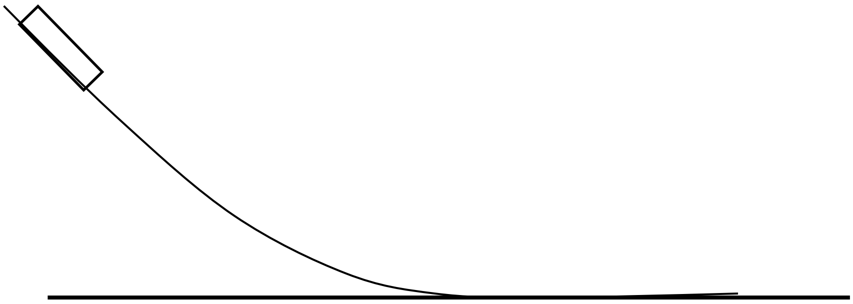
THE PR:IME! PACKAGE PART II

Mechanics 2 (Work, Energy & Power, Circular Motion and Gravitational Field)

MCQ

1 A 50 kg block is released from rest at a height of 5.00 m above the ground. It then travels a distance of 10.0 m along a curved slope to the ground as shown in the figure below. The final speed of the block at the end of the slope is 4.90 m s^{-1} because a constant resistive force acts on it during descent. What is the resistive force acting on the block?



A 185 N **B** 600 N **C** 2450 N **D** 22500 N

A

$$\text{Loss in GPE} = \text{Gain in KE} + \text{Work done against resistiv force}$$

$$50(9.81)(5.00) = 0.5(50)(4.90)^2 + f_{resistive}(10.0) \Rightarrow f_{resistive} = 185\text{ N}$$

[SAJC 2013]

2 A box of weight 100 N is pushed 10 m up a slope inclined at 30° above the horizontal by a force of 150 N along the slope. The frictional force opposing the motion is 60 N. How much of the work done by the 150 N force is converted into kinetic energy and internal energy?

	Kinetic Energy / J	**Internal Energy / J**
A	400	600
B	900	600
C	400	0
D	600	900

A

Work done by friction = 60 × 10 = 600 J = Increase in internal energy

$\Delta E_k + \Delta E_p + \Delta E_e = W_{supplied} - W_{dissipated}$

Kinetic energy = 150(10) – 600 – 100(10 sin30) = 400 J

[MJC 2013]

3 A mass *m*, attached to the end of an unstretched spring, is initially supported by a platform as shown in **Fig. (a)**. This platform is then removed and the mass falls, eventually coming to rest at the position shown in **Fig. (b)**.

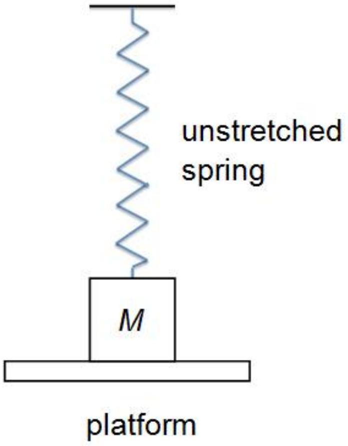


Fig. (a)

Fig. (b)

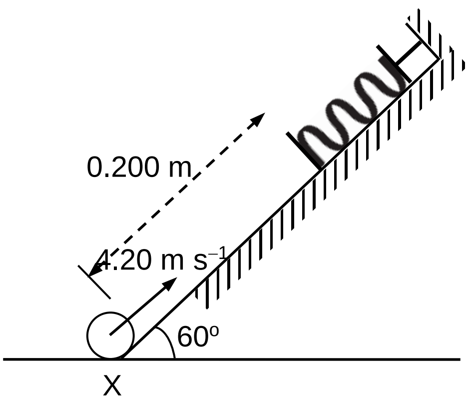
Which of the following correctly relates the changes in energy which may occur during this process?

A	decrease of gravitational potential energy	=	increase of elastic potential energy
B	decrease of gravitational potential energy	=	increase of elastic potential energy + energy dissipated as heat
C	decrease of gravitational potential energy + energy dissipated as heat	=	increase of elastic potential energy
D	decrease of gravitational potential energy + increase of elastic potential energy	=	energy dissipated as heat

B

[TJC 2013]

4 In the diagram below, a 50.0 g ball at point X is projected up a smooth slope with a velocity of 4.20 m s^{-1}, where it encounters a light spring after moving a distance of 0.200 m. The spring has a spring constant of 120 N m^{-1}.



What is the maximum compression of the spring?

A 1.21 mm **B** 5.93 mm **C** 7.36 cm **D** 7.70 cm

C

$$\text{Loss in KE} = \text{Gain in GPE} + \text{Gain in EPE}$$

$$\frac{1}{2}mu^2 - 0 = mgh + \frac{1}{2}kx^2 - 0$$

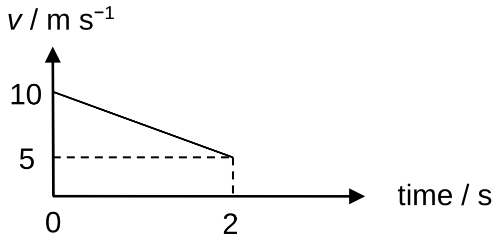
$$\frac{1}{2}(0.0500)(4.20)^2 = (0.0500)(9.81)[(0.200 + x)\sin 60) + \frac{1}{2}(120)x^2$$

$$60x^2 + 0.4248x - 0.35604 = 0$$

$$x = 0.0736 \text{ m} = 73.6 \text{ mm}$$

[RI 2013]

5 The following graph shows variation with time of the velocity v of a body when it is acted on by an external force.



Given that the mass of the body is 2.0 kg, what is the work done by the body?

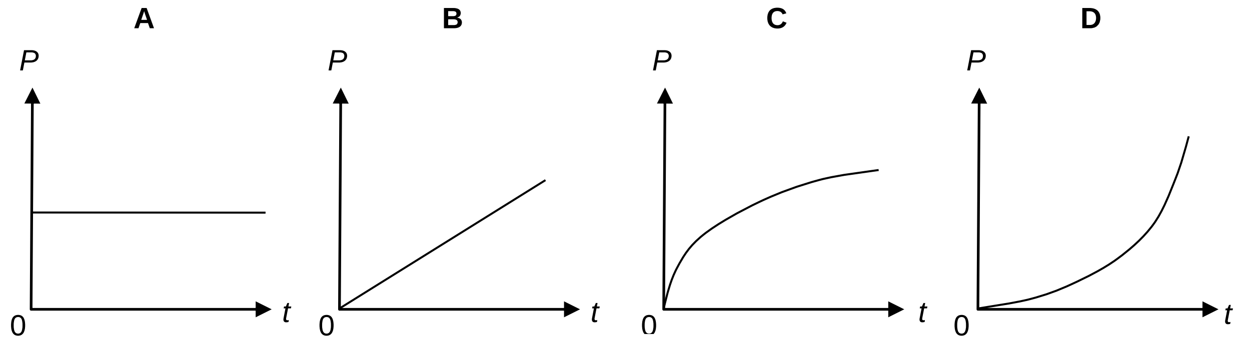
A −5 J **B** 5 J **C** −75 J **D** 75 J

D

$$\text{Work done by body} = -\text{Work done on body}$$
$$= -Fd = -(ma)d$$
$$= -2.0\left(\frac{5-10}{2}\right)[0.5 \times (10+5) \times 2] = 75 \text{ J}$$

[YJC 2012]

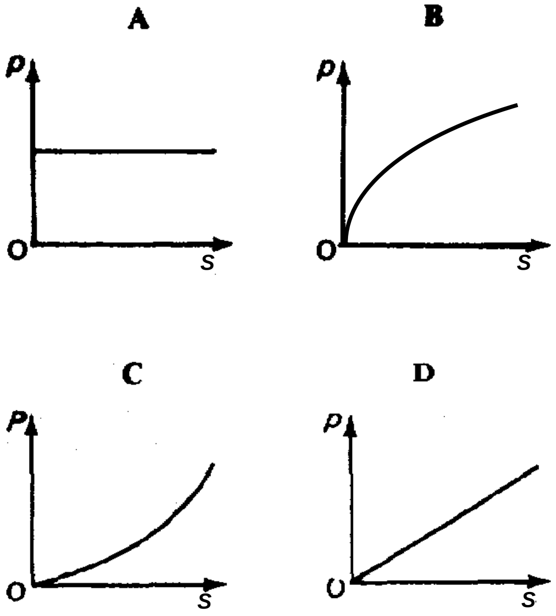
6 A constant force is applied to a body which is initially stationary but free to move in the direction of the force. Assuming that the effects of friction are negligible, which of the following graphs best represents the variation of P, the power supplied, with time t?



B
Constant force implies constant acceleration
Hence v increases uniformly
Since $P = Fv$, P increases proportionately with v

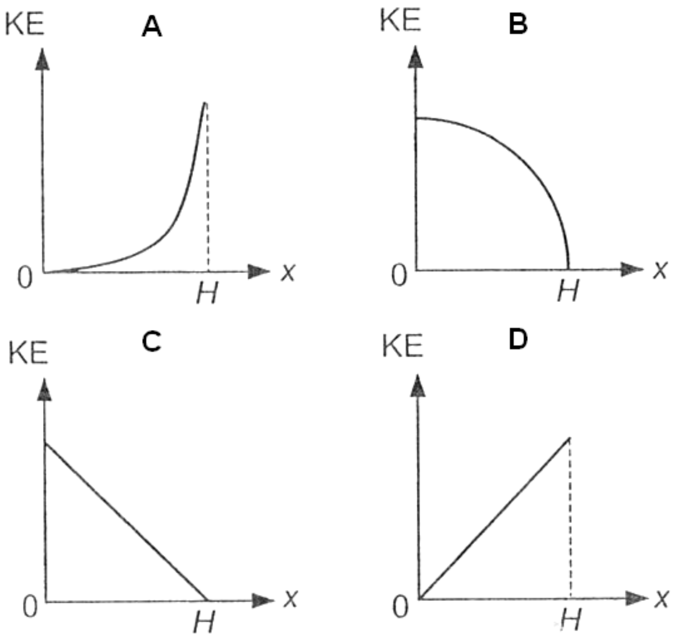
[SRJC 2012]

7 A constant force is applied to a body which is initially stationary but free to move in the direction of the force. Assuming that the effects of friction are negligible, which of the following graphs best represents the variation of P, the power supplied, with displacement s?



B
The power supplied, $P = Fv$
For a constant force, $v \propto \sqrt{s}$ (from $v^2 = 2as$)
So $P \propto \sqrt{s}$ **[VJC 2012]**

8 A ball is dropped from the top of a tower of height H. Which graph shows the variation of the kinetic energy of the ball with distance x measured from the ground?



C
$KE = ½ mv^2 = ½ m(u^2 + 2as) = mas$ *since* $u = 0$
This means that KE is directly proportional to displacement (linearly related)
Consider when $x=0$ (At ground level), KE will be maximum
Consider when $x=H$ (At top of tower), KE will be zero
Hence graph of KE vs x is best represented by C **[JJC 2013]**

9 A hydroelectric dam is built across a waterfall of height 100 m. 200 tonnes of water fall down the waterfall every second. The dam is able to generate electricity at a voltage of 400 kV, and delivers a current of 50 A.

What is the efficiency of the dam?

A 0.010% **B** 0.10% **C** 0.98% **D** 10%

D

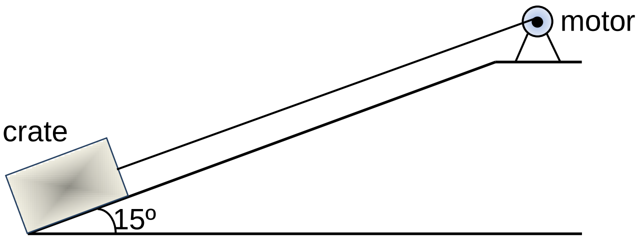
$$P_{in} = \frac{m}{t}gh = 200000(9.81)(100) = 1.962\times10^{8}\text{ W}$$

$$P_{out} = IV = 50(400000) = 2.0\times10^{7}\text{ W}$$

$$\text{Efficiency} = \frac{2.0\times10^{7}}{1.962\times10^{8}} = 10\%$$

[MI 2013]

10 An electric motor is used to pull a 120 kg crate up a ramp at a constant rate. The ramp is inclined at angle of 15º to the horizontal. The crate moves a distance of 25 m up the ramp in 35 s against a constant frictional force of 40 N.



Given that the efficiency of the motor is 30%, what is the electrical power supplied to the motor?

