rice only.
 10% of the students who took part in the survey are Holistic Club members.
 Holistic Club members had only fruits.
 There are 7 times as many students who ate as students who did not eat.

- (a) Draw a Venn diagram to illustrate this information.
 (b) Find the number of students who,
 (i) did not eat,
 (ii) had fruits,
 (iii) had noodles/rice and fruits.

6. Each of the 39 patients, who consulted a doctor, had at least one of the following symptoms: a runny nose (R), a headache (H) and a fever (F). The Venn diagram shows the number of patients in each subset of the sets R , H and F .



- (a) Express y in terms of x .
 (b) Given that the number of patients who had a headache is four times as many as those who had both a fever and a runny nose, find the value of x and y .
 (c) Write down the number of patients who had a fever.

7. In a Secondary Four class of 35 pupils, $C = \{\text{pupils who play chess}\}$ and $S = \{\text{pupils who play scrabble}\}$. Given that $n(C) = 24$ and $n(S) = 20$, find by drawing appropriate Venn diagrams or otherwise, the greatest and smallest possible values of $n(C \cap S)$.

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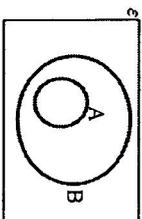
8. $E = \{x : x \text{ is a factor of } 24\}$,
 $P = \{x : x \text{ is a multiple of } 3\}$,
 $Q = \{x : x \text{ is divisible by } 12\}$,
 $R = \{1, 2, 4, 8\}$.
- (a) (i) List the elements of the sets E , P and Q .
 (ii) Find $n(P \cup Q)$.
 (iii) Find $P \cap R$.
 (iv) Is $\{3, 6\} \in P$?
 (v) Describe the set R by using the set-builder notation.

2E | EOY | 2007 | Cedar Girls' Sec Sch

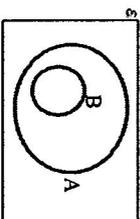
Answer Key:

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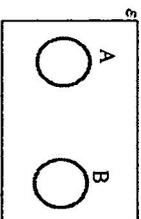
(a)



(b)



(c)



2. (a) $A \cup B$

(b) $(A \cap B)$

(c) $A \cap B'$

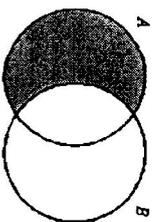
3. (a) Yes

(b) $(0.5, 2.5)$

(c) $q = -2$

(d) Line A // to Line C hence no intersection point

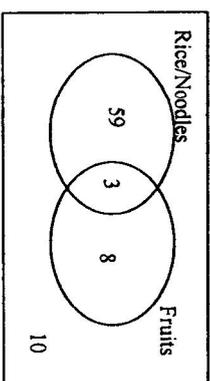
4. (a)



(b) $S \subset W$

(c) $S \cap E = \emptyset$

5.



5

(b) (i) 10 (ii) 11 (iii) 3

6. (a) $y = 23 - 3x$ (b) $x = 5$ and $y = 8$ (c) $3x + 1 = 16$

7. (a) 15 (b) 4

8. (ii) 6 (iii) \emptyset (iv) $3, 6 \in P$ but $\{3, 6\} \subset P$ (v) $R = \{x : x \text{ is a factor of } 8\}$



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REVISION WORKSHEET : CONGRUENCE & SIMILARITY

Prior Knowledge / Essential Concepts

- Congruent Triangles – Test for Congruent Triangles
 - SSS (3 sides)
 - SAS (2 sides, 1 angle)
 - ASA, AAS or SAA (1 side, 2 angles)
 - RHS (⊥, hypotenuse, 1 side)
- Similar Triangles – Test for Similar Triangles
 - 2 pairs of corresponding angles
 - 3 pairs of corresponding sides – same ratio
 - 2 pairs of corresponding sides – same ratio, and 1 pair of included angle

- Area of Similar Figures

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

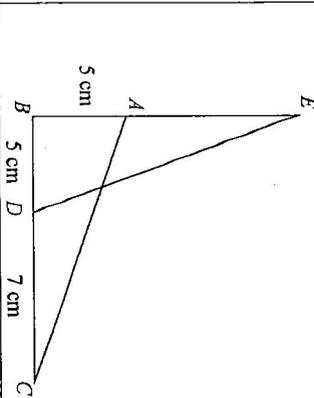
$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

Notes

- AAA proof applies for similar figures and not for congruent figures!
- Reasons to be provided at the end of the congruency test i.e. (_____)
- ASS is not a test for congruency!
- Parallel lines do not prove similarity!

What's wrong?

Given that $\angle ABC = 90^\circ$, BAE and BDC are straight lines and $DE = 13\text{cm}$, show that Triangles ABC and DBE are congruent.



Solution:

$$\angle ABC = \angle DBE = 90^\circ$$

$$AB = DB \text{ (given)}$$

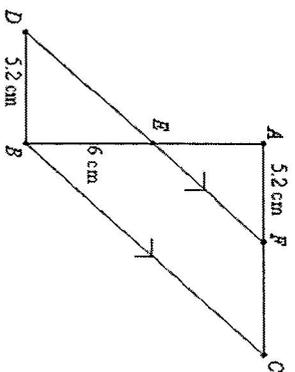
$$BE = \sqrt{13^2 - 5^2}$$

$$= 12$$

$$= BC$$

So, Triangles ABC and DBE are congruent (RHS).

1. In the diagram, $DF \parallel BC$, $AF = BD = 5.2\text{ cm}$, $BE = 6\text{ cm}$ and $\angle AFE = \angle BDE$.



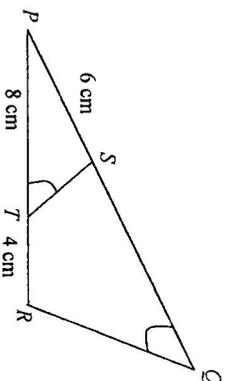
- (a) Show that $\triangle DBE$ is congruent to $\triangle FAE$.
 (b) Show that $\triangle AEF$ is similar to $\triangle ABC$.
 (c) Find the length of FC .
 (d) Calculate the area of $\triangle AEF$ if the area of $\triangle ABC$ is 50 cm^2 .
 (e) Hence, find the area of the trapezium $BCFE$.

3E | EOY | 2008 | Hong Kah Sec Sch

2. In the figure, RTP and QSP are straight lines, and $\angle PQR = \angle PTS$.

- (a) Stating your reasons clearly, show that $\triangle PQR$ and $\triangle PTS$ are similar.
 (b) Given that $PS = 6\text{ cm}$, $PT = 8\text{ cm}$ and $TR = 4\text{ cm}$, calculate

- (i) the length QS .
 (ii) $\frac{\text{area of } \triangle PQR}{\text{area of } \triangle PTS}$,
 (iii) $\frac{\text{area of } \triangle PQR}{\text{area of } \triangle QRTS}$.

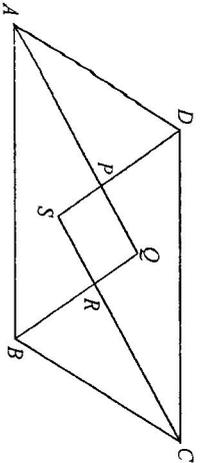


Zhonghua Sec School 2009

3. The volume of two gold spheres are 640 cm^3 and 1250 cm^3 .
 (a) Find, in its simplest form, the ratio of
 (i) the smaller radius to the larger radius,
 (ii) the smaller surface area to the larger surface area.
 (b) The larger sphere has a mass of 25 kg . Find the mass of the smaller sphere.

Nov 2009/1/15

4. The diagram shows a parallelogram, $ABCD$.

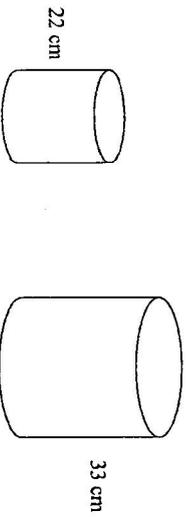


APQ , BKQ , CRS and DPS are straight lines which bisect angles A , B , C and D respectively.

- Show that $\angle PAD = \angle RCB$.
- Prove that triangles ADP and CBR are congruent.
- Show that
 - $\angle DPA = 90^\circ$,
 - $\angle PQR = 90^\circ$.

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5.

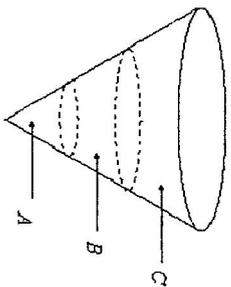


- The two containers shown above are geometrically similar. Their heights are 22 cm and 33 cm.
- The diameter of the base of the larger container is 12 cm. Calculate the diameter of the smaller container.
 - Every part of the surface of the container is painted orange. Given that it costs \$350 to paint the larger container, find the cost of painting the smaller container, correct to the nearest cents.
 - The containers are completely filled with oil. Given that the smaller container holds 5 litres of oil, find the amount of oil the larger container holds.

Ximmin Sec Sch 2009

6. The diagram shows a solid cone which is cut horizontally into three portions of equal thickness. Find the numerical value of the ratios

- $\frac{\text{Volume of } A}{\text{Volume of whole cone}}$
- $\frac{\text{Volume of } A}{\text{Volume of } C}$
- $\frac{\text{Curved Surface Area of } A}{\text{Curved Surface Area of } B}$



3E | EOY | 2007 | Canberra Sec Sch

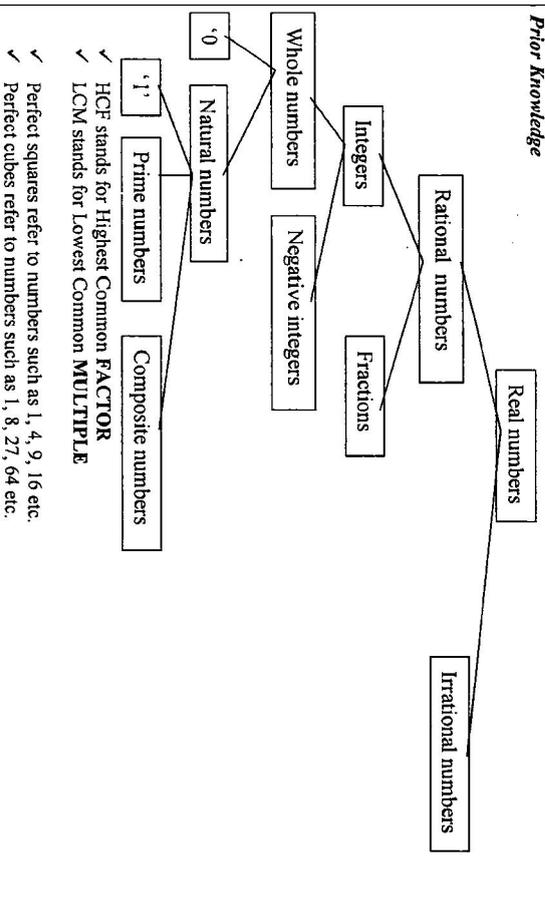
- Answers :**
- (a) 5.2 cm (d) 12.5 cm^2 (e) 37.5 cm^2
 - (b) 10 cm (bii) $\frac{2}{3}$ (biii) $\frac{1}{3}$
 - (ai) 4 : 5 (aii) 16 : 25 (b) 12.8 kg
 - (bi) 8 cm (bii) \$155.56 (biii) $16\frac{7}{8}$
 - (a) $\frac{1}{27}$ (b) $\frac{1}{19}$ (c) $\frac{1}{3}$



Name: _____ () Date: _____
 Class: Secondary 4 / _____

REVISION WORKSHEET : NUMBERS

Prior Knowledge



- Skills**
- ✓ Able to express a number as a product of its prime numbers
 - ✓ Able to recognise number patterns

- Objectives**
- ✓ Find HCF and LCM by prime factorization
 - ✓ Represent number patterns in the general n form

Key teaching points

- 1) **What's wrong?**
 Express 168 and 420 as products of the prime factors in index notation.

$$168 = 2^3 \times 3 \times 7$$

$$420 = 2^2 \times 3 \times 5 \times 7$$

$$\text{HCF} = 2^3 \times 3 \times 5 \times 7$$

Do not get confused between finding HCF and LCM.

Too tedious a method. Try using general n form.

- 2) Consider using algebraic representation.

Write down the 15th term in the sequence:

$$3^2, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2, 10^2, 11^2, 12^2, 13^2, 14^2, 15^2, 16^2, 17^2$$

15th term is 289.

Practice Questions:

1. Given that $132 = 2^2 \times 3 \times 11$ and $252 = 2^2 \times 3^2 \times 7$. Use these results to find
- the highest common factor of 132 and 252,
 - the smallest integer p , such that $132p$ is a perfect square,
 - the smallest integer k such that $132k$ is a multiple of 252.
 - the value of m given that the HCF and LCM of 132, 252 and m are 12 and 2 772 respectively.