A 190 W **B** 730 W **C** 820 W **D** 2800 W

C

$$P_{out} = \frac{\text{Gain in GPE} + \text{Work done against friction}}{t}$$

$$= \frac{mgh + fd}{t}$$

$$= \frac{120(9.81)(25\sin15) + 40(25)}{35} = 246.2\text{ W}$$

$$P_{in} = \frac{P_{out}}{0.30} = 820\text{ W}$$

[RI 2012]

11 An airplane has two jet engines. If each of the jet engines has an efficiency of 80%, what is the power input of each engine required to allow the plane to fly with a thrust of 200 kN at a speed of 250 m s^{-1}?

A 20.0 MW **B** 31.3 MW **C** 40.0 MW **D** 62.5 MW

B

For each jet engine, $\dfrac{\text{Power output}}{\text{Power input}} = 0.80$

$$\text{Power input} = \frac{\text{power output}}{0.8}$$

$$\text{Power input} = \frac{Fv}{0.8} \quad \text{where } F = \text{half of total thrust}$$

$$\text{Power input} = \frac{\left(\frac{1}{2} \text{ x } 200\,000\right)(250)}{0.8} \approx 31.3 \text{ MW}$$

[MJC 2013]

12 A speed-boat with three engines, each of power output 24 kW, can travel at a maximum speed of 12 m s^{-1}. The total drag D on the boat is related to the speed v of the boat by the relation shown.

$$D \propto v^2$$

What is the maximum speed of the boat when only two engines are working?

A 8.0 m s^{-1} **B** 9.5 m s^{-1} **C** 9.8 m s^{-1} **D** 10.5 m s^{-1}

D
$P = Fv$
At constant (maximum) speed, $F = D$ which is proportional to v^2.
Hence P is proportional to v^3
$P_1 / P_2 = v_1^3 / v_2^3$
$(24000 \text{ x } 2) / (24000 \text{ x } 3) = v_1^3 / (12)^3$
$v_1 = 10.483$ m s^{-1}
Maximum speed with one engine, $v_1 = 10.483 = 10.5$ m s^{-1}

[JJC 2012]

13 A car of mass 1200 kg travels along a horizontal road at a speed of 10 m s^{-1}. At the time it begins to accelerate at 0.20 m s^{-2}, the total resistive force acting on the car is 160 N.

What is the total output power developed by the car as it begins the acceleration?

A 0.8 kW **B** 1.6 kW **C** 2.4 kW **D** 4.0 kW

D

$$\Sigma F = ma$$

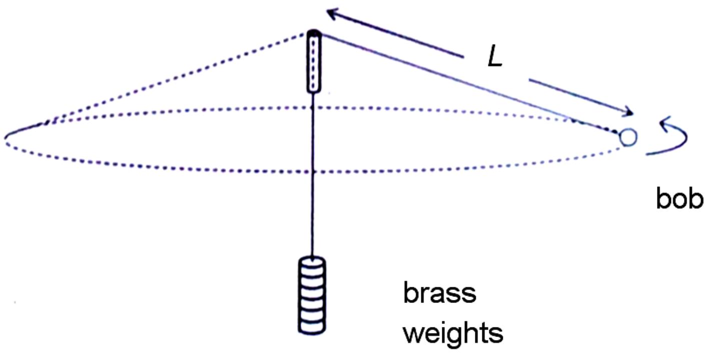
$$F_{drive} - F_{resistive} = ma$$

$$F_{drive} = 1200(0.20) + 160 = 400 \text{ N}$$

$$P_{out} = F_{drive} v = 400(10) = 4000 \text{ W} = 4.0 \text{ kW}$$

[DHS 2013]

14 A bob that is tied to a fixed set of brass weights is made to execute circular motion in a horizontal plane, so that the inelastic string traces out a cone, as shown in the diagram below. The string is passed through a smooth vertical glass tube so that the length L of the string from the top of the glass tube to the bob can vary freely as the speed of the bob changes.



What is the relationship between this length L and the frequency f of the circular motion?

A $L \propto f^2$ **B** $L \propto f$ **C** $L \propto \frac{1}{f}$ **D** $L \propto \frac{1}{f^2}$

D

Let M be the mass of the brass weights, m be the mass of the bob and θ be the angle between the string and the horizontal.

$\Sigma F_y = 0$

$Mg\sin\theta = mg \Rightarrow M\sin\theta = m$

$\Sigma F_c = mr\omega^2$

$Mg\cos\theta = m(L\cos\theta)(4\pi^2 f^2)$

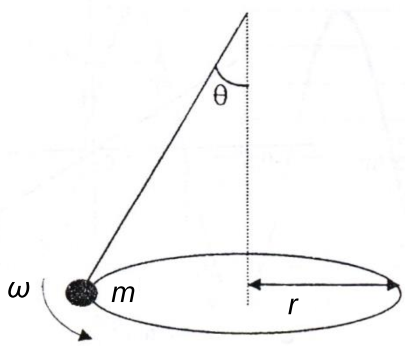
$Mg\cos\theta = M\sin\theta(L\cos\theta)(4\pi^2 f^2)$

$g = \sin\theta(L)(4\pi^2 f^2)$

$L = \dfrac{g}{\sin\theta(4\pi^2 f^2)} \Rightarrow L \propto \dfrac{1}{f^2}$

[TJC 2013]

15 A small bob of mass m which hangs from a light string is set to move in a horizontal circle of radius r with angular velocity ω. The string makes an angle θ with the vertical.



Which is the tension in the string?

A $\dfrac{mg}{\sin\theta}$ **B** $\dfrac{mg}{\tan\theta}$ **C** $\dfrac{mr\omega^2}{\tan\theta}$ **D** $\dfrac{mr\omega^2}{\sin\theta}$

D

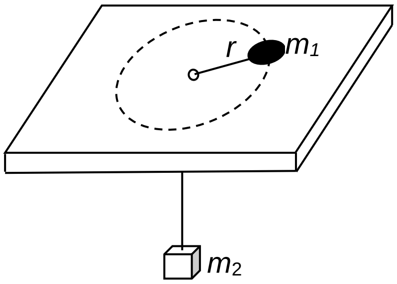
$\Sigma F_c = mr\omega^2$

$T\sin\theta = mr\omega^2$

$T = \dfrac{mr\omega^2}{\sin\theta}$

[NJC 2013]

16 Mass m_1 of 0.50 kg is tied to a string and made to revolve in a circle of radius r on a frictionless horizontal table with a speed of 2.0 m s^{-1}. A load m_2 of mass 0.80 kg is tied to the other end of the string and suspended below the table.



What is the radius r such that m_2 maintains its vertical position?

A 0.25 m **B** 0.33 m **C** 0.41 m **D** 0.65 m

A

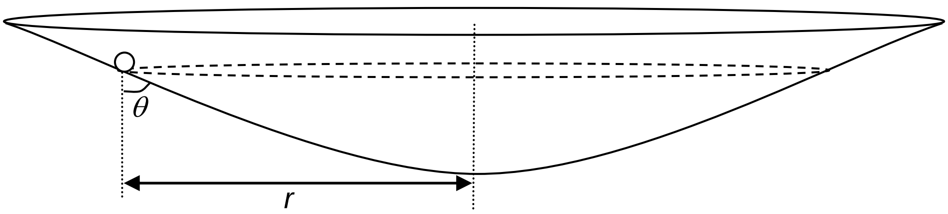
$\Sigma F_c = m_1\dfrac{v^2}{r}$

$m_2 g = m_1\dfrac{v^2}{r}$

$0.80 = 0.50\dfrac{2.0^2}{r} \Rightarrow r = 2.5 \text{ m s}^{-1}$

[RI 2012]

17 A sphere of mass m moves along a smooth horizontal circular path of radius r in a bowl with a constant linear speed v.



Which of the following gives the expression for angle θ?

A $\tan^{-1}\left(\dfrac{rg}{v^2}\right)$ **B** $\tan^{-1}\left(\dfrac{v^2}{rg}\right)$ **C** $\sin^{-1}\left(\dfrac{mv^2}{r}\right)$ **D** $\sin^{-1}(mg)$

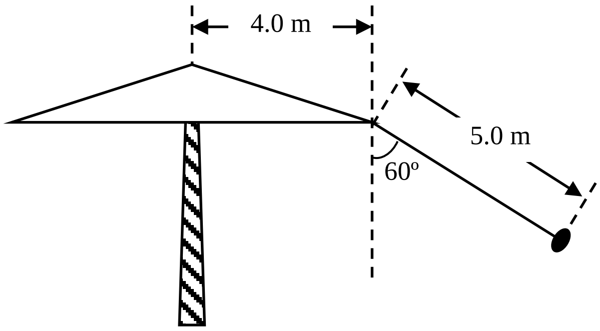
A

$N \sin\theta = mg$

$N \cos\theta = \dfrac{mv^2}{r}$

Therefore, $\tan\theta = \dfrac{rg}{v^2} \rightarrow \theta = \tan^{-1}\dfrac{rg}{v^2}$ **[PJC 2012]**

18 A theme park ride has a number of chairs each suspended from a cable from the edge of a circular canopy. The canopy revolves so that the chairs swing outwards as they move round in circles.



The canopy has radius 4.0 m. The cables are 5.0 m long. For safety, the angle θ of the cables with the vertical must not exceed 60º. The diagram above shows a chair swung outwards as the canopy revolves at the maximum safe speed.

What is the maximum angular speed of rotation?

A 1.42 rad s^{-1} **B** 1.62 rad s^{-1} **C** 1.98 rad s^{-1} **D** 2.61 rad s^{-1}

[PJC 2015]

A

Resolving the forces acting on the chair.

Vertically,

$T\cos\theta = mg$ --- (1)

Horizontally,

$T\sin\theta = mr\omega^2$ --- (2)

(2)/(1):

$\tan\theta = \dfrac{r\omega^2}{g}$

$\omega^2 = \dfrac{g\tan\theta}{r} = \dfrac{9.81\times\tan 60^\circ}{5.0\sin 60^\circ + 4.0}$

$\omega = 1.43$ rad s^{-1}

19 A mass m_1 is attached to one end of an elastic string of an unstretched length L. When the mass is rotating with a linear speed v on a smooth table in a horizontal circle, an extension e is obtained.
Which of the following shows the correct expression for mass m_2, if it is rotated with the same linear speed v but rotates at twice the radius as that produced by m_1?

A $m_2 = \dfrac{2m_1(L+2e)}{e}$ **B** $m_2 = \dfrac{2m_1(L+e)}{e}$

C $m_2 = \dfrac{2m_1(2L+e)}{e}$ **D** $m_2 = \dfrac{2m_1(2L+2e)}{e}$

A

For mass m_1:

$$\frac{m_1v^2}{L+e} = ke$$

$$v^2 = \frac{ke(L+e)}{m_1}$$

For mass m_2:

$$\frac{m_2v^2}{2(L+e)} = k(2L+2e-L)$$

$$\frac{m_2v^2}{2(L+e)} = k(L+2e)$$

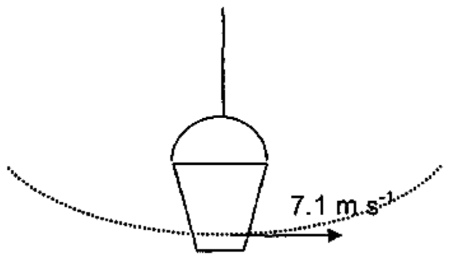
$$m_2 = \frac{2k(L+2e)(L+e)}{v^2}$$

$$= \frac{2k(L+2e)(L+e)m_1}{ke(L+e)}$$

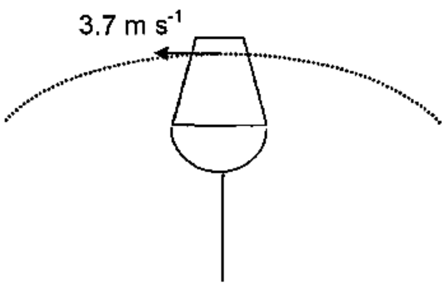
$$= \frac{2m_1(L+2e)}{e}$$

[PJC 2013]

20 A 1.2 kg bucket is held by a string and whirled in a vertical circle. The radius of the circle is 1.3 m. The speed of the bucket is 3.7 m s^{-1} at the top of the loop and 7.1 m s^{-1} at the bottom of the loop.



At the bottom of the loop



At the top of the loop

What is the difference in the tension in the string at these two positions?

A 34 N **B** 57 N **C** 59 N **D** 83 N

B

At the bottom, $\sum F_c = ma_c$

$$T_b - mg = m\frac{v^2}{r}$$

$$T_b = (1.2)\frac{(7.1)^2}{1.3} + (1.2)(9.81) = 58.2 \text{ N}$$

At the top, $\sum F_c = ma_c$

$$T_t + mg = m\frac{v^2}{r}$$

$$T_t = (1.2)\frac{(3.7)^2}{1.3} - (1.2)(9.81) = 0.868 \text{ N}$$

$$T_b - T_t = 57 \text{ N}$$

[SAJC 2012]

21 When a small mass tied to light inextensible string that is 20 cm long is hung onto a spring balance, the spring balance reads 50 g. (**Fig. 21.1**) The mass is then displaced from the vertical at an angle 30º and released from rest. (**Fig. 21.2**)

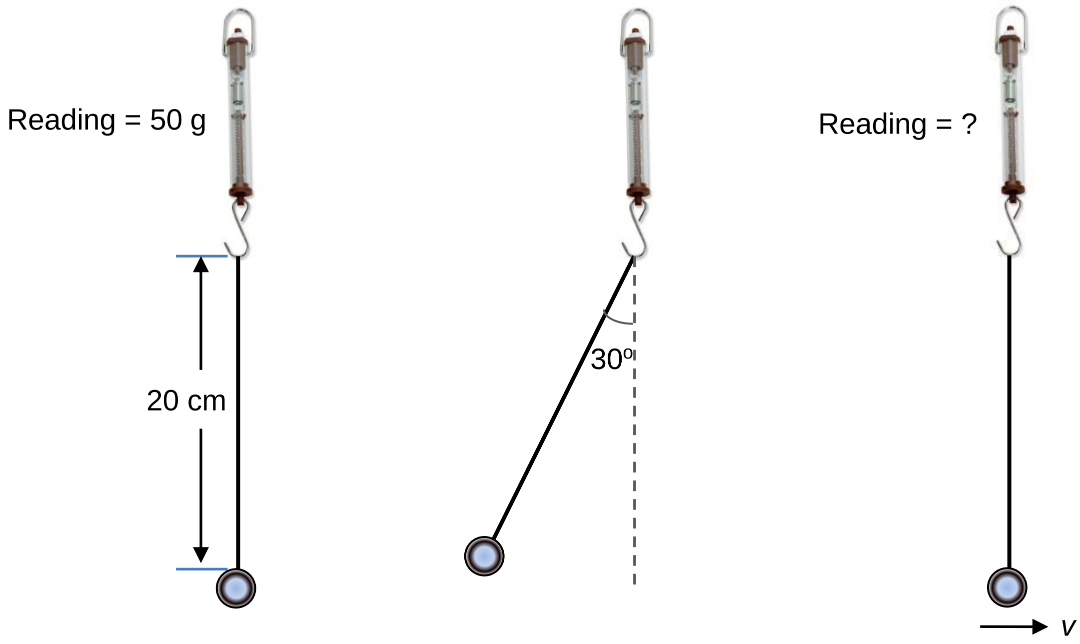


Fig. 21.1 **Fig. 21.2** **Fig. 21.3**

What is the reading on the balance when the mass is at the lowest point of its motion? (**Fig. 21.3**)

A 50 g **B** 56 g **C** 63 g **D** 68 g

C

Reading on spring balance = $\frac{T}{g}$

Mass is in circular motion: Consider the forces acting on the body at the lowest point,

$$T - mg = \frac{mv^2}{r} \quad \Rightarrow \quad \frac{T}{g} = \frac{mv^2}{gL} + m \quad ...(1)$$

To find v, consider conservation of energy,

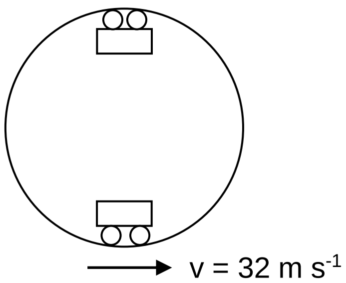
$$\frac{1}{2}mv^2 = mgh = mg(L - L\cos\theta) \quad \Rightarrow \quad mv^2 = 2mg(L - L\cos\theta) \quad ...(2)$$

Combining (1) and (2) we have

$$\frac{T}{g} = 2mL(1 - \cos 30^o) + m = 63 \text{ g}$$

[HCI 2012]

22 On one particular amusement park ride, passengers 'loop-the-loop' in a vertical circle as shown below.



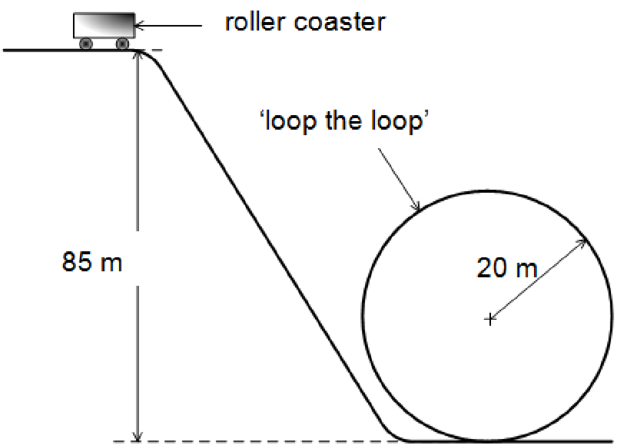
The loop has a radius of 20 m and the cart and the passenger, combined mass 120 kg, is travelling at 32 m s^{-1} at the bottom of the loop.
Assuming the cart experiences no frictional force with the track, what is the difference between the normal reaction forces acting on the cart at the top of the loop and the bottom of the loop?

A 0 N **B** 4800 N **C** 7100 N **D** 7300 N

C
By COE,
$E_{top} = E_{bottom}$
$\frac{1}{2} m v_{top}^2 + m(9.81)(40) = \frac{1}{2} m(32^2)$
$v_{top} = 15.5$ m s^{-1}
Force at top of loop,
$\frac{m v_{top}^2}{r} = N_{top} + W$ -----(1)
Force at bottom of loop,
$\frac{m v_{bottom}^2}{r} = N_{bottom} - W$ ----(2)
(2) – (1),
$N_{bottom} - N_{top} = \frac{m v_{bottom}^2}{r} - \frac{m v_{top}^2}{r} + 2W$
$= \frac{120 \times 32^2}{20} - \frac{120 \times 15.5^2}{20} + 2(120)(9.81)$
= 7100 N

[AJC 2013]

23 A roller coaster starts from rest on a hill-top. It accelerates along a frictionless track before entering a loop-the-loop of radius 20 m as shown below.



What is the minimum normal contact force that the roller coaster seat exerts on a passenger with weight *W*, as it passes through the 'loop the loop'?

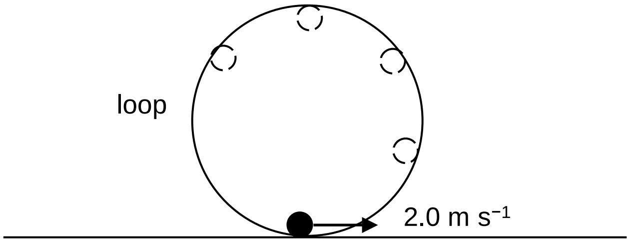
A 0 **B** 3.5 *W* **C** 4.5 *W* **D** 5.5 *W*

B
By Conservation of Energy,
Gain in KE = Loss in GPE
$\frac{1}{2} m v^2 - 0 = mg(h_0 - h)$
$v^2 = 2g(85 - 40) = 90g$

$F_{net} = \frac{mv^2}{r} = \frac{90mg}{20}$
$N + mg = \frac{9}{2} mg$
$N = 3.5mg = 3.5W$

[IJC 2013]

24 The diagram below shows a typical loop-the-loop experiment carried out on Earth where an object is projected with a minimum speed of 2.0 m s^{-1} at the bottom such that it can complete looping without leaving the track.



What is the minimum speed needed when the same experiment is performed on the moon, where the gravitational field strength is 1.6 N kg^{-1} on its surface?

A 0.33 m s^{-1} **B** 0.81 m s^{-1} **C** 2.0 m s^{-1} **D** 12 m s^{-1}

B

By Conservation of energy,

Loss in KE = Gain in GPE

$$\frac{1}{2}mv_{bot}^2 - \frac{1}{2}mv_{top}^2 = mg(2r)$$

$$v_{bot}^2 - v_{top}^2 = 4gr \text{ --- (1)}$$

By refering to free body diagram of the body at the top of motion,

$$mg + N = m\frac{v_{top}^2}{r}$$

$$g = \frac{v_{top}^2}{r} \text{ --- (2)} \quad (\text{since } N = 0)$$

For Earth, we have

$$4 - v_{top}^2 = 39.24r \text{ --- (3)}$$

$$9.81 = \frac{v_{top}^2}{r} \text{ --- (4)}$$

Solving (3) and (4), $r = 0.0815$ m

For Moon, we have

$$v_{bot}^2 - v_{top}^2 = 6.4r \text{ --- (5)}$$

$$1.6 = \frac{v_{top}^2}{r} \text{ --- (6)}$$

Sub $r = 0.0815$ m into (6),

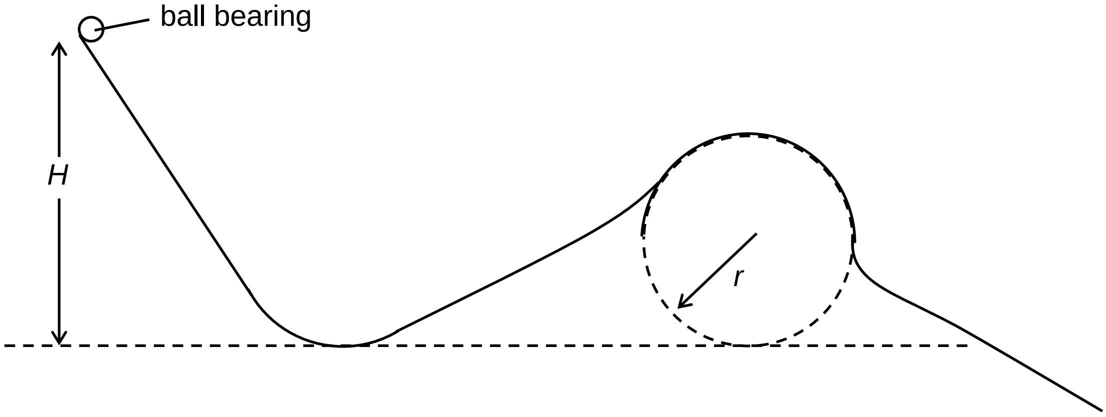
$$v_{top}^2 = 0.130 \text{ m s}^{-1}$$

Sub into (5), $v_{bot}^2 - 0.130 = 6.4(0.0815)$

$$v_{bot} = 0.81 \text{ m s}^{-1}$$

[RVHS 2012]

25 A ball bearing, initially at rest at a height *H* is being rolled down a slope as shown in figure below and goes through a trough and a crest in the form of a circular path of radius *r*.



What is the maximum height *H*, in terms of *r* that the ball bearing should start from in order for it to be always in contact with the floor surface?

A ½ *r* **B** 1.5 *r* **C** 2 *r* **D** 2.5 *r*

D
At the top of the crest, for the ball bearing to go in circular motion,
$mg - N = mv^2/r$ --- (1) where N is the normal contact force
For it to remain in contact with the floor means that $N \geq 0$.
Then, from (1), $mg - mv^2/r \geq 0$
$mv^2 \leq mgr$ --- (2)
Also, by conservation of energy when the ball rolls down the hill and then up the loop,
The loss in GPE = gain in KE
$mg(H - 2r) = \frac{1}{2} mv^2$
=> $mv^2 = 2mg(H - 2r)$ --- (3)
Subst (3) into (2), $2mg(H - 2r) \leq mgr$
$H \leq 0.5r + 2r = 2.5\,r$
Hence maximum height $H = 2.5r$

[VJC 2012]

26 A rigid track in a plane vertical to the ground consists of a semi-circular section of diameter 5.0 m and a straight horizontal section. A small object with a mass of 100 g is projected horizontally into the track at the top with a speed of 20 m s^{-1}. The track exerts an average resistive force of 1.0 N to the object. Point **P** is vertically below the projection point.

Assuming that the object always stays in contact with the track, calculate the distance from point **P** where the object comes to rest.

A 9.2 m **B** 17.1 m **C** 19.9 m **D** 24.9 m

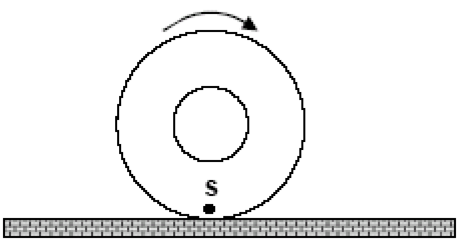
B
Arc length = $(2.5)\pi$
Energy lost along the arc = $(1)(2.5)\pi$

Initial total energy = $\frac{1}{2}(0.1)(20)^2 + (0.1)(9.81)(5) = 24.905$ J
Energy at P = $\frac{1}{2}(0.1)(20)^2 + (0.1)(9.81)(5) - (1)(2.5)\pi = 17.051$ J

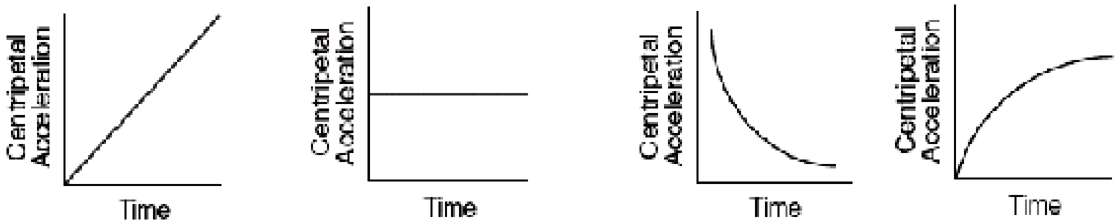
Object travels a distance of L before all the energy at P is fully discharged.
$(1)L = 17.051 \Rightarrow \quad L = 17.1$ m

[HCI 2012]

27 In the diagram below, S is a point on a car tire rotating at a constant rate.



Which graph best represents the magnitude of the centripetal acceleration of point *S* as a function of time?