2. At midnight, the temperature of a town is -24°C . At 11 00 hours the following day, the temperature is 18°C .
- Find the difference between the two temperatures.
 - The temperature increases at a steady rate of 0.4°C per hour after 11 00 hours. What is the temperature at 3 pm?

3. Refer to the numbers below.

$\sqrt{7}$	-1.18	0.333	0	13	$7\frac{2}{5}$	$\sqrt{0.0625}$
------------	-------	-------	---	----	----------------	-----------------

Without any repetition, write down all

- irrational numbers,
- natural numbers, and
- recurring decimals.

4. (a) The atomic radius of a carbon atom is 0.67×10^{-10} metre. Express the diameter of the carbon atom in picometre.

- (b) In 1960, the population of Singapore was 1 646 400. 1 646 400 can be written as k billion. Find k .

5. The first four terms of a sequence are 0, 1, 8 and 27.

- List the next 2 terms of the sequence.
- Write down, in terms of n , an expression for the n th term.
- The first four terms of another sequence are 2, 3, 10 and 29.
 - Using your answer in (b), deduce the n th terms of this sequence.
 - Hence, find the value of the 51st term of this sequence.

6. Consider the pattern:

Row	Mathematical sentence	Sum	No. of odd integers
1	$1+3$	4	2
2	$1+3+5$	9	3
3	$1+3+5+7$	16	4
4	$1+3+5+7+9$	25	5
...
n	$1+3+5+\dots+k$	S	...

- Is it possible to have a sum of 2007? Explain your answer.
- Express k in terms of n .
- Find the formula relating S and n .
- How many odd integers are there from 1 to 49?

7. A particular hard disk drive has 5.38×10^{10} bytes of available memory.

- Express the amount of available memory in terabytes.
- After saving 6.8×10^9 bytes of files into the hard disk, find the available memory in the hard disk, giving your answer in standard form.
- Express 20 picograms in nanograms, leaving your answer in standard form.

8. SBS bus services P , Q and R leave the bus interchange at regular intervals. Bus service P leaves every 6 minutes, bus service Q leaves every 8 minutes and bus service R leaves every 13 minutes. All three bus services start their first service together at 06 05. At what time do they next leave the bus interchange together?

Solutions:

1. (a) 12 (b) 33 (c) 21 (d) 12 2. (i) 42°C (ii) 19.6°C
3. (a) $\sqrt[3]{7}$ (b) 13 (c) -1.18
4. (a) 134 picometre (b) $k = 0.0016464$ or 1.6464×10^{-3}
5. (a) 64, 125 (b) $(n-1)^3$ (c) $(n-1)^3 + 2$ (ii) 125 002
6. (a) No, because 200 is not a perfect square. (b) $2n+1$ (c) $S = (n+1)^2$ (d) 25
7. (a) 0.0538 (b) 4.7×10^{10} (c) $2 \times 10^{-2} \text{ mg}$
8. 11 17



Name: _____ () Date: _____

Class: Secondary 4 / _____

REVISION WORKSHEET : RATIO, RATE, PROPORTION, MAP PROBLEMS

Prior Knowledge

✓ **Ratio**

- Ratio of a to b is written as a : b or $\frac{a}{b}$.

- No unit

✓ **Rate**

- Describe how a quantity is changing with another quantity

✓ **Proportion**

- Direct: corresponding pairs of values are in the same ratio; $\frac{x_1}{x_2} = \frac{y_1}{y_2}$.

- Indirect: one quantity increases in the same ratio as the other decreases; $\frac{x_1}{x_2} = \frac{y_2}{y_1}$.

✓ **Map Problems**

- Scale drawings are representation of the dimensions of a plan. The scale is usually expressed as a ratio 1 : n.

- Area scale of a map is the square of its linear scale. If the linear scale is 1 : n, then the area scale will be 1 : n².

Skills

✓ Able to express given quantities as a ratio

✓ Able to find one quantity given changes in the other quantity

✓ Able to solve problems involving ratio, rate and proportion

Objectives

✓ Solve problems involving ratio, rate, proportion and map.

Key teaching points

1) **What's wrong?**

Given that y is inversely proportional to $(x + 4)$. It is given that $y = 4$ when x is a particular value. Find y when x is doubled.

Solution: $y = \frac{k}{x + 4}$, where k is a constant

Let $x = 4$ when $y = 4$, then $4 = \frac{k}{4 + 4}$

$$k = 32$$

$$y = \frac{32}{8 + 4}$$

When $x = 8$, $y = \frac{32}{8 + 4}$

$$= \frac{2}{3}$$

(A) In-class Practice (20 min)

1. A map of a region is drawn on a scale of 1 : 20000.
 - (a) The length of a road is 6 km. Find the length, in cm, of the line on the map which represents this road.
 - (b) On the map, the area representing a park is 300 cm². Calculate the actual area of the park, giving your answer in km².

2. 4 men working 8 hours per day were to complete a renovation job in 4 days. At the end of the 3rd day, 1 worker became sick and was hospitalised. On average, how many more hours must each of the remaining men work for the remaining day in order to complete the job?

3. Given that F is directly proportional to G^2 , find the percentage increase in F if G is increased by 200%.

Answers:

1. (a) 30 cm (b) 12 km²
2. 2 $\frac{2}{3}$ hours
3. 800%

(B) Self Practice

4. The area of a piece of land is 2.5 km². On a map, the area of the piece of land is 10 cm².
 - (a) Express the scale of the map as a ratio in the form of 1 : n where n is a positive integer.
 - (b) Find in cm², the area representing the same piece of land on a map whose scale is 1 : 100,000.

5. Given that 5 men paved a stretch of road of length 200 metres in 4 hours. How long will it take 8 men to pave a 500 metres stretch of road?
6. 15 students packed a total of 120 kg of food to a camping trip. The food can last them for 8 days.
 - (a) Find how much food does one student consume a day?
 - (b) 3 students decide to pull out of the trip, how many more days can the remaining students extend their trip?

7. p is proportional to the cube of q . It is known that $p = 0.5$ for a particular value of q . Find the value of p when this value of q is tripled?
8. Two quantities, x and y are in inverse proportion. If the difference in the values of y when $x = 6$ and when $x = 14$ is 20, find
 - (a) an equation connecting x and y .
 - (b) the value of y when $x = 10$.

Answer:

4. (a) 1 : 50 000 (b) 2.5 km²
5. 6.25 hours
6. (a) 1 kg (b) 2 days
7. 13.5
8. (a) $y = \frac{210}{x}$ (b) 21

Solution:

$$\begin{aligned} \text{I(a) Length} &= \frac{6 \times 10^5}{20000} \\ &= 30 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{(b) } 20000 \text{ cm} &= 0.2 \text{ km} \\ \text{Area scale} &= 1 \text{ cm}^2 : (0.2)^2 \text{ km}^2 \\ &= 1 \text{ cm}^2 : 0.04 \text{ km}^2 \\ \text{Actual area} &= 0.04 \times 300 \\ &= 12 \text{ km}^2 \end{aligned}$$

$$\begin{aligned} 2 \quad \text{Total amt of work (men x hours)} &= 4 \times 8 \times 4 = 128 \\ \text{Total amt of work (men x hours) after 3 days} &= 4 \times 8 \times 3 = 96 \\ \text{Amt of work left} &= 128 - 96 = 32 \end{aligned}$$

$$\text{No. of hours each of the remaining 3 men must do} = 32 \div 3 = 10\frac{2}{3}$$

$$\text{No. of additional hours each man must do} = 10\frac{2}{3} - 8 = 2\frac{2}{3} \text{ h}$$

OR

$$\begin{aligned} \text{No of additional hours} &= \frac{4 \times 8 \times 1}{3} - 8 \\ &= 2\frac{2}{3} \text{ h} \end{aligned}$$

3 $F = kG^2$, where k is a constant

$$\begin{aligned} \text{New } F &= k(3G)^2 \\ &= 9kG^2 \\ &= 9F \end{aligned}$$

$$\begin{aligned} \% \text{ increase in } F &= \frac{9F - F}{F} \times 100\% \\ &= 800\% \end{aligned}$$

4(a)

$$\begin{aligned} 10 \text{ cm}^2 &= 2.5 \text{ km}^2 \\ 1 \text{ cm}^2 &= 0.25 \text{ km}^2 \\ 1 \text{ cm} &= \sqrt{0.25} \text{ km} \\ &= \sqrt{0.25} \times 10^5 \text{ cm} \\ &= 50,000 \text{ cm} \\ 1 : n &= 1 : 50,000 \end{aligned}$$

OR consider that there is only 1 day's work left

4(b)

$$\begin{aligned} 1 : 100,000 \\ 1 \text{ cm} : 10^5 \text{ cm} \\ 1 \text{ cm} : 1 \text{ km} \\ 1 \text{ cm}^2 : 1 \text{ km}^2 \end{aligned}$$

2.5 km² of land is represented by 2.5 cm² on the map.

$$\begin{aligned} 5 \quad 5 \text{ men paved a stretch of road of length 200 metres in 4 hours.} \\ 8 \text{ men paved a stretch of road of length 200 metres in } \frac{5}{8} \times 4 = 2.5 \text{ hours.} \\ 8 \text{ men paved a stretch of road of length 500 metres in } \frac{500}{200} \times 2.5 = 6.25 \text{ hours.} \end{aligned}$$

OR

$$\begin{aligned} \text{Time taken} &= 400 \times \frac{5}{8} \times 2.5 \\ &= 6.25 \text{ h} \end{aligned}$$

$$\begin{aligned} 6(a) \quad \text{Amount of food 1 student consumes in a day} &= 120 + 15 + 8 \\ &= 143 \text{ kg} \end{aligned}$$

$$\begin{aligned} (b) \quad \text{No. of days of extension for the trip} &= \frac{120}{12} - 8 \\ &= 2 \text{ days.} \end{aligned}$$

$$7 \quad p = kq^3, \text{ where } k \text{ is a constant}$$

$$\begin{aligned} \text{Let } q &= Q \text{ when } p = 0.5, \\ 0.5 &= kQ^3 \end{aligned}$$

$$k = \frac{1}{2Q^3}$$

$$\text{When } q = 3Q, p = k(3Q)^3$$

$$\begin{aligned} p &= \frac{1}{2Q^3} (27Q^3) \\ &= 13.5 \end{aligned}$$

$$8(a) \quad y = \frac{k}{x}, \text{ where } k \text{ is a constant.}$$

$$\text{When } x = 6, y = \frac{k}{6} \text{ and when } x = 14, y = \frac{k}{14}$$

Therefore,

$$\frac{\frac{k}{6}}{\frac{k}{14}} = 20$$

$$\frac{2k}{21} = 20$$

$$k = 210$$

The equation is $y = \frac{210}{x}$.

$$(b) \text{ When } x = 10, y = \frac{210}{10} = 21.$$



Name: _____ () Date: _____
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REVISION WORKSHEET : SOLVING LINEAR EQUATION, SOLVING QUADRATIC EQUATION

Prior Knowledge

1. Solving Linear Equations ($ax + b = 0$)

✓ Make the unknown variable the subject of equation:

$$3x - 6 = 0$$

$$3x = 6$$

$$x = 2$$

2. Simultaneous Linear Equations

$$50x + 3y = 3 \dots\dots\dots (1)$$

$$3x + 2y = -1 \dots\dots\dots (2)$$

✓ Elimination Method:

$$(1) \times 2 \quad 10x + 6y = 6 \dots\dots\dots (3)$$

$$(2) \times 3 \quad 9x + 6y = -3 \dots\dots\dots (4)$$

$$(3) - (4) \quad 10x - 9x = 6 - (-3)$$

$$x = 9$$

Sub. $x = 9$ into (1),

$$5(9) + 3y = 3$$

$$y = -14$$

$$\therefore x = 9, y = -14$$

✓ Substitution Method

$$\text{From (1), } y = \frac{3 - 5x}{3} \dots\dots\dots (3)$$

Sub. (3) into (2)

$$3x + 2\left(\frac{3 - 5x}{3}\right) = -1$$

$$x = 9$$

Sub. $x = 9$ into (3),

$$y = \frac{3 - 5(9)}{3}$$

$$y = -14$$

$$\therefore x = 9, y = -14$$

3. Solving Quadratic Equations ($ax^2 + bx + c = 0$)

✓ By factorisation

✓ Using the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

✓ By completing the square method:

$$x^2 + 8x + 9 = 0$$

$$x^2 + 8x + \left(\frac{8}{2}\right)^2 = -9 + \left(\frac{8}{2}\right)^2$$

$$(x + 4)^2 = -9 + 16$$

$$(x + 4)^2 = 7$$

$$x + 4 = \pm\sqrt{7}$$

$$x = -4 + \sqrt{7} \text{ or } x = -4 - \sqrt{7}$$

$$x = -1.35 \text{ or } x = -6.65 \text{ (3sf)}$$

Skills

- ✓ Able to manipulate algebraic expressions to form the appropriate equation
- ✓ Able to solve simultaneous linear equations
- ✓ Able to solve linear and quadratic equations by factorisation, completing the square or applying the quadratic formula

Objectives

- ✓ Form algebraic equations (linear or quadratic)
- ✓ Solve simultaneous linear equations
- ✓ Solve linear and quadratic equations by factorisation, completing the square or applying the quadratic formula
- ✓ Apply algebraic skills in real-life problems

What's wrong?