A B C D

B

Because the tyre is rotating at a constant rate, hence angular velocity is constant. Assuming no change in radius, centripetal acceleration remains constant with time.

[NJC 2013]

28 A small mass m is suspended from one end of a vertical string and then whirled in a horizontal circle, of radius r, at a constant speed *v*.

Which one of the following is true?

A The angle of inclination to the vertical of the string depends on *v* and *r* only.

B The angle of inclination to the vertical of the string depends on *v*, *r* and *g* only.

C The tension in the string depends on *m* only.

D The tension in the string depends on *m*, *r, g* and angle of inclination only.

B

$T \sin\theta = mv^2/r$ -----------(1)

$T \cos\theta = mg$ ------------(2)

(1)/(2) $\tan\theta = v^2/rg$

$(1)^2+(2)^2$ $T^2 = m^2g^2 + m^2v^4 / r^2$

[JJC 2013]

29 Two identical particles P and Q are set to travel in a circular path of the same radius. P moves in a vertical circle and Q moves in a horizontal circle. Both move with the same uniform speed.

Which of the following statements concerning the magnitude of the net force acting on P and Q towards the centre of the circular path is true?

A Both net forces on P and Q vary with time and are never equal in magnitude.

B Both net forces on P and Q vary with time and are equal in magnitude periodically.

C The net forces on P and Q are always equal in magnitude.

D The magnitude of the net force on P is always larger than that on Q.

C

Since mass, radius and speed is the same, then the magnitude of the centripetal force on P and Q is the same, regardless of orientation. Centripetal force is provided by the net force towards the centre of the circular path and thus the net forces of P and Q are equal in magnitude.

[PJC 2013]

30 Two stationary particles of masses M_1 and M_2 are a distance d apart. A third particle, lying on the line joining the particles, experiences no resultant gravitational force.

What is the distance of this particle from M_1?

A $d\left(\frac{M_2}{M_1}\right)$ **B** $d\sqrt{\left(\frac{M_1}{M_2}\right)}$ **C** $d\sqrt{\left(\frac{M_1}{M_1+M_2}\right)}$ **D** $d\left(\frac{\sqrt{M_1}}{\sqrt{M_1}+\sqrt{M_2}}\right)$

D

$F_1 = F_2$

$\frac{GM_1M_3}{r_1^2} = \frac{GM_2M_3}{r_2^2}$

$\frac{M_1}{r_1^2} = \frac{M_2}{(d-r_1)^2}$

$r_1 = d\left(\frac{\sqrt{M_1}}{\sqrt{M_1}+\sqrt{M_2}}\right)$

[RI 2013]

31 Two satellites A and B orbit the Earth in circular orbits, the radius of satellite A's orbit being 4 times that of satellite B. If the orbital period and tangential velocity of satellite A are T and v respectively, what are the corresponding values for satellite B?

	Period	Velocity
A	8T	2v
B	1/8T	2v
C	1/8T	½ v
D	8T	½ v

B

For satellite A, recall that

$mr\omega^2 = \frac{GMm}{r^2}$

$T = 2\pi\sqrt{\frac{r^3}{GM}}$

and

$\frac{mv^2}{r} = \frac{GMm}{r^2}$

$v = \sqrt{\frac{GM}{r}}$

So for satellite B, radius r' = ¼ r, then v' = $\sqrt{\frac{1}{1/4}}v = 2v$

And the period T' = $\sqrt{\left(\frac{1}{4}\right)^3}T = \frac{1}{8}T$

[VJC 2012]

32 Two objects of different masses falling freely from the same heights above the Earth's surface will experience the same

A rate of change of momentum

B gravitational field strength

C gravitational force

D gravitational potential energy

B

As the calculation of the rate of change of momentum , gravitational force and potential energy involved mass.

[YJC 2012]

33 A satellite moves from one circular orbit around earth into another circular orbit with a smaller orbital radius. Which of the following is true?

A Kinetic energy of the satellite decreases.

B Angular velocity of the satellite decreases.

C Potential energy of the satellite decreases.

D Gravitational pull on the satellite by earth decreases.

C

Consider the various energies associated with an orbiting satellite:

$KE = \frac{GMm}{2R}$ $\quad GPE = -\frac{GMm}{R}$ $\quad TE = -\frac{GMm}{2R}$

So when R decreases, kinetic energy increases while gravitational potential energy and total energy decrease.

Consider the forces on the satellite:

$\frac{GMm}{r^2} = mr\omega^2 \quad \Rightarrow \quad \omega^2 \propto r^{-3}$

So when r decreases, both gravitational pull on the satellite and the its angular velocity increase.

[HCI 2013]

34 An artificial satellite travels in a circular orbit about the Earth. Its rocket engine is then fired and produces a force on the satellite exactly equal and opposite to that exerted by the Earth's gravitational field.

The satellite would then start to move

A along a spiral path towards Earth

B along the line joining it to the centre of the Earth (i.e. radially)

C along a tangent to the orbit

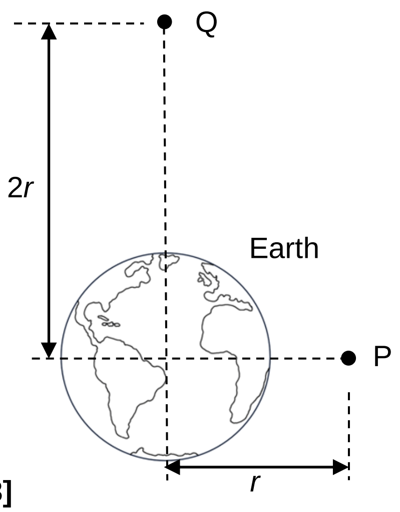
D in a circular orbit with a longer period

C

Net force acting on the satellite at that instant is now zero. Since the satellite is already moving with a speed tangent to the orbit, Newton's 1st law says that it will continue to move along the tangent to the orbit.

[MJC 2013]

35 P and Q are two points above Earth's surface at distances r and $2r$ respectively from the centre of the Earth.



The gravitational potential at P is –800 kJ kg^{-1}. When a 1.00 kg mass is taken from P to Q, what is the work done on the mass?

A -400 kJ **B** -200 kJ

C 400 kJ **D** 800 kJ

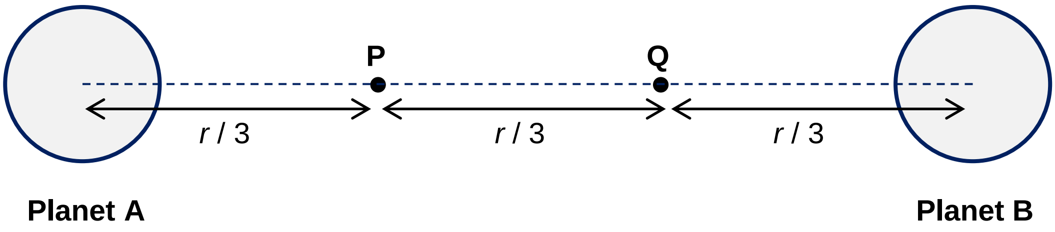
C

Since potential at P is -800 kJ kg^{-1}, Q, which is further, will have a potential that is less negative, hence -400 kJ kg^{-1}.
The difference in the potential energy (since the mass is 1.00 kg) is 400 kJ.
From P to Q, there is a gain in potential energy, hence +400 kJ.

[DHS 2013]

36 Planet A, of mass M, and planet B, of unknown mass, are found to be at a fixed separation of r, measured from their respective centres of mass. When a spacecraft moves from point P to point Q, its gravitational potential energy changes from U to $1.25U$.

What is the mass of planet B in terms of M?



A $5M/4$ **B** $4M/3$ **C** $3M/2$ **D** $2M$

D

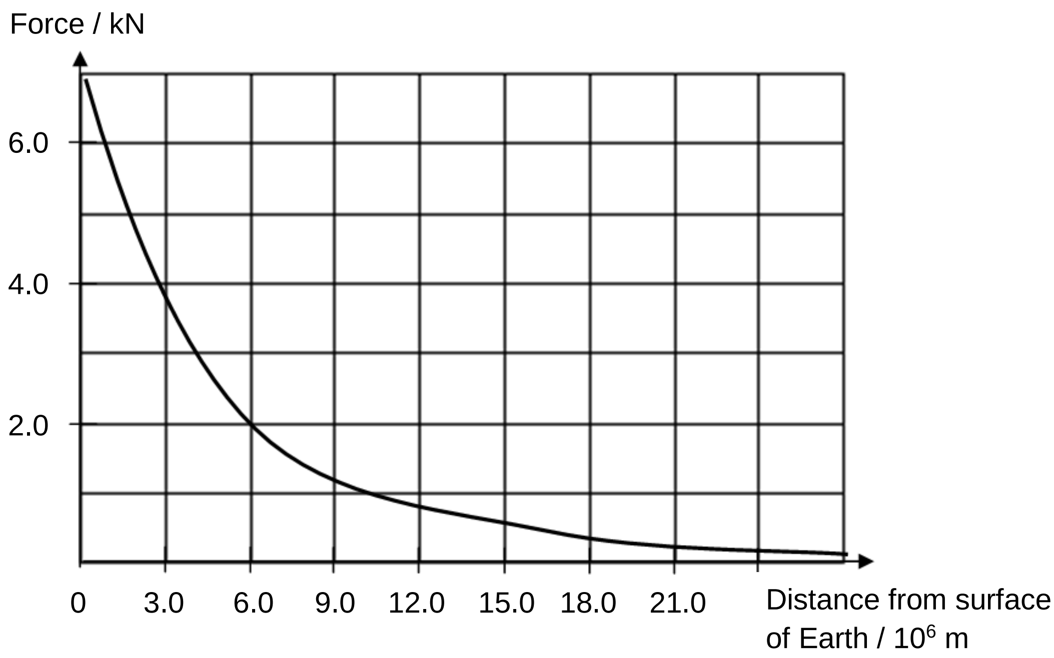
At point P: $\Phi_P = \left(-\dfrac{GM}{r/3}\right) + \left(-\dfrac{GM_B}{2r/3}\right) = -\dfrac{G}{r}(3M + 1.5M_B)$

At point Q: $\Phi_Q = \left(-\dfrac{GM}{2r/3}\right) + \left(-\dfrac{GM_B}{r/3}\right) = -\dfrac{G}{r}(1.5M + 3M_B)$

$\Phi_Q = 1.25\ \Phi_P \Rightarrow (1.5M + 3M_B) = 1.25(3M + 1.5M_B) \Rightarrow M_B = 2M$

[HCI 2013]

37 The Mars Odyssey spacecraft was launched from Earth in 2001. The graph below shows the variation of the gravitational force acting on the 700 kg spacecraft with distance above the Earth's surface.



Based on information from the graph, what is the minimum launch speed needed for the spacecraft to escape the Earth's gravitational field?

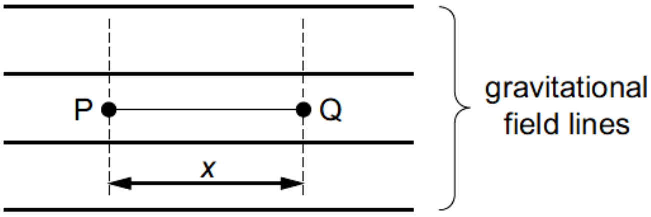
A 3.1×10^{-1} m s^{-1} **B** 9.9×10^{3} m s^{-1} **C** 2.7×10^{7} m s^{-1} **D** 3.3×10^{10} m s^{-1}

B

This is the only logical answer, the other values are either too large or too small.