Eq 1) Complete the square for $y = -x^2 + 3x - 2$

$$y = -x^2 + 3x - 2$$

$$y = -(x^2 - 3x + 2)$$

$$y = -\left(x - \frac{3}{2}\right)^2 - \left(-\frac{3}{2}\right)^2 + 2$$

$$y = -\left(x - \frac{3}{2}\right)^2 - \frac{1}{4}$$

What's missing?

Eq 2) A shop owner spent \$540 on computer keyboards. If the price of each keyboard had been reduced, he could have bought 3 more keyboards. Find the price of one keyboard.

- What information do you know from the problem?

- What else do you need to solve the problem?

- Pick a reasonable number for the information you need.

- What is the price of one keyboard? Show your workings.

Misconception - Wrong cancellation of terms

$$\frac{x+y}{2x+2y} = \frac{x+y}{2(x+y)}$$

Corrections - Right cancellation of terms

$$\frac{x+y}{2x+2y} = \frac{(x+y)}{2(x+y)} = \frac{1}{2}$$

✓ To strive for accuracy, adopt a good practice of **expanding your expression** that is in the form $y = a(x-h)^2 + k$ after completing the square method to check that you arrive at your initial expression of $y = ax^2 + bx + c$. For example,

$$y = -\left(x - \frac{3}{2}\right)^2 + \frac{1}{4}$$

$$y = -\left(x^2 - 3x + \frac{9}{4}\right) + \frac{1}{4}$$

$$y = -x^2 + 3x - \frac{9}{4} + \frac{1}{4}$$

$$y = -x^2 + 3x - 2 \longleftarrow \text{My initial expression!}$$

This is just a side step! Remember to leave your answer in the form $y = a(x-h)^2 + k$ as specified in the qn.

(A) In-class Practice (20 min)

1. Solve the simultaneous equations.

(a) $2x - 5y = -1$

(b) $\frac{4}{x} + \frac{3}{y} = 1$

$6x - 4y = 30$

$\frac{5}{x} - \frac{7}{y} = \frac{13}{2}$

2. The total cost of 3 mangoes and 5 apples is \$7. The total cost of 6 mangoes and 2 apples is \$10. Find the cost of a mango and the cost of an apple.

3. A shopkeeper planned to sell T-shirts at x dollars each in order to receive \$2800 from the sales. Because of poor business, he reduced the price of each T-shirt by \$1 and finally received \$2730 from the sales.

- (i) Write down an expression in terms of x for
 (a) the number of T-shirts he originally planned to sell,
 (b) the number of T-shirts he finally sold.

(ii) Given that the shopkeeper sold 10 more T-shirts than he originally planned, write down an equation in x and show that it reduces to $x^2 + 6x - 280 = 0$.

(iii) Solve the equation $x^2 + 6x - 280 = 0$ and find the number of T-shirts the shopkeeper finally sold.

4. A motorist rode 90 km from P to Q at an average speed of x km/h.

(a) Write down an expression, in terms of x, for the number of hours taken to travel from P to Q. He returned from Q to P by the same route. His average speed for the return journey was 10 km/h less than on outward journey.

(b) Write down an expression, in terms of x, for the number of hours taken to travel from Q to P. He took 1 hour 30 minutes more on the return journey than on the outward journey.

(c) Write down an equation in x, and show that it reduces to $x^2 - 10x - 600 = 0$.

(d) Solve the equation $x^2 - 10x - 600 = 0$.

(e) Find the total duration he was travelling.

Answers:

1. (a) $x = 7, y = 3$

(b) $x = 2, y = -3$

2. Cost of a mango is \$1.50 and cost of an apple is \$0.50.

3. (i) (a) $\frac{2800}{x}$ (b) $\frac{2730}{x-1}$ (ii) $\frac{2800}{x} + 10 = \frac{2730}{x-1}$ (iii) 210

4. (a) $\frac{90}{x}$ (b) $\frac{90}{x-10}$ (d) $x = 30$ or $x = -20$ (rejected) (e) 7.5 hrs

(B) Self-Practice

5. The total mass of two chemicals A and B is 40g. The mass of A is 4g more than the mass of B . Find the mass of A and the mass of B .
6. The total hourly wage of three technicians, Robert, Sulaiman and Tiong Heng, is \$74. Robert's hourly wage is 20% more than Sulaiman's. Tiong Heng's hourly wage is \$4 more than half of Robert's hourly wage. Suppose Sulaiman's hourly wage is \$ x .
 - (a) Express, in terms of x , the hourly wage of Robert and Tiong Heng.
 - (b) Calculate the Robert's hourly wage.
7. In January, Anthony invested an amount of \$0.5 million for a piece of land in a prime area, when the rate at that time was \$ x /m². In July of the same year, due to an economic downturn, the price of land in the same prime area had depreciated to \$($x - 2.50$)/m². At this time, another investor, Robert paid \$0.5 million for a piece of land in the same area.
 - (a) Write down an expression, in terms of x , the area of the piece of land that Anthony bought.
 - (b) Write down an expression, in terms of x , the area of the piece of land that Robert bought.
 - (c) Given that the land area Robert had purchased was 800 m² more than that of Anthony, form an equation in x and shows that it reduces to $2x^2 - 5x - 3125 = 0$.
 - (d) Solve this equation, giving your answers correct to 2 decimal places.
 - (e) Hence, calculate the area of the land bought by Robert.
 - (f) If Anthony sold his piece of land in July, calculate the loss incurred in this transaction. Leave your answer to the nearest cents.
8. Peter drove from Singapore to Kuala Lumpur and then returned to Singapore. The distance between Singapore and Kuala Lumpur is approximately 340 km via the North-South Highway. During the journey from Singapore to Kuala Lumpur, he drove at an average speed of x km/h.
 - (a) Write down the time, in hours, taken for the journey from Singapore to Kuala Lumpur.
 - (b) On the return journey, his average speed was reduced by 15 km/h. Write down the time, in hours, taken for his return journey.
 - (c) Given that the difference in time for the two journeys was 50 minutes, form an equation in x and show that it reduces to $x^2 - 15x - 6120 = 0$. Hence, solve the equation, giving your answer correct to one decimal place.
 - (d) Peter paid a total toll of RM \$71.40 for the two journeys. Calculate the total toll he paid in Singapore dollars given that the exchange rate was S\$50 = RM \$117.50. Give your answer correct to the nearest cent.
 - (e) Peter set off at 22.15 hrs on Saturday for the Singapore-Kuala Lumpur journey. Find the day on which he arrived in Kuala Lumpur and also give the time of arrival, correct to the nearest minute.

Answers:

5. Mass of A is 28g. Mass of B is 12g.
6. (a) Robert's hourly wage = \$1.2 x ; Tiong Heng's hourly wage = \$(0.6 x + 4)
(b) Robert's hourly wage = \$30.
7. (a) $\frac{500\,000}{x}$ (b) $\frac{500\,000}{x-2.5}$ (d) 40.80 or -38.30 (e) 13100 m² (f) \$30637.25
8. (a) $\frac{340}{x}$ (b) $\frac{340}{x-15}$ (c) 86.1 (d) S\$ 30.38 (e) 02.12 hrs, Sunday



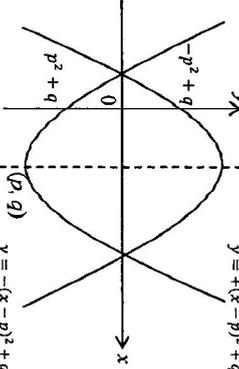
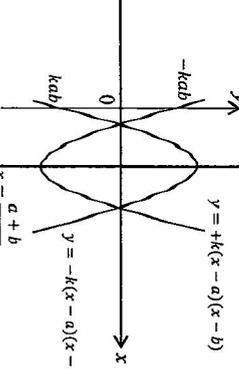
Name: _____ () Date: _____
 Class: Secondary 4 / _____

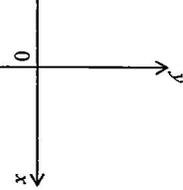
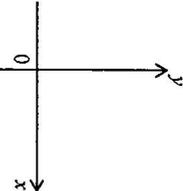
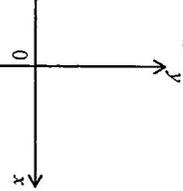
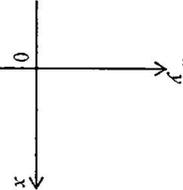
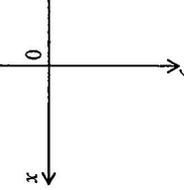
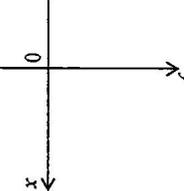
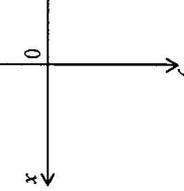
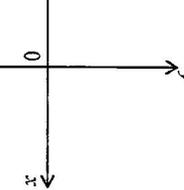
REVISION WORKSHEET : COORDINATE GEOMETRY, FUNCTIONS & GRAPHS

Prior Knowledge - Coordinate Geometry

- Given two points $A(x_1, y_1)$ and $B(x_2, y_2)$,
- ✓ Length of line segment $AB: \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ (Application of Pythagoras' Theorem)
 - ✓ Gradient of $AB: \frac{y_2 - y_1}{x_2 - x_1}$
 - ✓ Midpoint: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
 - ✓ Equation of straight line: $y = mx + c$, where m is the gradient and c is the y -intercept
 - ✓ Horizontal straight line equation is $y = c$ and gradient is zero.
 - ✓ Vertical straight line equation is $x = k$, where k is the x -intercept and gradient is undefined.

Prior Knowledge - Functions & Graphs

<p>✓ Quadratic graphs</p> 	<p>$y = \pm(x - p)^2 + q$</p> <ul style="list-style-type: none"> ✓ Parabolic curve ✓ For $y = +(x - p)^2 + q$, minimum point is at (p, q), y-intercept is $p^2 + q$ ✓ For $y = -(x - p)^2 + q$, maximum point is at (p, q), y-intercept is $-p^2 + q$ ✓ Line of symmetry for both graphs is $x = p$
	<p>$y = \pm k(x - a)(x - b)$</p> <ul style="list-style-type: none"> ✓ Parabolic curve ✓ For $y = +k(x - a)(x - b)$, y-intercept is kab. ✓ For $y = -k(x - a)(x - b)$, y-intercept is $-kab$. ✓ Line of symmetry for both graphs is $x = \frac{a+b}{2}$

<p>✓ Other functions and graphs</p> 	<p>$y = -kx^3$</p> 	<p>$y = \frac{k}{x}, x \neq 0$</p> 	<p>$y = -\frac{k}{x}, x \neq 0$</p> 
<p>$y = \frac{k}{x^2}, x \neq 0$</p> 	<p>$y = -\frac{k}{x^2}, x \neq 0$</p> 	<p>$y = kx^x, k > 1$</p> 	<p>$y = -kx^x, k > 1$</p> 

✓ Graph sketching

1. Predict if parabola is an 'open upwards' curve or 'open downwards' curve by the coefficient of x^2
2. Find Maximum/Minimum point
3. Find x -intercepts (sub. $y = 0$)
4. Find y -intercept (sub. $x = 0$)
5. Find line of symmetry

Skills

- ✓ Able to solve problems in coordinate geometry
- ✓ Able to identify different functions, their graphs and features of the graph
- ✓ Able to sketch graphs of different functions

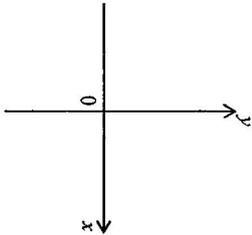
Objectives

- ✓ Solve problems in coordinate geometry
- ✓ Sketch graphs of different functions
- ✓ Apply knowledge of graphs in real-life problems

Key teaching points

1) What if?

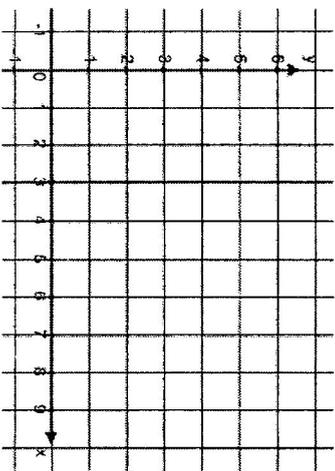
Sketch the graph of $y = x^2$.



- (a) What if $y = \frac{1}{4}x^2$?
- (b) What if $y = -x^2$?
- (c) What if $y = x^2 + 3$?
- (d) What if $y = x^2 - 3$?
- (e) What if $y = \frac{1}{2}x^2$?

Practice Questions

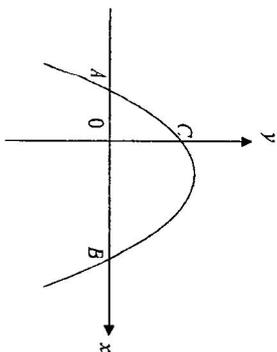
1.



- (a) On the diagram above, plot and label the points $A(3, 4)$, $B(5, 6)$, $C(7, 4)$ and $D(5, 0)$.
- (b) Draw the kite $ABCD$ and write down the equation of the line of symmetry.
- (c) Calculate the area of triangle ABD .
- (d) Calculate the area of the kite $ABCD$.

2.

The diagram shows part of the graph of $y = (x + 1)(2 - x)$, where A and B lie on the x -axis and C lies on the y -axis.



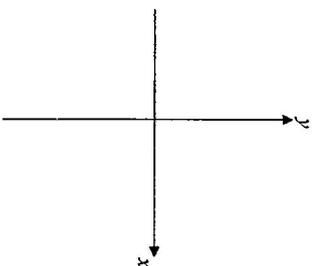
Write down,

- (a) the coordinates of A , B and C ,
- (b) the coordinates of the maximum point,
- (c) the equation of the line of symmetry of the graph.

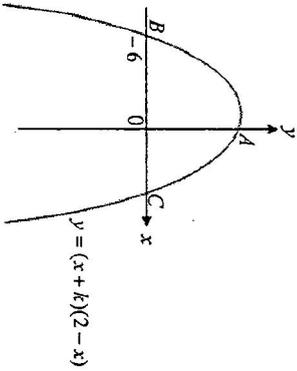
3.

The graph of $y = (x - 4)^2 - 9$ cuts the x -axis at points A and B . It cuts the y -axis at C .

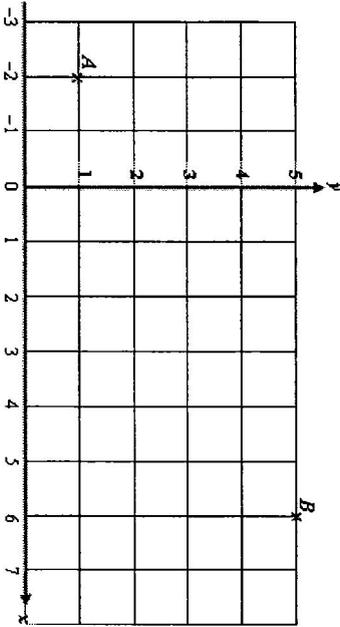
- (a) Find the coordinates of points A , B and C .
- (b) Find the coordinates of the maximum/minimum point.
- (c) Sketch the graph of $y = (x - 4)^2 - 9$ on the axes provided, showing clearly points A , B and C .



4. The diagram shows the graph of the curve $y = (x + k)(2 - x)$, where k is a constant. The curve meets the y axis at A and the x axis at B and C as shown.
- Write down the coordinates of B and find the value of k .
 - Find the coordinates of A .
 - Write down the equation of the line of symmetry of the curve.
 - Find the length of AC .
 - Find the perimeter of triangle OAC , where O is the origin.



5. Point A and point B lie on the grid above



- Join point A to point B and write down the y -intercept of line AB .
- Find the gradient of line AB and hence write down the equation of line AB .
- Draw a line, EF , on the same grid such that it is parallel to AB and has the same length.
- Mark a point C on the grid such that area of triangle ABC is 20 cm^2 .

6. Answer the whole of this question on a sheet of graph paper.

The table below shows the number of electronic devices (n) produced by a factory in certain number of hours (t) for $0 \leq t \leq 200$.

t	0	20	40	60	80	100	120	140	160	180	200
n	0	2200	4000	5400	6400	7000	7200	7000	6400	5400	4000

- Using a horizontal scale of 2 cm to represent 40 hours and a vertical scale of 2 cm to represent 1000 electronic devices, draw the graph of n against t .
- Use your graph to estimate
 - the number of electronic devices produced in 70 hours,
 - the number of hours to produce 3500 electronic devices.
- By drawing a tangent, find the gradient of the curve at $t = 80$.
- Explain what this gradient indicates.
- The number of toys produced by another factory is given by $n = 45t$. On the same axes, draw a graph to represent the number of electronic devices produced in this factory.
- Find the number of hours required for the two factories to produce the same number of electronic devices.

Answers:

- (b) $x = 5$ (c) 6 units² (d) 12 units²
- (a) $A(-1, 0), B(2, 0), C(2, 0)$ (b) $\left(\frac{1}{2}, 2\frac{1}{4}\right)$ (c) $x = \frac{1}{2}$
- (a) $(1, 0), (7, 0), (0, 7)$ (b) $(4, -9)$
- (a) $x = -2$ (b) $(0, 12)$ (c) 12.2 units (d) (e) 26.2 units
- (a) 2 (b) gradient = $\frac{1}{2}$, Equation of line $AB: y = \frac{1}{2}x + 2$ (d) C must be at $(6, 0)$
- (b) (i) 6000 (ii) 34 (c) 36.9 (accept 33 to 41) (f) 150 hours (accept 146 to 154 hours)



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REVISION WORKSHEET : STATISTICS & PROBABILITY

Prior Knowledge - Statistics

- ✓ **Quartiles and Percentiles**
 - For a set of N observations, the median is the value in the $\left(\frac{N+1}{2}\right)^{th}$ position when the _____.
 - For _____ data sets, the lower quartile, median and upper quartile is estimated using the $\left(\frac{N}{4}\right)$, $\left(\frac{N}{2}\right)$ and $\left(\frac{3N}{4}\right)$ values. For example, in the _____ an estimate of the median is the data value in which 50% of the total frequency $\left(\frac{N}{2}\right)$ have this data value or less i.e. 50th percentile)
 - ✓ **Mode**
The mode is the _____ value in a set of observations. There may be zero, one or more modes in a data set.
 - ✓ **Standard Deviation**
Interpretation of Standard Deviation (SD): a small value for SD indicates that the data are concentrated around the mean. Hence the mean is a good representative of the data set.
Compare 2 machines: smaller SD → _____
Compare 2 modes of transport → _____

Histogram

Advantages	Disadvantages
<ul style="list-style-type: none"> • Shows data that is comparable to normal curve 	<ul style="list-style-type: none"> • Cannot read exact values because data is grouped into categories • More difficult to compare two data sets

Stem and Leaf Diagram

Advantages	Disadvantages
<ul style="list-style-type: none"> • It groups the data and yet retains the actual value of each piece of data • Shows range, minimum & maximum, gaps & clusters, and outliers easily • The data is arranged compactly since the stem is not repeated in multiple data points 	<ul style="list-style-type: none"> • Does not easily indicate measures of centrality for large data sets

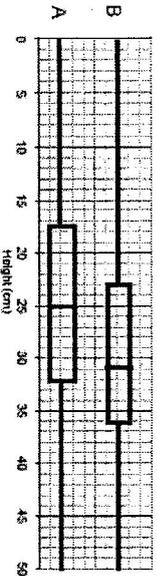
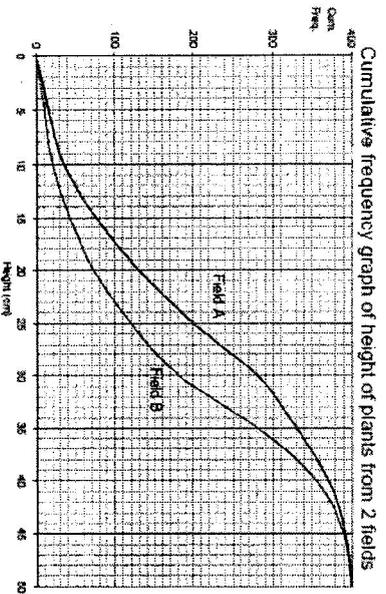
Box and Whisker Plot

Advantages	Disadvantages
<ul style="list-style-type: none"> • Summarises a set of data that are measured in intervals. Hence, it is useful when comparing sets of data that involves large amounts of information • Gives a feeling for how data is clustered or dispersed between different values 	<ul style="list-style-type: none"> • Original data is not preserved. • Mean and mode cannot be identified. • Easy to misinterpret if not truly understood – there is a tendency to think a bigger space means more numbers in it

Cumulative Frequency Graph

Advantages	Disadvantages
<ul style="list-style-type: none"> • Easy to interpret information at any percentile • Provides an easier way to compare different sets of data 	<ul style="list-style-type: none"> • Original data is not preserved.

Comparing between Box and Whisker Plot and the Cumulative Frequency



Advantages of boxplots over cumulative frequency curves	Disadvantages of boxplots compared to cumulative frequency curves
<ul style="list-style-type: none"> The 5 numbers are more easily compared for different data sets Easy comparison of > 2 data sets Easy comparison between data sets of _____ Can compare discrete data sets 	<ul style="list-style-type: none"> Can be used to compare any _____ not just the _____ Boxplot is only a _____ does not display the data distribution

Skills

- ✓ Read and interpret tables and statistical diagrams: tables, pie charts, dot diagrams, box-and-whisker plots, stem and leaf diagrams, histograms, cumulative frequency curves
- ✓ Find mean, median, mode to distinguish between the purposes for which they are used
- ✓ Construct tables, pie charts, dot diagrams, box-and-whisker plots, stem and leaf diagrams, histograms, cumulative frequency curves and be aware of their purposes and uses, advantages and disadvantages.

Objectives

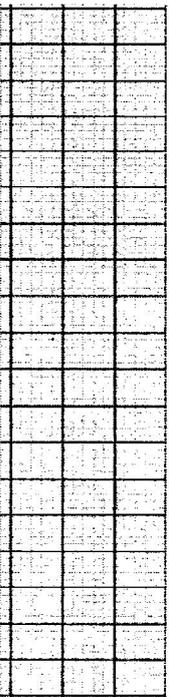
- ✓ Use cumulative frequency curves to estimate median, percentiles, quartiles, interquartile range
- ✓ Calculate mean and standard deviation for grouped and ungrouped data
- ✓ Identify modal class from a grouped frequency distribution

1. The masses (in kg) of 12 girls in a class are shown in the stem-and-leaf diagram below.

Stem	Leaf
4	2 3 3 5 7 8
5	0 2 2 5 9
6	1

Key: 4 | 0 means 40 kg

- (a) Find the median of this distribution.
 (b) Draw a box-and-whisker plot to represent the above distribution.

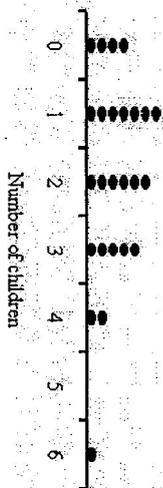


2. The distribution of the examination marks of a group of students is shown in the table.

Marks (m)	$0 < m \leq 20$	$20 < m \leq 40$	$40 < m \leq 60$	$60 < m \leq 80$	$80 < m \leq 100$
No. of students	40	100	90	60	

- (a) Draw a histogram to show this information. Label your axes clearly. Use a scale of 2cm to 20 marks on the horizontal axis and 2 cm to 10 students on the vertical axis.
 (b) Calculate an estimate of the mean mark for the group of students.
 (c) The passing mark is 50. Are there more than half or less than half of the students who passed the test? Explain your answer.
 (d) What is the modal class of marks?

3.



The dot diagram shows the number of children living in the houses in a certain region.

- Find the mode, median and mean number of children.
- Find the standard deviation of the number of children.
- A child is chosen at random, find the probability that the child belongs to the house which has 3 children living in it.

The table below shows the heights of the same group of children living in the region.

Height (cm)	$50 < x \leq 60$	$60 < x \leq 70$	$70 < x \leq 80$	$80 < x \leq 90$	$90 < x \leq 100$
No. of children	3	18	18	6	3

- Find the mean height of the children.
- Find the standard deviation of the heights of the children.

4. The students in two schools took the same standing broad jump test. Information relating to the results is shown in the tables below.

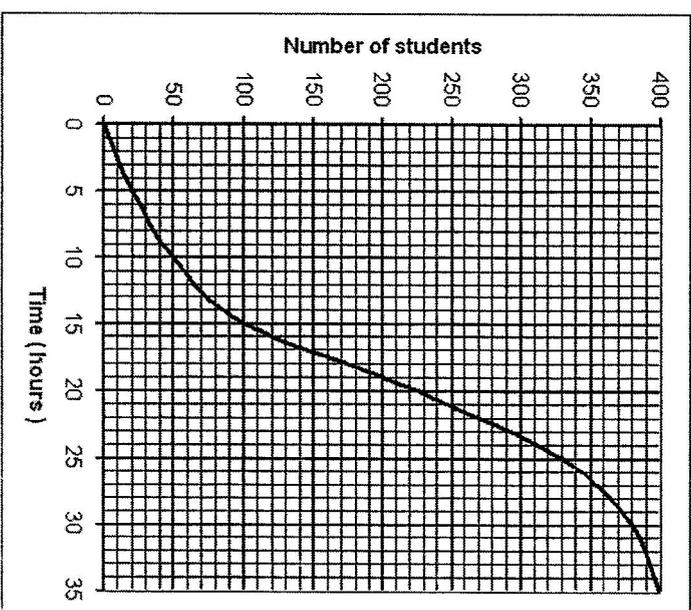
School A	
Jumping distance (cm)	Frequency
$80 < x \leq 100$	20
$100 < x \leq 120$	16
$120 < x \leq 140$	12
$140 < x \leq 160$	2

School B	
Mean = 110.5	Standard Deviation = 15

- For School A, calculate
 - the mean,
 - the standard deviation.
- Compare briefly, the results for the two schools.