[RVHS 2012]

38 A mass m is situated in a uniform gravitational field.



When the mass moves through a displacement x, from P to Q, it loses an amount of potential energy E. Which row correctly specifies the magnitude and the direction of the acceleration due to gravity in this field?

	magnitude	direction
A	$\frac{E}{mx}$	$\rightarrow$
B	$\frac{E}{mx}$	$\leftarrow$
C	$\frac{E}{x}$	$\rightarrow$
D	$\frac{E}{x}$	$\leftarrow$

A

GPE decreases from P to Q implies the direction of the G-field is towards $\rightarrow$

Since mgx = E, g = E / mx

[AJC 2015]

39 A satellite orbits a planet at a distance r from its centre. Its gravitational potential energy is –3.2 MJ.
Another identical satellite orbits the planet at a distance 2r from its centre.
What is the sum of the kinetic energy and the gravitational potential energy of this second satellite?

A –0.40 MJ

B –0.80 MJ

C –1.6 MJ

D –6.4 MJ

[IJC 2015]

B

Recall:

$\text{GPE} = -\dfrac{GMm}{r}$ (1)

$\text{KE} = \dfrac{GMm}{2r}$ (2)

$\text{Total E} = -\dfrac{GMm}{2r}$ (3)

GPE of new satellite = $-\dfrac{GMm}{r} = -3.2 \text{ MJ}$

From eqn (3), Total Energy of new satellite $= -\dfrac{GMm}{2(2r)} - 0.8 \text{ MJ}$

40 Planets P and Q have equal mass. The radius of P is twice that of Q. The minimum kinetic energy needed by a body to escape from the surface of planet P is K_P.
What is the minimum kinetic energy needed by the same body to escape from the surface of planet Q?

A $0.25K_P$

B $0.5K_P$

C $2K_P$

D $4K_P$

[PJC 2015]

C

For an object to escape from the Earth's surface,

loss in K.E. = gain in G.P.E.

$$\frac{1}{2}mv^2 = \frac{GMm}{R}$$

$$\text{KE} \propto \frac{1}{R}$$

$$\frac{K_Q}{K_P} = \frac{R_P}{R_Q} = 2$$

$$K_Q = 2K_P$$

Short structured questions

1 **(a)** Derive the equation $E_p = mgh$ for the potential energy change of a mass *m* moved through a vertical distance *h* near the Earth's surface. [2]

Assumption: *g* is constant as *h* is very small compared to the radius of the earth.
Work done by external agent in moving a mass upward goes to the gain in GPE of the object
Hence, GPE gain = Work Done = Force by external agent × Distance moved = *mgh*

(b) **Fig. 1.1** shows the variation with velocity *v* of the force *F* applied to an object. *F* and *v* are in the same direction.

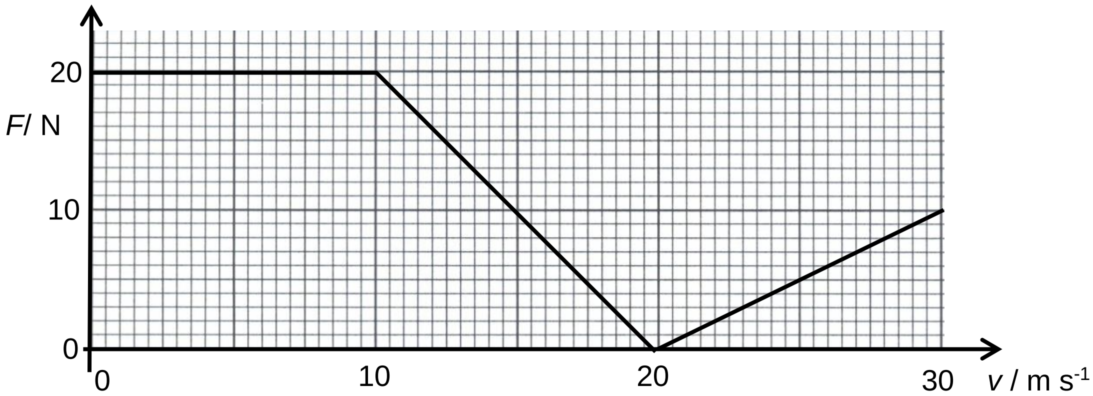
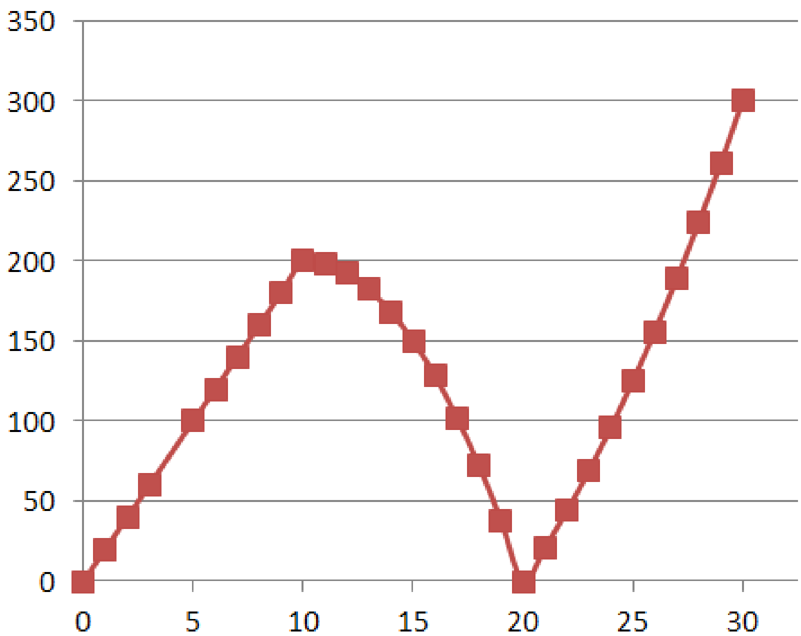


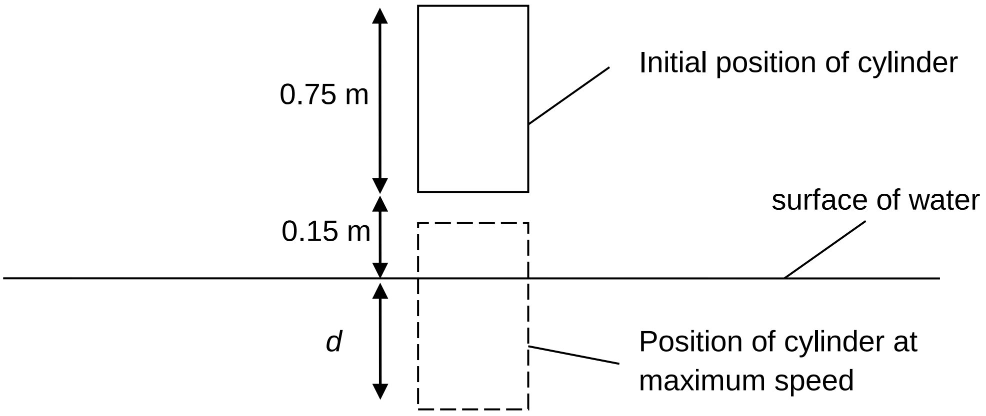
Fig. 1.1

On **Fig 1.2**, draw a graph showing the variation with *v* of the rate of work done. [3]



[NJC 2013]

2 A uniform cylinder of length 0.75 m, cross sectional area of 119 cm^2 and mass 8.48 kg is dropped vertically from a height of 0.15 m above the water surface as shown below. The density of the cylinder and that of the water are 950 kg m^{-3} and 4200 kg m^{-3} respectively.
You may assume that there is no significant rise in water level after the cylinder has entered the water, and that there is negligible drag force acting on the cylinder.



(a) **(i)** Show that when the bottom of the cylinder is about to touch the surface of the water, its kinetic energy is 12.5 J. [1]

Loss in GPE = Gain in KE

$KE = 8.48 \times 9.81 \times 0.15 = 12.5$ J

(ii) The cylinder then enters the water, and reaches a submerged length of *d* when it attains maximum speed as shown in the figure above.
Determine the submerged length *d*. [3]

Max speed occurs when $F_{net} = 0$

$U = mg$

$\rho_w V_w g = \rho_c V_c g$

$\rho_w \left(A l_w\right) g = \rho_c \left(A l_c\right) g$

$l_w = \dfrac{950}{4200} \times 0.75$

$= 0.170$ m

(iii) The cylinder eventually comes to a momentary stop, while it is still not entirely submerged.

Show that when the cylinder comes to a momentary stop, it has a submerged length of 0.452 m.
[Hint: When the cylinder is submerged in the water, it does work of *W* against upthrust.

$$W = \frac{1}{2} k x^2$$

where
k = density of fluid × cross sectional area of object × gravitational acceleration
x = submerged length of object] [3]

Method 1

Consider from position of release to position of momentary rest.

Loss in GPE = Work done against upthrust

$$mgh = \frac{1}{2}kx^2$$

$$(\rho_c A l_c) g (0.15 + x) = \frac{1}{2}(\rho_w A g) x^2$$

$$(950 \times 0.75)(0.15 + x) = \frac{1}{2}(4200) x^2$$

$x = -0.113$ m (rej) or 0.452 m

Method 2

Consider from position before touching water to position of momentary rest.

Loss in KE + Loss in GPE = Gain in EPE

$$KE + mgh = \frac{1}{2}kx^2$$

$$KE + (\rho_c A l_c) gh = \frac{1}{2}(\rho_w A g) x^2$$

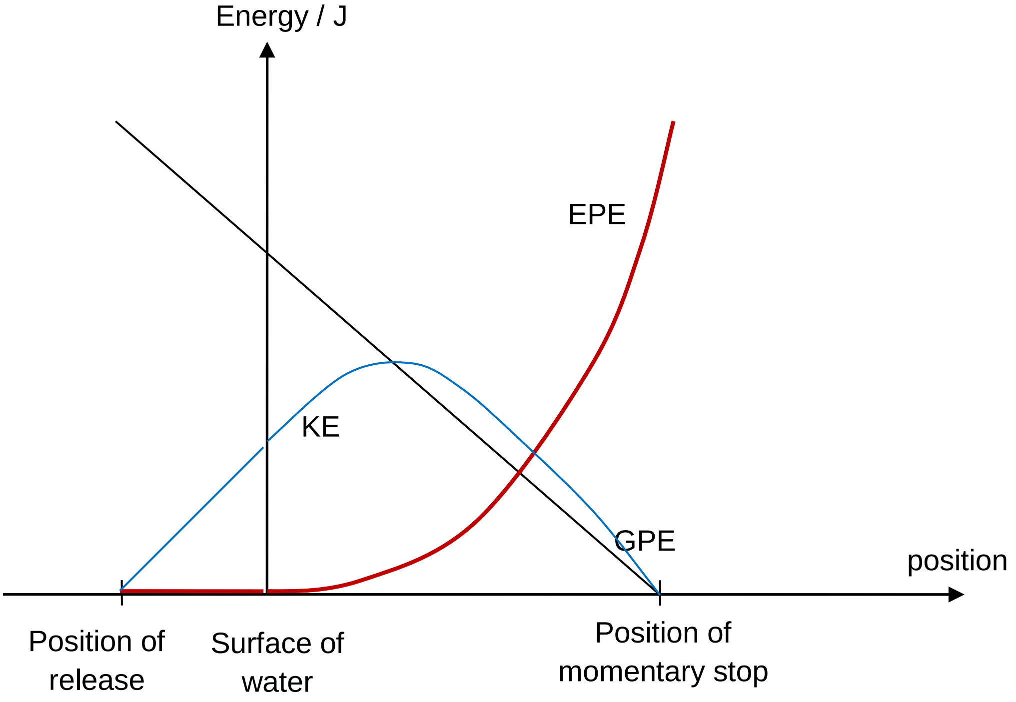
$$12.5 + (950 \times 0.75)(9.81) h = \frac{1}{2}(4200 \times 9.81) x^2$$

$x = -0.113$ m (rej) or 0.452 m

(iv) The table below shows the various energy values of the cylinder at different positions. Fill in the missing values in the table. [3]

	Kinetic Energy / J	Gravitational Potential Energy /J	Work done against upthrust / J
Position of Release	0	50.1	0
Position of Maximum Speed	$50.1 - 23.5 - 7.08$ $= 19.5$	$mg(0.452 - 0.170)$ $= 23.5$	$\frac{1}{2}(\rho_w A g) x^2$ $= \frac{1}{2}(4200 \times 0.0119 \times 9.81)$ $\times (0.170)^2$ $= 7.08$
Position of Momentary Stop	0	0	50.1

(v) In the graph below, sketch the variation of the kinetic energy (KE) and work done against upthrust (W) of the cylinder, with respect to its position.
The graph of gravitational potential energy (GPE) has been sketched for you. [3]



[SRJC 2012]

3 An advertisement claims that a certain 1200 kg car can accelerate uniformly from rest to a speed of 25 m s^{-1} in a time of 8.0 seconds.