5. The cumulative frequency curve below shows the duration of time spent on the Heymath portal by 400 students during the E-Learning week.

- Use the graph to find
 - the median,
 - the interquartile range,
 - the percentile of students who spent more than 12.5 hours each.
- If two students are chosen at random, calculate the probability that
 - the two chosen students spent more than 30 hours each during E-Learning week,
 - one student took at most 10 hours and the other took more than 15 hours but not more than 25 hours.

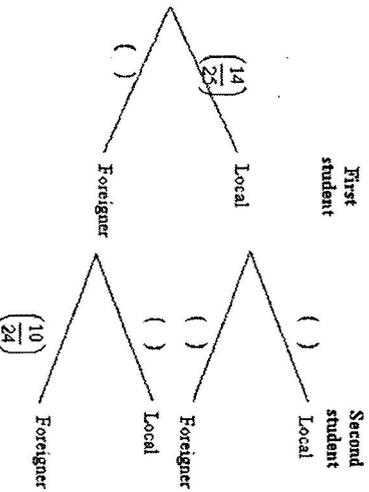


6. There are three identical balls in each of the two bags. The balls in Bag A are marked with numbers 1, 3 and 5 respectively, while the numbers in Bag B are marked with numbers -1, 0 and respectively. One ball is selected from each bag at random and the numbers of the two balls are added up.

(a) Copy and complete the possibility diagram.

+	-1	0	1
1			
3			
5			

- (b) What is the probability that the sum will be lesser than 4?
 (c) What is the probability that the sum is a prime number?
7. In a class of 25 students, 14 are locals and 11 are foreigners. A teacher selects a student at random from the class. Another student is selected at random from the remaining students.



- (a) Copy and complete the tree diagram.
 (b) Find the probability that
 (i) the first student selected is a foreigner and the second a local,
 (ii) at most one of the students selected is a foreigner,
 (iii) the second student selected is a local.
8. A two-digit number is formed at random in a single event using the digits 3, 2 and 4 with repetition of digits allowed. Find
 (a) the set of Sample Space S,
 (b) the probability of forming an even number,
 (c) the probability of forming a number x , such that $32 < x < 44$,
 (d) the probability of not forming a prime number.

Answers:

- (a) 49 kg
- (b) 60 (d) $40 < m \leq 60$
- (a) 1, 2, 1.92 (b) 1.44 (c) $\frac{1}{5}$ (d) 72.5 cm (e) 9.68 cm
- (a) (i) 108.4 cm (ii) 17.8 cm
 (b) School B's students have a higher mean than School A's.
 Hence, School B's scored better in the test. Also, School B's lower standard deviation indicates that their performance is more consistent than students in School A.
- (a) (i) 19 (ii) 8.2 to 8.5 (iii) 82.5% (b) (i) $\frac{1}{420}$ (ii) $\frac{115}{798}$
- (b) $\frac{5}{9}$ (c) $\frac{1}{3}$
- (a) $\frac{11}{25}; \frac{13}{24}; \frac{11}{24}; \frac{14}{24}$ (b) (i) $\frac{77}{300}$ (ii) $\frac{49}{60}$ (iii) $\frac{14}{25}$
- (a) S = {22, 23, 24, 33, 32, 34, 42, 43, 44} (b) $\frac{2}{3}$ (c) $\frac{4}{9}$ (d) $\frac{7}{9}$



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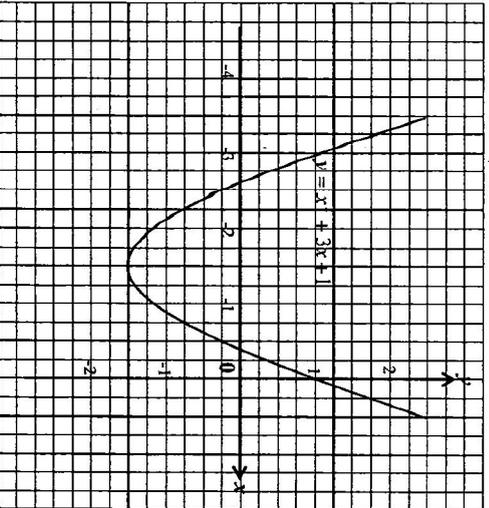
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REVISION WORKSHEET : GRAPHICAL SOLUTION

Prior Knowledge - Graphical Solution

✓ *Solving Non-linear Equations by Graphical Method*

Consider the graph of $y = x^2 + 3x + 1$.



From the graph, when $y = 0$, $x \approx -0.4$ and $x \approx -2.6$ are approximate solutions to the equation $0 = x^2 + 3x + 1$.

How do we solve the equation $x^2 + 3x + 1 = -1$ graphically?

- The equation $x^2 + 3x + 1 = -1$ means that $y = -1$.
- Thus, from the graph, when $y = -1$, $x \approx$ _____ and $x \approx$ _____.
- Using the same graph $y = x^2 + 3x + 1$, how do we solve the equation $x^2 + 3x - 1 = 0$?
- There is a need to manipulate the equation until LHS = $x^2 + 3x + 1$.

- Similar to above, when $y =$ _____, the corresponding x -coordinates are $x \approx$ _____ and $x \approx$ _____.

Think! How about solving the equation $2x^2 + 5x = 0$ using the same graph above?

Skills

- ✓ Able to solve simultaneous equations by graphical method
- ✓ Able to read off x - and/or y - values accurately and with precision from the graph
- ✓ Able to estimate gradients of curves by drawing tangent to the curve

Mathematics Department revision- To Nurture Creative and Confident Problem Solvers

✓ Able to solve equations by graphical method

Objectives

✓ Apply knowledge of graphs and solve real-life problems involving graphs

Practice Questions:

1. **Answer the whole of this question on a piece of graph paper.**

The following table gives corresponding values of x and y which are connected by the equation $y = x + \frac{6}{x} - 1$.

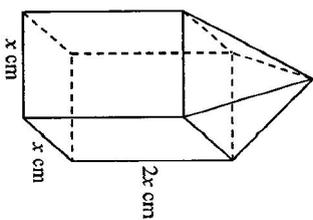
x	0.5	1	1.5	2	3	4	5	6
y	11.5	6	a	4	4	4.5	b	6

- Calculate the values of a and of b .
- Using the scale of 2 cm to represent 1 unit on the horizontal x -axis and 2 cm to represent 2 units on the vertical y -axis, draw the graph of $y = x + \frac{6}{x} - 1$ for $0.5 \leq x \leq 6$.
- Using your graph, find the values of x when $y = 5$.
- By drawing a line, find the x -coordinate of the point on the curve $y = x + \frac{6}{x} - 1$ at which the tangent is parallel to the line $\frac{1}{5}y = -x + 1$.
- By drawing a suitable straight line on the same axes, find the solutions of the equation $x + \frac{6}{x} - 1 = 6$.

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2. Answer the whole of this question on a single sheet of graph paper.

A container is made by joining together a pyramid with a square base of length x cm and a cuboid of dimension x cm by x cm by $2x$ cm. The total height of the container is $2(x + 3)$ cm.



(a) If the volume of the container is y cm³, show that $y = 2x^3 + 2x^2$.

(b) The table shows the corresponding values of x and y . Using a scale of 4 cm to 1 unit on the x -axis and 4 cm to 50 units on the y -axis, draw the graph of $y = 2x^3 + 2x^2$ for $0 \leq x \leq 5$.

x	0	1	2	3	4	5
y	0	4	24	72	160	300

(c) Use your graph to estimate the volume of the container if the length of the square base is 4.5 cm.

- (d) By drawing a tangent, find the gradient of the graph when the length of the square base is 2.5 cm.
 (e) A second container, which is a cuboid, has dimensions $(4 - x)$ cm long, 7 cm wide and 10 cm high. On the same axes, draw the graph of $y = 280 - 70x$.
 (f) Using your graph, solve $2x^3 + 2x^2 + 70x - 280 = 0$. Explain what the solution to the equation represents.

[TKSS Prelim 2011]

[Answer Key]

- (c) $220 \sim 225$ cm³ (d) 49.4
 (f) Solution represents the length (value of x) when the volumes of the two containers are equal.



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REVISION WORKSHEET : VECTORS

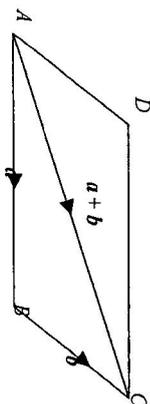
Prior Knowledge

Definition of Vectors

- ✓ A vector is a quantity which has both m and d.
- ✓ Magnitude of $\vec{AB} = \begin{pmatrix} u \\ v \end{pmatrix}$ is $|\vec{AB}| = \sqrt{\dots}$

Addition of vectors

- ✓ $\vec{AB} + \vec{BC} = \vec{AC}$ (Triangle law of vector addition)
- ✓ $\vec{AB} + \vec{AD} = \dots$ (Parallelogram law of vector addition)



Subtraction of vectors

- ✓ To subtract a vector is to add the negative of that vector.
- ✓ $\vec{AB} - \vec{AC} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b})$

Scalar Multiplication of Vectors

- ✓ When $k > 0$, $k\vec{a}$ are in the _____ direction and $|k\vec{a}| = k|\vec{a}|$.
- ✓ When $k < 0$, $k\vec{a}$ are in _____ direction and $|k\vec{a}| = -k|\vec{a}|$

Position Vector

- ✓ \vec{OP} is the position vector of a point P with reference to a fixed point O .
- ✓ If $\vec{OP} = \begin{pmatrix} p \\ q \end{pmatrix}$, then the coordinates of P are (_____ , _____), the converse is true.

Column Vectors

$$\begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} p+r \\ q+s \end{pmatrix}, \begin{pmatrix} p \\ q \end{pmatrix} - \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} p-r \\ q-s \end{pmatrix}, k \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} kp \\ kq \end{pmatrix}$$

Proving Parallel Lines

- ✓ Show that the two vectors are scalar multiples of each other (e.g., $\vec{AB} = 2\vec{CD}$) or show that the two vectors have the same gradient
- ✓ Conclude that the two vectors are parallel
- ✓ Hence, conclude that the two lines are parallel

Proving Collinear Points

- ✓ Show that the two vectors are scalar multiples of each other (e.g., $\vec{AB} = 2\vec{CD}$) or show that the two vectors have the same gradient
- ✓ Conclude that the two vectors are parallel
- ✓ State that since one point is common and the vectors are parallel, the three points must be collinear

Skills

- ✓ Represent a vector by a directed line segment, use equal vectors and find the negative of a vector.
- ✓ Add and subtract vectors including using scalar multiplication of vectors.
- ✓ Calculate magnitude of a vector.
- ✓ Solve geometric problems involving position vectors and column vectors.

What's the question if the following information is given:

$$\vec{AB} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}, \vec{BC} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \text{ and } A \text{ is a point } (-2, 1).$$

Write down 2 possible questions with the above information:

- 1.
- 2.

My solutions:

- 1.
- 2.

What's Wrong? What should the solution be?

Find the value of k if $\vec{XY} = \begin{pmatrix} 9 \\ k \end{pmatrix}$ and \vec{XY} is parallel to $\vec{AB} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$.

Solution:

$$\text{Since } XY \parallel AB, \begin{pmatrix} 9 \\ k \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}.$$

$$\therefore k = -4$$

Explain the error in the solution:

Show how you would find the answer:

Practice Questions:

1. $OXYZ$ is a parallelogram in which $\overline{OX} = p$ and $\overline{OY} = q$. N is a point on YZ such that it divides YZ in the ratio 2 : 3.

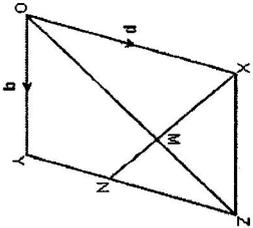
(a) Express in terms of p and q , in the simplest form:

(i) \overline{ON} (ii) \overline{NX} (iii) \overline{OM}

(b) Find the numerical value of

(i) $\frac{OM}{MZ}$ (ii) $\frac{\text{Area } \Delta XOM}{\text{Area } \Delta MNZ}$

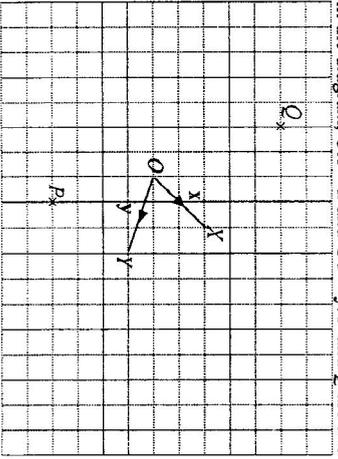
(iii) $\frac{\text{Area } \Delta XMZ}{\text{Area } MNYO}$



(c) Lines XN and OY are produced to meet at H .

- (i) find the ratio of $\frac{OY}{HY}$ (ii) \overline{HZ} in terms of p and q

2. In the diagram, $\overline{OX} = x$ and $\overline{OY} = y$. P and Q are two points on the diagram.



(a) Mark clearly on the diagram a point

- (i) A such that $\overline{OA} = x + 2y$
 (ii) B such that $\overline{OB} = -2x + y$
 (iii) C such that $\overline{OC} = x - 2y$
 (b) Express \overline{PQ} in terms of x and y .

- 3.

(a) Given that $\overline{OP} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$, $\overline{OQ} = \begin{pmatrix} -1 \\ -7 \end{pmatrix}$ and $\overline{OR} = \begin{pmatrix} 2 \\ k \end{pmatrix}$, find

- (i) the column vector representing \overline{QP} ,
 (ii) the magnitude of \overline{QP} .

(b) Find the value of k if \overline{QP} and \overline{OR} are parallel sides of a trapezium.

4. The area of the quadrilateral $PQRS$ is 30 units².

(a) If $\overline{PQ} = -\overline{RS}$, $\overline{PS} = -\overline{RQ}$, what type of quadrilateral is $PQRS$?

(b) Suppose that $|\overline{PQ}| = 12$ units and $|\overline{RQ}| = 5$, find

- (i) the perimeter of $PQRS$,
 (ii) the size of $\angle QRS$ if it is obtuse.

(c) Hence, find $|\overline{QS}|$.

5. (a) Given that $\overline{AB} = \begin{pmatrix} 3p \\ 4p-15 \end{pmatrix}$ and $\overline{BC} = \begin{pmatrix} 20-3p \\ -4p \end{pmatrix}$ where p is a constant, show that $|\overline{AC}| = 25$ units for any value of p .

(b) If $|\overline{BC}| = 20$ units,

- (i) find the values of p and the corresponding values of $|\overline{AB}|$,
 (ii) hence, show that $|\overline{AB}| + |\overline{BC}| > |\overline{AC}|$.

(c) If $|\overline{AB}| = 9$ units,

- (i) find the value of p and the corresponding value of $|\overline{BC}|$,
 (ii) hence, determine, stating reasons, if $|\overline{AB}| + |\overline{BC}| > |\overline{AC}|$.

6. $ABCD$ is a parallelogram whose diagonals, AC and BD , intersect at O . E is a point on AB such that $AE = 2EB$. DE intersects AC at F .

(a) If O is the point $(0, 0)$ on the coordinate plane, A is the point $(-7, -5)$ and D is the point $(-4, 5)$,

- (i) express \overrightarrow{AD} as a column vector.
 (ii) find $|\overrightarrow{AD}|$.

(b) Given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$, express the following vectors in terms of \mathbf{a} and/or \mathbf{b} , giving each of your answers in its simplest form.

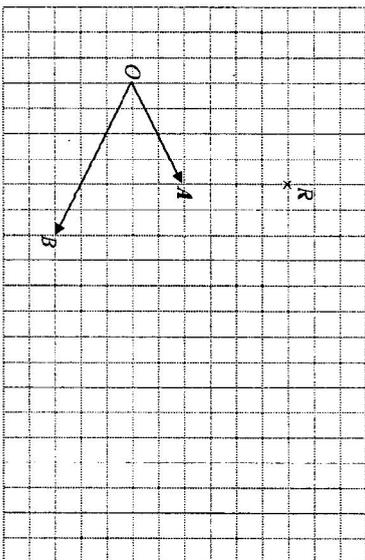
- (i) \overrightarrow{AC} (ii) \overrightarrow{CD}

(c) Show that $\overrightarrow{DE} = \frac{1}{3}(\mathbf{a} + 5\mathbf{b})$

(d) Given that $\overrightarrow{FA} = \frac{4}{5}\overrightarrow{OA}$, find \overrightarrow{FE} and hence, show that D , E and F lie on the same straight line.

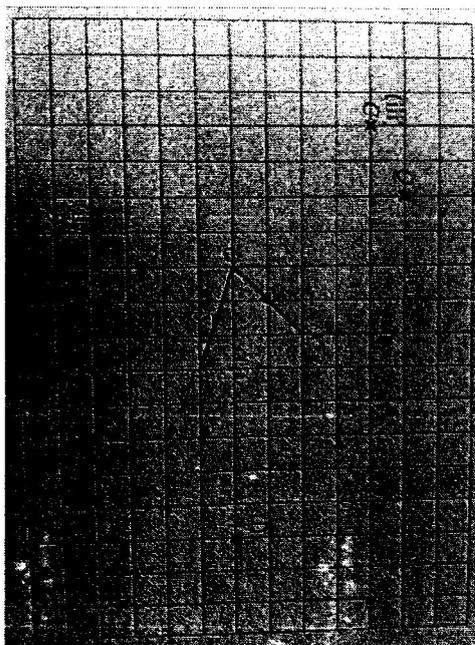
(e) Find the numerical value of $\frac{\text{area of } \triangle DOF}{\text{area of } \triangle AEF}$.

7. In the diagram given below, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.



- (a) Plot the points P and Q such that $\overrightarrow{OP} = 2\mathbf{a}$ and $\overrightarrow{OQ} = \frac{1}{3}\mathbf{b}$.
 (b) Express \overrightarrow{OR} in the form $m\mathbf{a} + n\mathbf{b}$.
 (c) Given that O_1ARC is a parallelogram, mark the point C .

- Answers :
- (a)(i) $\mathbf{q} + \frac{2}{5}\mathbf{p}$ (ii) $\frac{3}{5}\mathbf{p} - \mathbf{q}$ (iii) $\frac{5}{8}(\mathbf{p} + \mathbf{q})$ (b)(i) $1\frac{2}{3}$ (ii) $2\frac{7}{9}$ (iii) $\frac{15}{31}$
 (c)(i) $\frac{3}{2}$ (ii) $\mathbf{p} - \frac{2}{3}\mathbf{q}$
 - (a)



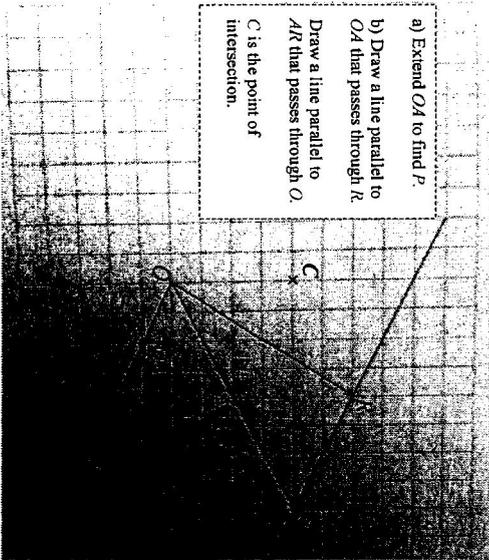
(b) $PQ = 3x - 3y$

- (a)(i) $\begin{pmatrix} 5 \\ 12 \end{pmatrix}$ (ii) 13 units (b) 4.8
- (a) parallelogram (b)(i) 34 units (ii) 150° (c) 16.5 units
- (b)(i) $p = 0$ or $4\frac{4}{5}$, 15 (c)(i) $2\frac{2}{5}$, 16
- (a)(i) $\begin{pmatrix} 3 \\ 10 \end{pmatrix}$ (ii) 10.4 units (b)(i) $-2\mathbf{a}$ (ii) $\mathbf{a} - \mathbf{b}$ (d) $\frac{2}{15}(\mathbf{a} + 5\mathbf{b})$ (e) $\frac{3}{8}$

7.

(a)

- a) Extend OQ to find P .
 - b) Draw a line parallel to OQ that passes through R .
- Draw a line parallel to QR that passes through O .
- C is the point of intersection.



(b) $\overline{OR} = 2a - \frac{2}{3}b$.



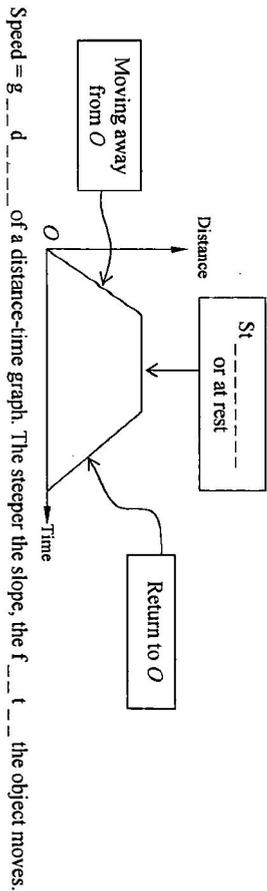
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REVISION WORKSHEET : GRAPHS IN PRACTICAL SITUATIONS

Prior Knowledge

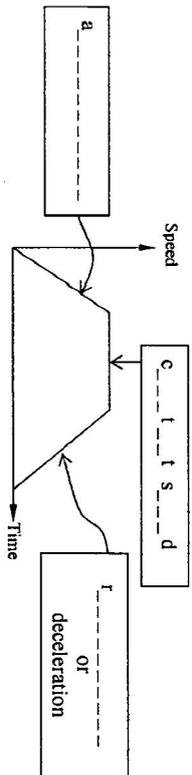
Distance-time Graph

✓ It depicts the distance of an object from a reference point at time t .



Speed-time Graph

✓ It shows how the speed of a moving object changes with time.



Acceleration = g of a s-time graph.

Distance traveled = a under the speed-time graph.

Skills

- ✓ Interpret and draw distance-time graphs
- ✓ Interpret and draw speed-time graphs

Lesson Objectives

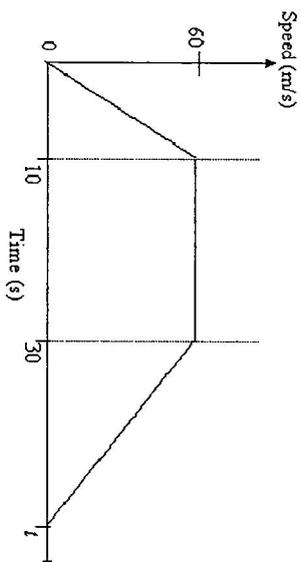
- ✓ Interpret and use graphs in practical situations

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1

Key teaching points

The diagram below shows the speed-time graph of a particle.



Calculate

- (a) the acceleration for the first 4 s,
- (b) the time taken for the particle to come to rest if the retardation is 3 m/s^2 .

1) What's wrong?

$$(a) \frac{4}{10} = \frac{x}{60}$$

$$x = 24$$

\therefore Acceleration is 24 m/s^2

What is the x that we have found? Is it acceleration at $t = 4$?

$$(b) \frac{0 - 60}{t - 30} = 3$$

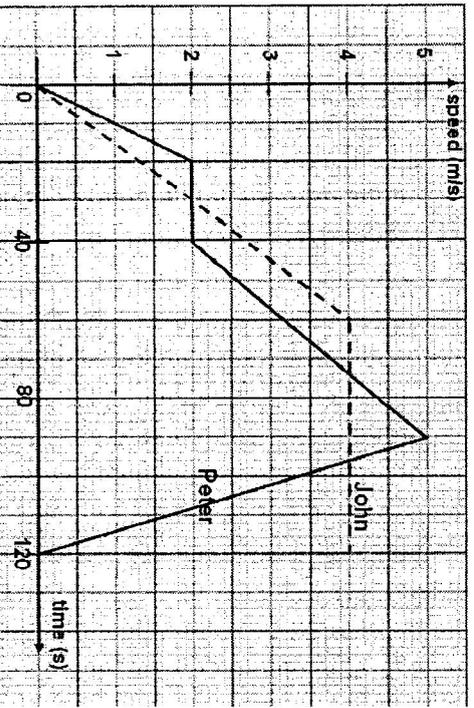
$$t = 10 \text{ s}$$

Since $t = 10 \text{ s}$ is impossible based on the diagram, where could the mistake be?

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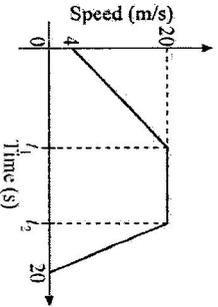
2

1. The diagram shows the speed-time graphs of two runners, John (dotted lines) and Peter (solid lines), for the first 120 seconds of their run.

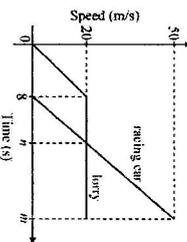


- Calculate the total distance run by John.
- At three instances the two men were running at the same speed. What were the times?
- Peter ran the last 30 second stretch at reducing speed. Calculate his deceleration.