(a) Calculate the distance that the car will travel during the duration of 8.0 seconds. [2]

Assuming constant acceleration a.

$$s = \frac{1}{2}(v+u)t$$
$$= \frac{1}{2}(25)8$$
$$= 100 \text{ m}$$

(b) Determine the average power that the motor must produce to cause this acceleration. You may ignore any frictional loss. [2]

Change in kinetic energy = $\frac{1}{2}mv^2 - 0$
= $\frac{1}{2}(1200)(25)^2$
= 375 kJ

$$\text{average power} = \frac{\Delta KE}{\Delta t}$$
$$= \frac{375}{8}$$
$$= 46.9 \text{ kW} = 4.69 \times 10^4 \text{ W}$$

(c) Determine the average power if the car is going up a slope inclined at 15° instead with the same acceleration. [2]

Change in KE = 375 kJ
Change in GPE = (1200)(9.81)(100sin15°)

$$\text{Average Power} = \frac{\Delta KE + \Delta GPE}{\Delta t}$$

$$= \frac{375 \times 10^3 + (1200)(9.81)(100 \sin 15°)}{8}$$

$$= \frac{679682}{8}$$

$$= 8.5 \times 10^4 \text{ W}$$

(d) The energy obtained for every litre of regular petrol burned is 34.8 MJ. Given that the car uses 1.24 litres of petrol every kilometre travelled in **(c)**, determine the efficiency of the car as it accelerates up the slope. [3]

From (a) distance travelled is 0.10 km.
From (c) kinetic energy gained over the distance is (85.0)(8) = 680 kJ
Energy supplied from fuel consumed = (0.10)(1.24)(34.8)
= 4.32 MJ

$$\text{Efficiency of car} = \frac{680 \times 10^3}{4.32 \times 10^6} \times 100\%$$

$$= 15.6\%$$

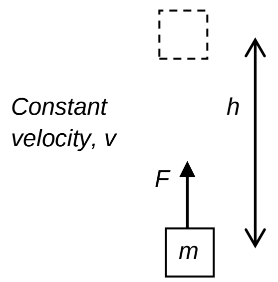
(e) In real life, it is found that a car consumes more fuel for every kilometre travelled when the car is moving at a fast speed.
Suggest a reason for the increase in fuel consumption. [1]

Drag force on the car increases with speed. Work is done against drag force.
OR
At higher speed, the more power is required since $P = Fv$.

[MJC 2012]

4 **(a)** By referring to work done being the product of force and the displacement in the direction of the force, derive the formula $E_p = mgh$ for potential energy changes near the Earth's surface. [3]

A force F acts on a mass m to move it vertically upwards **at constant speed** (so that no change in E_k) **by a displacement *h* in the direction of the force**.



Since the object moves at constant speed, **the upward force, *F*, must be equal to the weight of the object, *mg***. (no resultant force).

Using **Work done on object = *Fh* = *(mg)h***

Since Fh is the work done on the object and is equal to the increase in potential energy, $E_p = mgh$

(b) A typical escalator at Jurong Point Shopping Mall rises at an angle of 30º to the horizontal. It lifts people through a vertical height of 15 m in 0.50 minute. Assuming all the users stand still while on the escalator, 60 users can get on at the bottom and get off at the top in 0.50 minute. The average mass of a user is 55 kg.

(i) Determine the average power needed to lift the users when the escalator is fully laden. Assume that any kinetic energy transferred to the users by the escalator is negligible. [2]

$$\text{Power} = \frac{n \times mgh}{t} = \frac{60 \times (55)(9.81)(15)}{1 \times 30}$$

$= 1.62 \times 10^4$ W

(ii) The frictional force in the escalator system is 1.0×10^4 N when the escalator is fully laden.
Calculate the power to overcome friction. [3]

v (in direction parallel to escalator) = $\dfrac{15}{\sin 30^o} \div 30 = 1.0 \text{ m s}^{-1}$

Power = $Fv = 1.0 \times 10^4 \times 1.0$
$= 1.0 \times 10^4$ W

(iii) When there are 60 users walking up the moving escalator, instead of standing still, at any point in time, explain whether more or less power is required by the motor to maintain the escalator at the same speed. [2]
To maintain constant speed, the escalator has to supply **more power** to **overcome the (downward) force exerted on escalator steps by users** moving up.
OR higher rate of increase of GPE of the users

[JJC 2012]

5 A ball bearing of mass 5.00 g rests on a Hooke's spring of elastic constant k = 500 N m^{-1}. The ball is pushed down 2.0 cm from its equilibrium position and then released as shown in **Fig. 5.1** and **Fig. 5.2**.

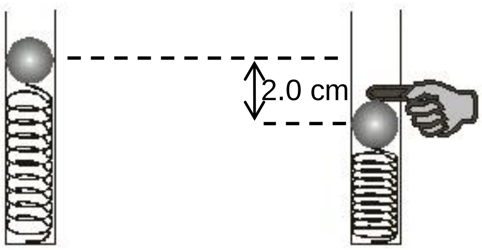


Fig. 5.1

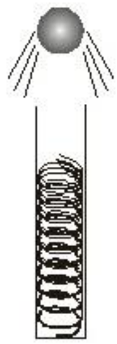
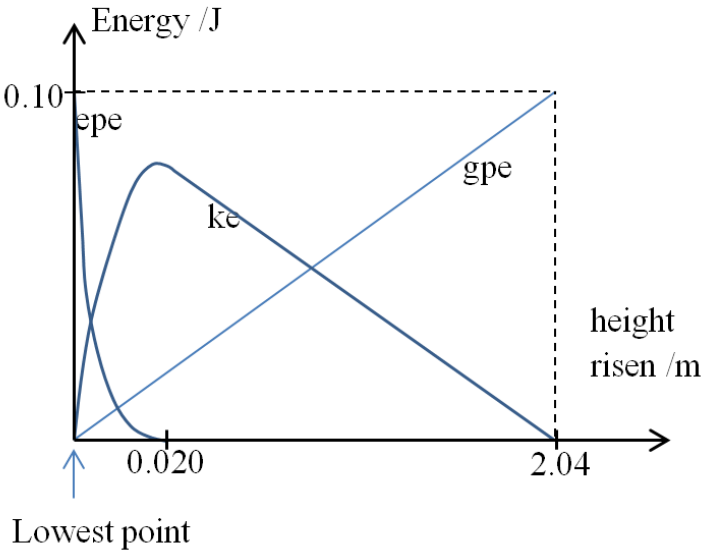


Fig. 5.2

(a) Calculate the maximum height H to which the ball bearing will rise from its lowest position . [2]
From conservation of energy, the elastic energy stored in the spring will be transformed into gravitational potential energy. Hence:
$\frac{1}{2} kx^2 = mgH$
$\rightarrow H = \frac{1}{2} kx^2/(mg)$
$= \frac{1}{2} (500)(0.020^2)/(0.005 \times 9.81)$
$= 2.04$ m = 204 cm

(b) (i) Explain clearly the transformation of energy of the ball bearing when it is released from the depressed position of the spring until it reaches the maximum height H. Include discussion of the energy transformation during the time interval when the ball is rising. [2]
As the spring recovers from its 2 cm depression, the stored elastic potential energy will be transformed to kinetic energy as well as gravitational potential energy.
The ball bearing's kinetic energy will finally be all transformed to gravitational potential energy when the ball bearing reaches its maximum height H.

(ii) Sketch the variations of the different energies in **(b)(i)** as a function of height starting from the ball bearing's lowest depressed position to its highest point. Label the sketch appropriately. [3]



[VJC 2013]

6 **(a)** A student is hoisting a flag attached to a rope at constant speed up a pole of height 6.5 m above the ground. The flag of mass 0.60 kg was initially at rest on his shoulders 1.0 m above the ground.

(i) Calculate the increase in potential energy of the flag. [1]

mgh = (0.60)(9.81)(5.5)= 32J

(ii) Estimate the work done by the student in hoisting the flag to the top of the flagpole. The total resistive force is 1.5 N and is taken to be constant. You may assume the flag rises at constant speed. [2]

T = *mg* + *f* and WD = *Td*

WD = (*mg* + *f*) 5.5 = (5.9 +1.5)(5.5) = 41 J

(b) The student releases the flag at the same constant speed down the flagpole. He catches hold of the flag on his shoulders. Estimate the work done by the student, assuming the total resistive force is the same as in **(a)**. [2]

T + *f* = *mg* and WD = *Td* cosθ

WD = (*mg* – *f*)(5.5)(-1) = (5.9 - 1.5)(5.5)(-1) = -24 J

(c) The time taken for the flag to rise up and down the flagpole is 8.0 s. Calculate the average power. [1]

power = (41 - 24)/8

=2.1 W

(d) Discuss whether the instantaneous power is constant throughout the rising and falling of the flag. [2]

No. The power is greater on the way up and smaller on the way down

[YJC 2013]

7 A mass *M* is moving at a speed of 5.00 m s^{-1} along a horizontal frictionless guide which bends into a vertical circle of radius *r*, as illustrated in **Fig. 7.1**.

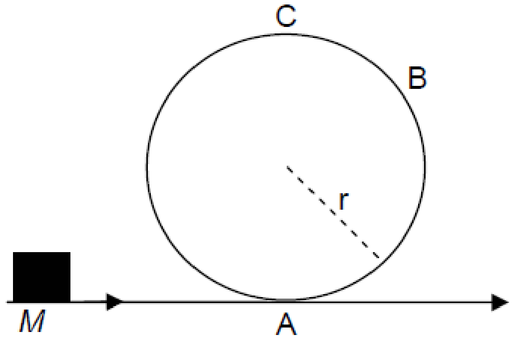


Fig. 7.1

Fig. 7.2 shows the the variation of the horizontal velocity of the mass with time along the section ABC of the curve.

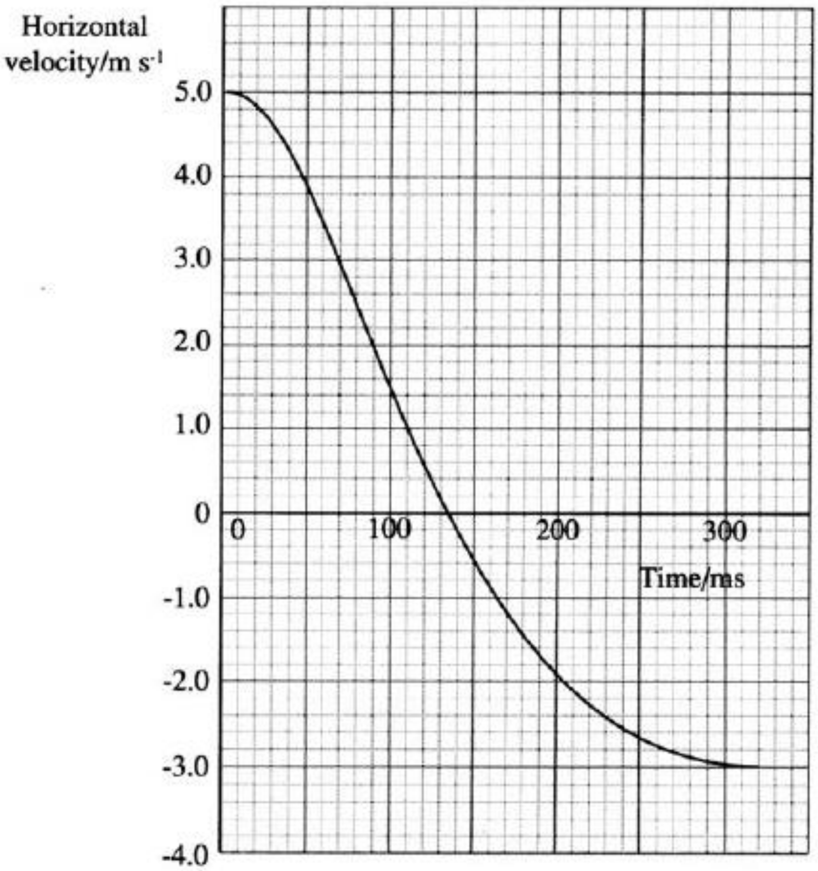


Fig. 7.2

(a) **(i)** Using the principle of conservation of energy and with the aid of **Fig. 7.2**, find an appropriate value for the height of the vertical circle. [2]

Check:
Using COE: Total Energy at C = Total Energy at A

$$\frac{1}{2}mv_c^{\ 2} + mgh = \frac{1}{2}mv_A^{\ 2}$$

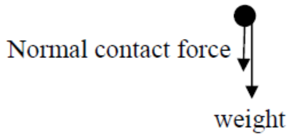
$$\frac{1}{2}v_c^{\ 2} + gh = \frac{1}{2}v_A^{\ 2}$$

$$h = \frac{v_A^{\ 2} - v_c^{\ 2}}{2g} = \frac{5^2 - 3^2}{2(9.81)} = 0.81549 \text{ m}$$

(ii) Hence, find the value for the radius *r* of the vertical circle. [1]

$$\therefore r = \frac{h}{2} = 0.408 \text{ m (3sf)}$$

(iii) Show that the minimum speed v_c for the mass *M* to remain in contact with the track when it is at point C is $v_c = \sqrt{gr}$, where *g* is the acceleration of free fall. [2]



At point C, both the normal contact force and the weight are acting downwards. To calculate the minimum speed at C, the contact force at C is zero.

$$N + mg = m\frac{v_C^{\,2}}{r}$$

$$mg = m\frac{v_C^{\,2}}{r}$$

$$v_C = \sqrt{gr}$$

(b) Another mass 8*M* is moving with the same kinetic energy along the same horizontal frictionless guide which bends into the same vertical circle of radius *r*, as illustrated in **Fig. 7.3**.

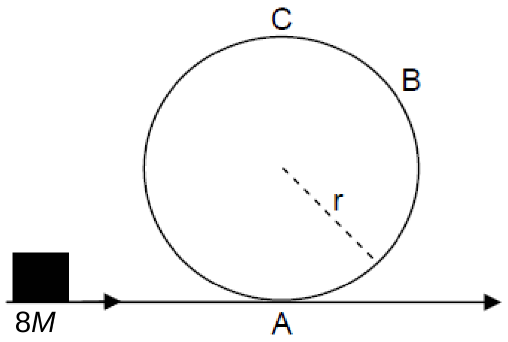


Fig. 7.3

With reference to your answer in **(a)(iii)** and the principle of conservation of energy, explain whether the mass 8*M* is able to pass through point C and travel back to point A. [3]

Speed of the 8*M* mass = $[\frac{1}{2}M(5)^2/(8M/2)]^{0.5}$ = 1.776 m s^{-1}
Since this speed is less than the min speed required in (a)(iii), the 8*M* mass will not reach point C.

[IJC 2013]

8 In a front load home laundry dryer, a cylindrical tub containing wet clothes is rotating steadily about the axis directed into the plane of the paper. As the clothes dry uniformly, they will tumble.

The rate of rotation of the smooth-walled tub is chosen so that a small piece of cloth will lose contact with the tub when the cloth is at an angle of θ = 63.0° above the horizontal.

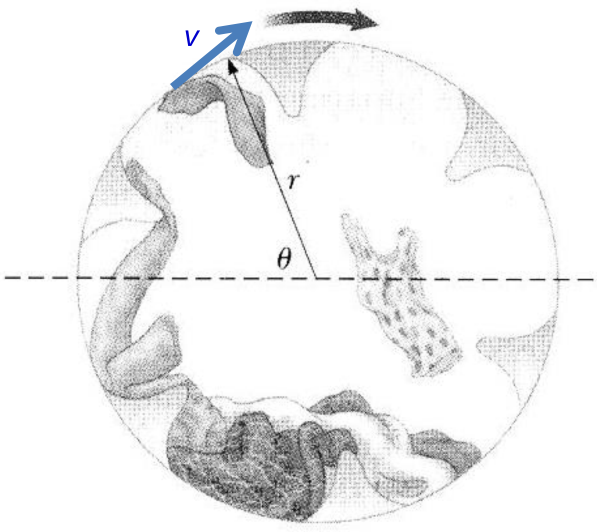
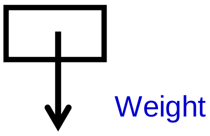


Fig. 8.1

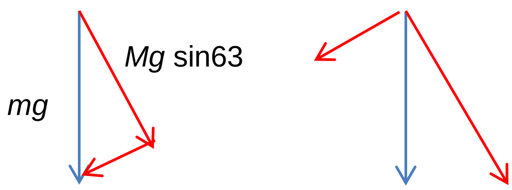
(a) On **Fig. 8.1** above, draw an arrow to indicate the direction of the velocity of the cloth before it loses contact with the tub. Label the arrow ***v***. [1]
(Arrow tangential to the circle. Must within θ. Need to read question clearly. **It loses contact at 63 degrees.** It asked for the arrow to indicate the velocity **before** it loses contact with the tub.)

(b) Draw the free body diagram of the cloth when θ = 63.0° above the horizontal. [1]



(Only weight is acting on the cloth since it loses contact at this point.
Must be labelled
No normal contact force, no centripetal force. Centripetal force is a label.)

(c) If the radius of the tub is r = 0.380 m, calculate the chosen rate of revolution. [3]
Since the weight is the only force acting on it at θ = 63°,



$mg \sin 63° = mr\omega^2$ [1]
$\omega = (g \sin 63°/0.380)^{0.5}$ = 4.796 rad s^{-1}
f = 0.763 Hz

[NJC 2013]

9 Domestic washing machines such as that in **Fig. 9.1** often incorporate washing, rinsing, spinning and drying. This question is about the spinning action.

Fig. 9.1

(a) The inner drum of a washing machine into which clothes are placed has large holes around it. Suggest how, when clothes are being spin-dried, water is removed from the clothes through the holes. [3]

Inner drum provides the centripetal force in the form of normal contact force as it is the only **predominant** force **(besides weight)** acting on the wet clothes. .

At the holes, there is nothing to provide the centripetal force.

The fabric will undergo circular motion together with the spinning drum, poking through a little at the holes. Water will exit from the inner drum tangentially through the holes.

OR Water did not stay on clothes due to absence of normal force at the holes, thus the water leaves through the holes

(b) One of the spin speeds of a model of washing machine was listed as 1000 rpm (revolutions per minute). Calculate the maximum contact force that could be exerted on a single wet blanket of mass 1.50 kg being spin-dried in the machine. The radius of the spinning drum is 22.5 cm. [3]

Consider at the bottom of the spin (result in largest *N*)

ω = 1000*(2π)/60 = 104.72 rad s^{-1}

$\Sigma F = mr\omega^2$, thus $N - mg$ = (1.5)(0.225)(104.72)^2 = 3701.1

***N* = 3701.1 + (1.5*9.81) = 3715.8 N = 3720 N (3 s.f)**

(c) If clothes are unevenly distributed in the machine, it vibrates slightly as it rotates. The outer drum within which the spinning drum rotates is attached to the rest of the framework of the washing machine by springs. Suggest briefly the purpose of these springs. [2]

They allow the spinning drum to vibrate a little within the rest of the framework rather than pass on the vibrations through the framework to the surroundings.

OR Springs cause damping/ Absorb/reduce impact force which reduces the amplitude of vibration/ prevent from vibrating too much/ smooth spinning

[YJC 2013]

10 **(a)** A moon is in a circular orbit of radius *r* about a planet. The planet and its moon may be considered to be point masses that are isolated in space. Show that, for the moon in circular orbit, the period *T* of the orbit is given by the expression

$$T^2 = \alpha r^3$$

where α is a constant. Explain your working. [3]

gravitational force provides the centripetal force

$GMm / r^2 = m\, r\, \omega^2$ (must be in terms of ω)

$T^2 = (4\pi^2 / GM)\, r^3$ = <u>and</u> $(4\pi^2 / GM)$ is a constant

(b) Phobos and Deimos are moons that are in circular orbits about the planet Mars. Data for Phobos and Deimos are shown in **Fig. 10.1**.

Moon	Radius of orbit / 10^6 m	Period of rotation about Mars / hours
Phobos	9.39	7.65
Deimos	19.9	T

Fig. 10.1

(i) Using the expression in **(a)** and the data from **Fig. 10.1**, determine the period of Deimos, T, in its orbit about Mars. [2]

$(9.39 \times 10^6)^3 / (7.65)^2 = (1.99 \times 10^7)^3 / T^2$

T = 23.6 hours

(ii) The period of rotation of Mars about its axis is 24.6 hours. Deimos is in an equatorial orbit, orbiting in the same direction as the spin of Mars about its axis. Use your answer in **(i)** to comment on the orbit of Deimos. [1]

Almost 'geostationary' or
satellite would take a long time to cross the sky or
the moon has the same angular speed as the rotation of the planet.

(c) A binary star consists of two stars that orbit about a fixed point C, as shown in **Fig. 10.2**.

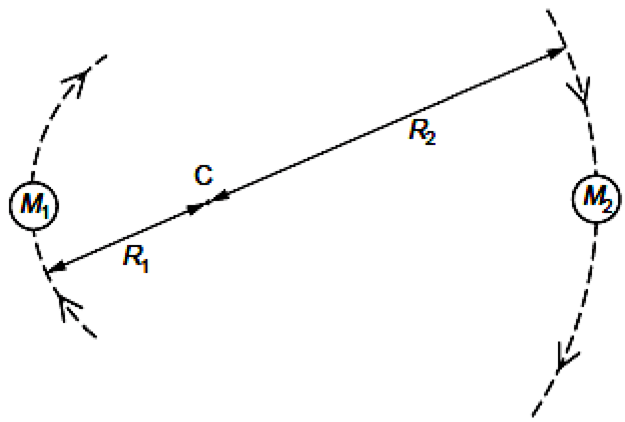


Fig. 10.2

The star of mass M_1 has a circular orbit of radius R_1 and the star of mass M_2 has a circular orbit of radius R_2. Both stars have the same angular speed ω, about C.

(i) State the formula, in terms of G, M_1, M_2, R_1, R_2 and ω for

1 the gravitational force between the two stars, [1]

$$\frac{GM_1M_2}{(R_1+R_2)^2}$$

2 the centripetal force on the star of mass M_1. [1]

$M_1R_1\omega^2$.

(ii) Suggest why M_1 is of a bigger mass compared to M_2. [2]
M_1 has a slower speed due to its smaller orbital radius and hence has a smaller acceleration.
By Newton's second law, for the same gravitational force between M_1 and M_2, M_1 must be of a bigger mass.
Alternatively, since both masses experience the same gravitational force (N3L) → same centripetal force → since R_1 is smaller than R_2, M_1 has to be larger than M_2.
[IJC 2013]

11 **(a)** State Newton's law of gravitation and hence show that the gravitational field strength g at a distance R from a point mass M is given by $g = \dfrac{GM}{R^2}$. [3]

Newton's law of gravitation states that the gravitational force between 2 point masses is directly proportional to the product of their masses and inversely proportional to the square of their separation, R. i.e. $F = \dfrac{GMm}{R^2}$

$$g_M = \frac{F}{m} = \frac{GM}{R^2}$$

where g is defined as force per unit mass and m is a test mass.

(b) A neutron star has mass 5.2×10^{30} kg and radius 1.7×10^4 m.

(i) Calculate the gravitational field strength at the surface of the star. [2]

$$g = \frac{GM}{R^2} = \frac{(6.67\times10^{-11})(5.2\times10^{30})}{(1.7\times10^4)^2} = 1.2\times10^{12}\ \text{N kg}^{-1}$$

(ii) State the assumption made in your calculation in part **(b)(i).** [1]
The neutron star is assumed to be a point mass.

(iii) Determine the centripetal acceleration of a particle moving in a circular path of radius 1.7×10^4 m and with a period of rotation of 0.21 s. [2]

$$a = R\omega^2 = R\left(\frac{2\pi}{T}\right)^2 = (1.7\times10^4)\left(\frac{2\pi}{0.21}\right)^2 = 1.52\times10^7\ \text{m s}^{-2}$$

(iv) The star rotates about its axis with a period of 0.21 s.
Use your answer to **(i)** and **(iii)** to suggest whether particles on the surface of the star leaves the surface owing to the high speed of rotation of the star. [2]
On the surface of the star, the gravitational field strength is much <u>greater</u> (approximately 10^5 times) than the centripetal acceleration of the particle. Hence the <u>gravitational force on the particle is sufficient to provide the centripetal force</u> to maintain the particle in circular orbit on the surface of the star.
This is why a particle <u>will not leave the surface</u> of the star.
[JJC 2013]

12 **(a)** Define *gravitational field strength*. [1]
Gravitational field strength <u>at a point</u> in a gravitational field is defined as the <u>gravitational force per unit mass</u> acting on a body placed at that point.

(b) Two stars of equal masses are in circular orbit about a common centre as shown in **Fig 12.1**.

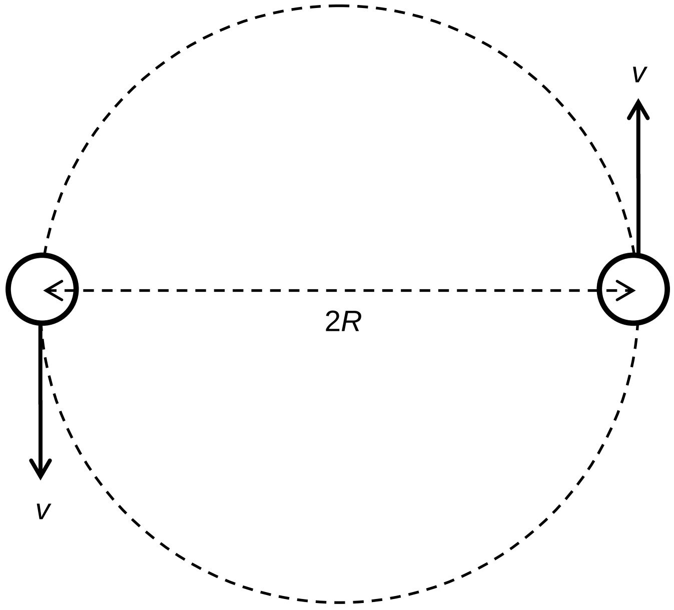


Fig. 12.1

The mass of each star is M and their separation is $2R$. The speed of each star is v.