2. The diagram shows the speed-time graph of a car during the first 20 seconds of a journey. From $t = 0$ to $t = t_1$, the car accelerates from 4 m/s to 20 m/s. From $t = t_1$ to $t = t_2$, the car travels at constant speed and from $t = t_2$ to $t = 20$, the car decelerates before coming to rest.
- Calculate the value of t_1 , if the acceleration of the car in the first t_1 seconds is 2 m/s^2 .
 - Calculate the value of t_2 if the distance covered by the car in the first t_2 seconds is 256 m.
 - Calculate the average speed of the car during the 20s.



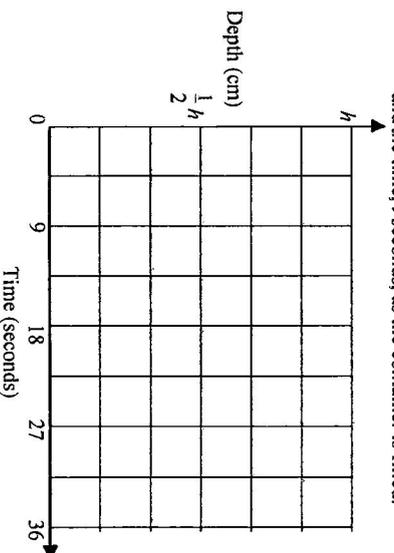
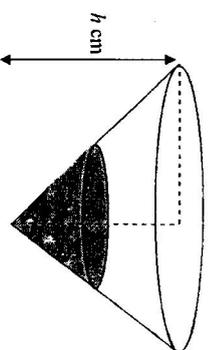
3. The diagram shows the speed-time graph of a lorry and a racing car that travelled along the same road during an interval of m seconds. Both vehicles started from the same place but the racing car started 8 seconds later than the lorry.



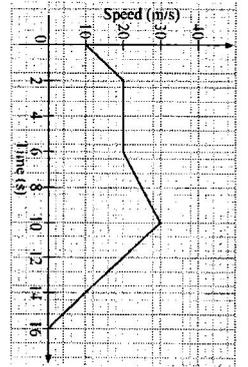
- At the m th second, the distance covered by both vehicles is the same.
 - Form an equation in m and solve it.
 - Hence, find the distance covered by each vehicle during the m seconds.
- Calculate the acceleration of the racing car.
- Find the value of n if the speed of both vehicles is 20 m/s at the n th second.

4. The diagram shows a container in the shape of a triangular pyramid. The height of the container is h cm. If it takes 36 seconds to fill the container to its brim at a constant rate,

- find the time taken to fill to half its height
- sketch the graph to show the relationship between the depth of the water, h cm, and the time, t seconds, as the container is filled.

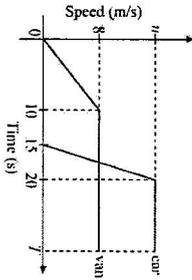


5. The diagram shows the speed-time graph for the first 16 seconds of a motorcyclist's journey from a point A.



- (a) Find the distance covered in the interval when
 (i) the acceleration of the motorcyclist is zero,
 (ii) the motorcyclist is decelerating,
 (iii) the motorcyclist is accelerating.
- (b) Find the average speed of the motorcyclist during the 16 seconds.
- (c) Sketch the distance-time graph of the motorcyclist during the 16 seconds.

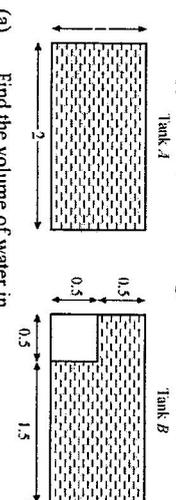
6. The diagram shows the speed-time graphs of a van and a car.



- (a) The van, starting from rest, accelerates uniformly for 10 s until it reaches a speed of 8 m/s. It then continues to travel at this constant speed.
 (i) Find the acceleration of the van during the first 10 s.
 (ii) Show that the total distance travelled by the van in T s is $(8T - 40)$ m where $T > 10$.
- (b) The car, starting from rest at the same place as the van but 15 s later, accelerates uniformly for 5 s until it reaches a speed of u m/s ($u > 8$). Find the value of u if the acceleration is 3.2 ms^{-2} .
- (c) The car continues to travel at the constant speed of u m/s until it overtakes the van. If the overtaking occurs when the van has travelled for T s, find the value of T .

- 7.

The diagrams show the cross-section views of two completely filled water tanks A and B. Tank A is 2 m long, 3 m wide and 1 m high. For the lower 0.5 m, tank B is 1.5 m long and 3 m wide, for the upper 0.5 m, it is 2 m long and 3 m wide.



- (a) Find the volume of water in
 (i) tank A,
 (ii) tank B.
- (b) Suppose that the tanks are filled by pouring water into them at the constant rate of 1.5 m^3 per minute. Calculate in minutes, the time needed to completely fill
 (i) tank A,
 (ii) tank B.
- (c) Sketch on the same diagram, the relationship between the depths of the water, y cm, measured from the bottom of the tanks and the time, t minutes, as the tanks are being filled with water.

Answers:

- (a) 360 m (b) 30 s, 74 s, 96 s (c) $\frac{1}{6} \text{ ms}^{-2}$
- (a) 8 (b) 16 (c) 14.8 m/s
- a(i) 24 (ii) 400m (b) $3\frac{1}{8} \text{ ms}^{-2}$ (c) 14.4
- 4.5 s
- (a)(i) 80 m (ii) 90 m (iii) 130 m (b) $18\frac{3}{4} \text{ m/s}$
- (a)(i) 0.8 ms^{-2} (b) 16 (c) 30
- a) 6 aii) 5.25 b) 4 min bii) 3.5 min

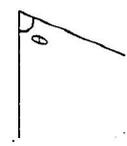
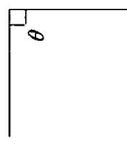
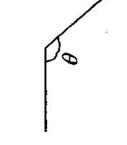


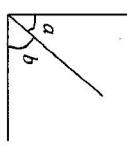
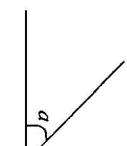
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REVISION WORKSHEET : PROPERTIES OF ANGLES, ANGLE PROPERTY OF POLYGONS, GEOMETRICAL CONSTRUCTION

Prior Knowledge

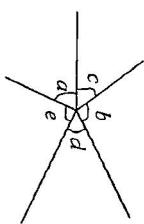
✓ Name the following angles:

 $0^\circ < \theta < 90^\circ$	 $\theta = 90^\circ$	 $90^\circ < \theta < 180^\circ$	 $180^\circ < \theta < 360^\circ$
--	--	--	---

 $\angle a + \angle b = 90^\circ$	 $\angle a + \angle b = 180^\circ$
---	--

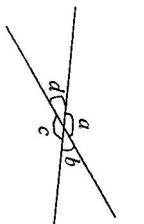
✓ **Angles at a Point**

Sum of angles at a point = 360°



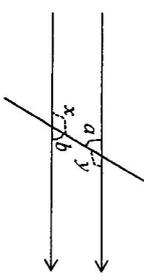
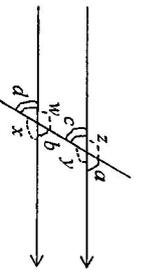
$\angle a + \angle b + \angle c + \angle d + \angle e = 360^\circ$

Vertically opposite angles are equal



$\angle a = \angle c; \angle b = \angle d$

✓ **Angles on Parallel Lines**

 $\angle a = \angle b; \angle x = \angle y$ angles are equal	 $\angle a = \angle b; \angle c = \angle d$ $\angle x = \angle y; \angle w = \angle z$ angles are equal
---	---

$\angle a + \angle b = 180^\circ$
 $\angle x + \angle y = 180^\circ$

angles between two parallel lines are supplementary.

✓ **Angle Property of Polygons**

Exterior Angles

- Sum of exterior angles of any polygon is 360°
- For an n -sided regular polygon, each exterior angle = $\frac{360^\circ}{n}$

Interior Angles

- Sum of interior angles of any n -sided polygon = $(n - 2) \times 180^\circ$

Skills

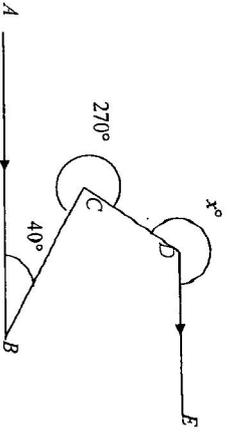
- ✓ Able to identify the different properties of angles and angles formed by parallel lines and transversal
- ✓ Able to calculate the angle sum of interior and exterior of any convex polygon
- ✓ Able to identify quadrilaterals and polygons on the basis of their properties
- ✓ Able to construct simple geometrical figures from given data

Objectives

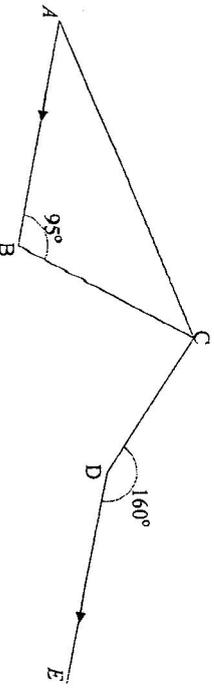
- ✓ Apply knowledge of properties of angles to solve for unknown angles
- ✓ Apply knowledge of angle properties of polygons to solve for unknown sides/angles of polygons
- ✓ Apply knowledge of perpendicular and angle bisectors using Mathematical tools to construct geometrical figures

Practice Questions:

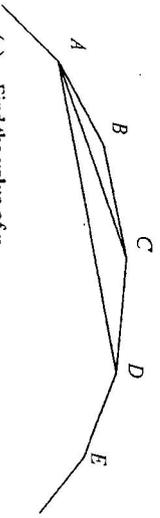
1. In the diagram, AB is parallel to DE . $\angle BC$ is 40° and the reflex angle at C is given as 270° . Find the value of x .



2. In the figure, $\angle ABC = 95^\circ$, $\angle CDE = 160^\circ$ and AB is parallel to DE . Find $\angle BCD$.



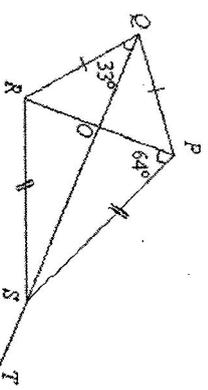
3. The diagram shows part of a regular polygon with n sides. Each interior angle of this polygon is 150° .



- (a) Find the value of n .
 (b) Find $\angle BAC$.
 (c) Find $\angle CAD$.

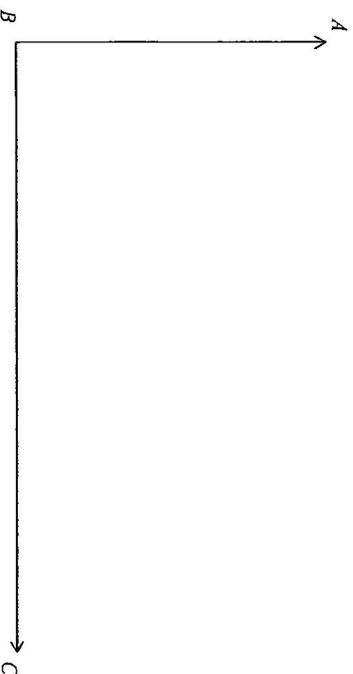
4. The ratio of an interior angle to an exterior angle of a regular polygon is $13 : 2$. Find the number of sides of the polygon.

5. In the diagram below, $\angle OQR = 33^\circ$ and $\angle SPR = 64^\circ$.



- (a) State the special name of the quadrilateral $PQRS$.
 (b) Find
 (i) obtuse $\angle TSP$,
 (ii) $\angle QRS$.

6. Answer the whole of this question in the spaces below...
 The scale drawing in the answer space below shows the positions of the towns A , B and C . A is due North of B .



- (a) Find the bearing of C from A .
 (b) The town D is on a bearing of 052° from A and on a bearing of 335° from C . Find and label the position of the town D .
 (c) A television mast is to be erected equidistant from A , B and C . By constructing perpendicular bisectors, find and label the position of the mast M . Given that $AB = 60$ km, calculate the distance from D to the mast.

Answer Key: 1. 230° 2. 75° 3.(a) 12 sides (b) 15° (c) 15° 4. 15 5.(a) kite 5.(b) 154° 5.(c) 121°
 6.(a) 116° (d) distance from D to the mast = 83 km



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REVISION WORKSHEET (16): MENSURATION

Prior Knowledge

1. Formulae for Area of Plane Figures

- ✓ Area of rectangle = Base \times Height
- ✓ Area of triangle = Base \times \perp Height = $\frac{1}{2} ab \sin C$
- ✓ Area of trapezium = $\frac{1}{2} \times$ Sum of parallel sides \times \perp Height
- ✓ Area of parallelogram = Base \times \perp Height
- ✓ Area of Rhombus and Kite = $\frac{1}{2} \times$ Product of diagonals

2. Formulae for Surface Area of Solids

- ✓ Surface area of cone = $\pi r l + \pi r^2$ ($l = \sqrt{h^2 + r^2}$)
- ✓ Surface area of sphere = $4\pi r^2$

3. Formulae for Volume of Solids

- ✓ For solids with uniform cross-section (i.e. prisms):
 Volume = Base Area \times Height
- ✓ For cone and pyramid:
 Volume = $\frac{1}{3} \times$ Base Area \times Height
- ✓ For sphere with radius r :
 Volume = $\frac{4}{3} \pi r^3$

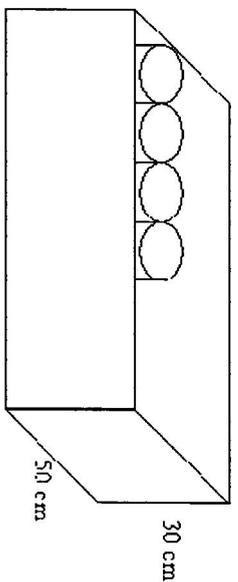
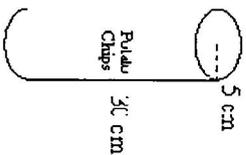
Skills

- ✓ Able to calculate area and perimeter of figures
- ✓ Able to calculate surface area and volume of solids
- ✓ Able to convert between cm^2 and m^2 , and between cm^3 and m^3
- ✓ Able to solve problems involving volume and surface area of composite solids

Objectives

- ✓ Calculate area of parallelogram and trapezium
- ✓ Solve problems involving perimeter and area of composite plane figures (including triangle and circle)
- ✓ Calculate volume and surface area of cube, cuboid, prism, cylinder, pyramid, cone and sphere
- ✓ Conversion between cm^2 and m^2 , and between cm^3 and m^3
- ✓ Solve problems involving volume and surface area of composite solids

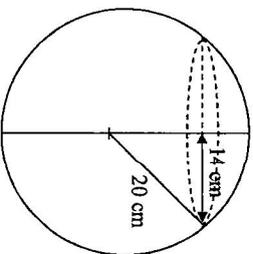
1. Potato chips are sold in cylindrical tins of radius 5 cm and height 30 cm. (Use $\pi = 3.142$)



- (a) Calculate the volume of a can of potato chips.
- (b) Calculate the surface area of the rectangular piece of label wrapping the can.
- (c) Cans of potato chips are packed into a rectangular box measuring 100 cm \times 50 cm \times 30 cm.
 - (i) Find the maximum number of cans that can be packed into the box.
 - (ii) Find the volume of empty space left in the box after packing the maximum number of cans in the box. Leave your answer to the nearest whole number.

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2.



- (a) Calculate
 - (i) the area of the horizontal water surface,
 - (ii) the height of this water surface above the centre of the ball,
 - (iii) the depth of water in the ball.
- (b) If 500 drops of water is required to fill the ball to the brim and each drop is approximately 2.5 millilitres, calculate the original volume of water in the ball.

3.

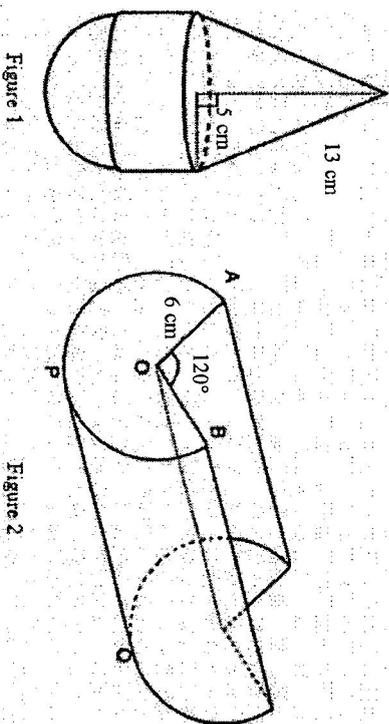


Figure 1 consists of a cone, cylinder and a hemisphere. The base radius of the cone and the cylinder, and the radius of the hemisphere is 5 cm. The slant height of the cone is 13 cm.

- (a) Show that the volume of the cone is $100\pi \text{ cm}^3$.
- (b) Given that the sum of the volumes of the cylinder and the hemisphere is twice the volume of the cone, find the height of the cylinder in Figure 1.

- 2. Figure 1 is melted down completely and all the metal was used to form the prism in Figure 2. Figure 2 with a uniform cross-sectional area has a base of a sector of circle ABO , where O is the centre of the circle and radius 6 cm. Calculate
 - (c) the length PQ in Figure 2,
 - (d) the volume of metal needed in Figure 2 to form a complete cylinder.

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4.

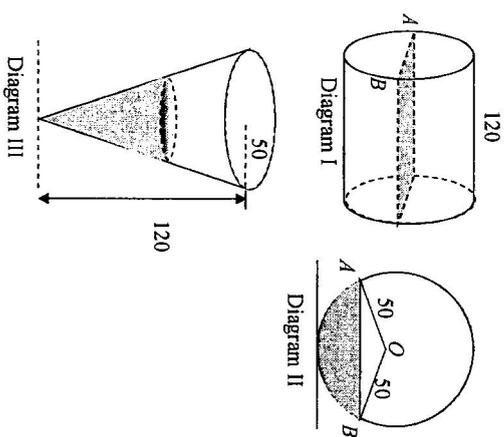
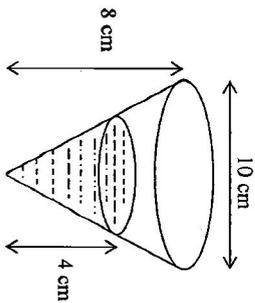


Diagram I shows a cylindrical tank of radius 50 cm and length 120 cm. The tank is partially filled with water and placed with its curved surface on a horizontal floor.

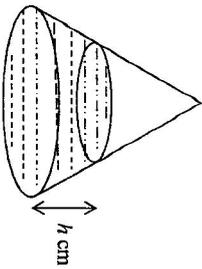
Diagram II shows the circular cross-section of the cylinder. O is the centre of the circle. $\angle AOB = 2$ radians.

- (a) Find the area of the shaded region in Diagram II.
- (b) Find the area of the internal surface of the tank which is in contact with the water in Diagram I.
- (c) The water in the cylindrical tank is now poured into an inverted cone of radius 50 cm and of height 120 cm. Find.
 - (i) the volume of the inverted cone,
 - (ii) the height of the water in the cone.

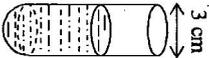
5. A conical container with a diameter of 10 cm and a depth of 8 cm is filled with water to a depth of 4 cm.



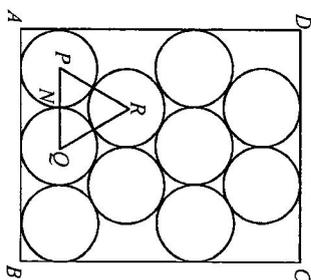
- (a) Show that the volume of water in the container, in terms of π , is $8\frac{1}{3}\pi \text{ cm}^3$.
 (b) Find the surface area of the container in contact with the water.
 (c) The container is inverted so that the water is now at the base of the cone with height h cm. Calculate the value of h .



The water in the cone is poured into a test tube with a diameter 3 cm. The test tube consists of a cylinder with a hemispherical base. Calculate the height of the water in the test tube.



6. $ABCD$ is a rectangular piece of paper. Mary wants to cut out 10 identical circles of radius 1 cm, as shown in the diagram. The points P , Q and R are the centres of the circles. The midpoint of PQ is N .



- (a) Write down the length of PR .
 (b) Calculate
 (i) the length of RN ,
 (ii) the length of AD ,
 (iii) the area of the paper wasted.

Answers:

- (a) 2356.5 cm^3 (b) 942.6 cm^2 (c) (i) 50 cans (ii) $32\ 175 \text{ cm}^3$
 (a) (i) 615.832 cm^2 (ii) 14.3 cm (iii) 34.3 cm (b) 32300 cm^2
- (a) $4\frac{2}{3} \text{ cm}$ (c) 12.5 cm (d) 471 cm^3
- (a) 1360 (b) 14700 (c) (i) 314000 (ii) $h = 96.5 \text{ cm}$
- (b) 37.0 cm^2 (c) $0.348, 4\frac{11}{54} \text{ cm}$
- (a) 2 cm (b) 1.73cm (bii) 7.20cm (biii) 11.8 cm^2



Name: _____ () Date: _____
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REVISION WORKSHEET 1: MATHEMATICS IN PRACTICAL SITUATIONS – CURRENCY EXCHANGE, INTEREST RATE, PROFIT/LOSS, HIRE PURCHASE, TAX

Prior Knowledge

- ✓ Profit or loss in a business transaction is calculated with reference with the cost price:
 - Profit = selling price – cost price
 - Loss = cost price – selling price
 - ✓ Calculate simple interest based on the formula $I = \frac{Prt}{100}$, where I : Simple Interest,
 P : amount (initial investment),
 r : interest rate per annum in percent and
 t : time in years
 - ✓ Calculate compound interest based on the formula $A = P(1 + \frac{r}{100})^t$, where I : Compound Interest,
 P : Principal amount (initial investment),
 r : interest rate per period in percent and
 t : amount after time t .
 *Note: If rate is given per annum time t would be in number of years.
 If rate is given per month, time t would be in number of months.
 - ✓ Calculate monthly repayment (instalment) based on hire-purchase where

$$A = \frac{\text{loan} + \text{total interest}}{\text{total no of months in a period}}$$
 - ✓ Determine tax after calculating taxable amount based on the following:
 Taxable amount (chargeable income) = total income – reliefs
 Tax = Tax rate \times Taxable amount,
- Skills**
- ✓ understand the calculation of profit and loss
 - ✓ distinguish between simple interest and compound interest (i.e. unlike simple interest which is based on a fixed principal, in compound interest, interest is also paid for the interest generated in the previous period)
 - ✓ understand the calculation of interest in hire-purchase plans
 - ✓ understand the calculation of income tax and property tax
- Objectives**
- ✓ apply the above to solve problems in daily life, involving utility bills income tax and property tax, money exchange and taxation

Practice Questions:

1. The selling price of a hand phone is $x\%$ lower than the cost price. If the cost price and selling price are \$200 and \$180 respectively, find x .
 2. Mr Raju, a salesman gets 7% commission on his sales. If his commission for selling a product is \$686, calculate the selling price of the product.
 3. The rate of exchange between Singapore dollars (S\$) and British pounds (£) is S\$2.6940 = £1. Calculate the number of nearest pounds that can be obtained in exchange for S\$85.
 4. John borrows \$ x from a finance company which charges at a compound interest rate of 5% per annum for the whole repayment period. If he plans to repay his loan in 30 months, he has to pay a total interest of \$1297.26. Find x . Give your answer as a whole number.
 5. The table below shows the money changing rates against the Singapore Dollar (SGD).
- | Currency | Code | Unit | Selling | Buying |
|-------------------|------|------|---------|--------|
| Malaysian Ringgit | MYR | 1 | 0.4287 | 0.4083 |
| Indonesian Rupiah | IDR | 1000 | 0.1560 | 0.1420 |
- (a) Calculate, in Malaysian Ringgit, the amount received for SGD 214.35.
 - (b) Calculate, in Singapore Dollar, the amount received for IDR 1 695 000.
 6. A married man with 3 children earns a gross annual income of \$132 000. He is entitled to the following reliefs and given the following tax rates.
- | Personal | Reliefs |
|------------|-----------|
| Personal | \$ 3 000 |
| Wife | \$ 2 000 |
| Each child | \$ 4 000 |
| CPF | \$ 25 080 |
| Donations | \$ 800 |
- | | Tax rates |
|-----------------------------------|-----------|
| First \$40 000 of taxable income: | 4% |
| Next \$50 000 of taxable income | 7% |
- (a) the total reliefs the man is entitled to,
 - (b) the amount of chargeable income,
 - (c) the amount of income tax payable.
 7. Susan invested \$10 000 over a period of 4 years into two different investment plans. Plan A offers 6.25% per annum compound interest compounded annually. Plan B offers 6% per annum compound interest compounded monthly. Which plan is a better choice? Show your working clearly.

8. The selling price of a Honda Stream 1.8XA is \$58 900 and there are two different payment scheme available to John.

Payment Scheme A: 1.5% discount if payment by cash.
Payment Scheme B: 20% down payment, balance at simple interest of 2.25% per annum payable in 3 years.

- (a) For Payment Scheme B, calculate
 (i) the down payment payable.
 (ii) John's monthly instalment.

- (b) How much will he save if he chooses Payment Scheme A.
 Leave your answers (a)(i) and (b) correct to two decimal places.

Answers: 1. $x = 10$ 2. \$9 800 3. £317 4. $x \approx 10\,000$ 5. (a) MYR 500 (b) SGD 240.69
 6. (a) \$42 880 (b) \$89 120 (c) \$5 038.40 7. Plan A
 8. (a) (i) \$11 780 (ii) \$1397.24 (b) \$4064.10