(i) Determine the speed of each star, v, in terms of G, M and R. [2]

Gravitational force provides for centripetal force.

$$F = ma$$

$$\frac{GMM}{(2R)^2} = \frac{Mv^2}{R}$$

$$v = \sqrt{\frac{GM}{4R}}$$

(ii) Hence or otherwise, show that the period of revolution T of each star is given by the expression $T = \sqrt{\frac{16\pi^2 R^3}{GM}}$. [2]

$$v = R\omega = R\frac{2\pi}{T}$$

$$R\frac{2\pi}{T} = \sqrt{\frac{GM}{4R}}$$

rearranging,

$$T = \sqrt{\frac{16\pi^2 R^3}{GM}}$$

(c) The total energy E of the star system is given by the expression $E = -\frac{GM^2}{4R}$.

This particular star system is unstable and loses energy over time.

(i) Using conservation of energy, show that $E = -\dfrac{GM^2}{4R}$. [1]

By conservation of energy,

$E_{total} = E_k + E_p$

$$= \frac{1}{2}(2M)(\frac{GM}{4R}) + (-\frac{GMM}{2R})$$

$$= -\frac{GM^2}{4R}$$

(ii) Explain how the loss of energy implies that the orbital period of the stars will decrease. [2]

From the equation, a loss of energy means that $-\dfrac{GM^2}{4R}$ decreases, hence, $\dfrac{GM^2}{4R}$ increases and R decreases.

From the equation for T, when R decreases, T decreases.

(iii) For an unstable star system with $R = 6.5 \times 10^5$ km and $M = 3.0 \times 10^{30}$ kg, the rate of decrease of the period of star system is 7.0×10^{-5} s per year.

Estimate the time, in years, when the stars will crash into each other. [2]

$T = \sqrt{\dfrac{16\pi^2 R^3}{GM}}$ = 14721 s

time to crash = 14721 / (7 × 10^{-5}) = 2.1 × 10^8 years

[MJC 2013]

13 A certain planet has a radius of 1150 km. **Fig. 13.1** below shows the variation with the distance r from the centre of this planet, of the gravitational potential ϕ near it.

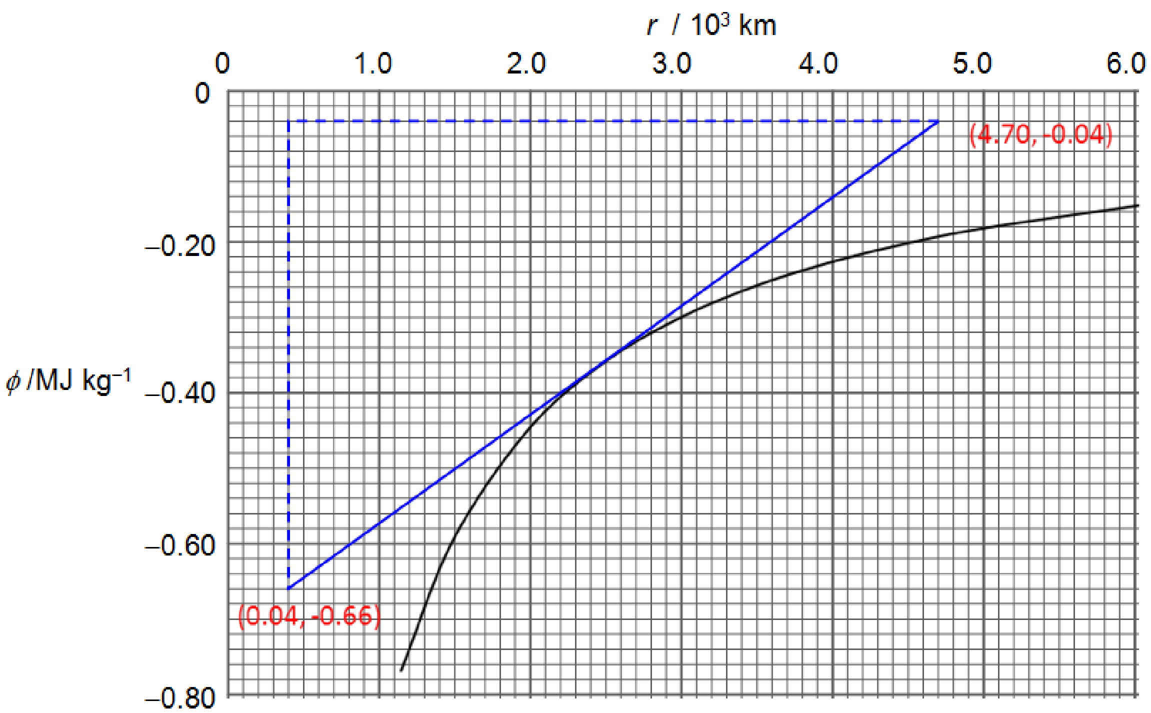


Fig. 13.1

(a) Explain why gravitational potential has a negative value. [2]
Gravitational potential at infinity is defined as being zero. Work done by an external force is negative as the direction of the force is opposite to the direction of displacement of the small mass from infinity to that point.

(b) **(i)** On **Fig. 13.1**, draw a tangent to the graph at r = 2500 km. [2]

(ii) The gradient of this tangent represents the magnitude of a particular vector quantity. State what this physical quantity is. [1]
Gravitational field strength

(iii) Calculate the gradient of this tangent and hence state the magnitude of the physical quantity that you have identified in **(b)(ii),** together with its S.I. unit. [3]

$$\text{Gradient } = \frac{[-0.04-(-0.66)]\times 10^{6}}{[4.70-0.04]\times 10^{3}\times 10^{3}} = 0.133\ \text{N kg}^{-1}$$

(c) Use the graph of **Fig. 13.1** to determine the escape velocity for an object at the surface of the planet. [2]

$(E_p + E_k)_{\text{initial}} = (E_p + E_k)_{\infty}$

$$m\phi + \frac{1}{2}mv^2 = 0 + 0$$

$$v^2 = -2\phi$$

At the surface of the planet, $\phi = -0.77 \times 10^6$ J kg^{-1}

v = 1240 m s^{-1}

(d) An object of mass 20.0 kg is projected from the surface of the planet and reaches a point 3500 km from the centre of the planet at the highest point in its trajectory. Use the graph of **Fig. 13.1** to determine the total energy (gravitational potential energy plus kinetic energy) of the object. [2]

Total energy (TE)

$= (E_p + E_k)_{3500\text{km}}$

$= -0.26 \times 10^6 \times 20 + 0 = -5.2 \times 10^6$ J

[RI 2013]

14 **(a)** Newton's law of gravitation applies to point masses.

(i) State Newton's law of gravitation. [1]
The (mutual) gravitational force of attraction between two point masses is proportional to the product of their masses & inversely proportional to the square of their separation.

(ii) Explain why, although the planets and the Sun are not point masses, the law also applies to planets orbiting the Sun. [1]
Separation much greater than size of Sun & of planet.

(b) The orbit of the Earth around the Sun can be assumed to be circular with a radius of 1.49×10^8 km. The period of the orbit is 365 days.

(i) Calculate the angular speed of the Earth in its orbit around the Sun. [1]

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{365 \times 24 \times 3600} = 1.99 \times 10^{-7}\ \text{rad s}^{-1}$$

(ii) Calculate the mass of the Sun. [2]

The gravitation force provides the centripetal force (for orbit)

$$\frac{GMm}{r^2} = mr\omega^2$$

$$M = \frac{r^3\omega^2}{G}$$

$$= \frac{(1.49 \times 10^{11})^3(1.99 \times 10^{-7})^2}{6.67 \times 10^{-11}}$$

$$= 1.96 \times 10^{30} \text{ kg}$$

(c) The actual orbit taken by the Earth is elliptical in shape with a distance of 1.47×10^{11} m to 1.52×10^{11} m from the Sun. Calculate the change in kinetic energy of the Earth from the largest to the shortest distance from the Sun. The mass of the Earth is taken to be 6.0×10^{24} kg. [2]

Change in KE = Change in GPE

$$= -\frac{GMm}{r_f} - \frac{GMm}{r_i}$$

$$= -(6.67 \times 10^{-11})(1.96 \times 10^{30})(6 \times 10^{24})\left(\frac{1}{1.52 \times 10^{11}} - \frac{1}{1.47 \times 10^{11}}\right)$$

$$= 1.76 \times 10^{32} \text{ J}$$

[SAJC 2012]

15 **(a)** Astronomers found that the Sun is located about 30000 light years from the centre of its galaxy, with an orbital speed of about 250 km s^{-1}.

(i) Given that 1 light year = distance traveled by light in a year, show that the radius of the Sun's orbit around the galactic centre is 2.8×10^{20} m. [1]

30000 light years = $c \times t = 3.0 \times 10^8 \times (30000 \times 365 \times 24 \times 3600)$

$= 2.8 \times 10^{20}$ m (shown)

(ii) Calculate the period of Sun's orbit around the galactic centre. [2]

$$\omega = \frac{v}{r} = \frac{250 \times 10^3}{2.8 \times 10^{20}} = 8.9 \times 10^{-16} \text{ rad s}^{-1}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{8.9 \times 10^{-16}} = 7.0 \times 10^{15} \text{ s}$$

(b) **(i)** Given that the mass of our Sun is 2.0×10^{30} kg, show that the galaxy is made up of more than 10^{11} stars if the average mass of a star is the same as the mass of our Sun. [3]

Since our sun is going round in orbit about the centre of the galaxy then the centripetal force is provided by the gravitational force

$$mr\omega^2 = \frac{GMm}{r^2}$$

$$\Rightarrow T^2 = \frac{4\pi^2}{GM} r^3$$

$$\Rightarrow M = \frac{4\pi^2}{G}\frac{r^3}{T^2} = \frac{4\pi^2 \times (2.8 \times 10^{20})^3}{6.67 \times 10^{-11} \times (7.0 \times 10^{15})^2}$$

$$= 2.7 \times 10^{41} \text{ kg}$$

No. of suns = $2.7 \times 10^{41} / 2.0 \times 10^{30} = 13 \times 10^{11}$

(ii) Calculate the gravitational potential due to the Milky Way galaxy on Earth. [2]

$$\varphi = -\frac{GM}{r}$$

$$= -\frac{6.67\times10^{-11}\times2.7\times10^{41}}{2.8\times10^{20}} = -6.4\times10^{10}\ \text{J kg}^{-1}$$

(iii) NASA wants to design a space probe to carry information about humans to other galaxies to contact other forms of intelligent life in those galaxies. Calculate the initial velocity that the space probe needs to acquire immediately leaving Earth such that it can drift out of Milky Way without any further propellant. Ignore contributions from Earth and Sun. [2]

To escape, the spacecraft need to be given an amount of KE = PE_∞ - PE_i

$$\tfrac{1}{2}mv^2 = 0 - \left(-\frac{GMm}{r}\right)$$

$$\tfrac{1}{2}v^2 = \frac{GM}{r}$$

$$\tfrac{1}{2}v^2 = 6.4\times10^{10}$$

$$v = 3.6\times10^{5}\ \text{m s}^{-1}$$

[VJC 2012]

16 **(a)** A space shuttle is to orbit around Earth at an altitude of 350 km (or 350 km above the Earth's surface). When its final construction is completed, it has a weight (measured at the Earth's surface) of 4.22 × 10^6 N.

Calculate its weight when it is in orbit.
(Given radius of Earth = 6.37 × 10^6 m, mass of Earth = 5.98 × 10^{24} kg) [3]

$$m = \frac{F_g}{g} = \frac{4.22\times10^6}{9.81} = 4.30\times10^5\ \text{kg}$$

$$g = \frac{GM_E}{(R_E+h)^2} = \frac{(6.67\times10^{-11})(5.98\times10^{24})}{(6.37\times10^6+0.350\times10^6)^2} = 8.83\ \text{m s}^{-2}$$

$$mg = (4.30\times10^5)(8.83) = 3.80\times10^6\ \text{N}$$

(b) **(i)** When the space shuttle is in orbit around the Earth, show that its total energy E is given by $E = -\dfrac{GMm}{2r}$. [2]

$$\frac{GMm}{r^2} = ma = \frac{mv^2}{r}$$

$$\frac{1}{2}mv^2 = \frac{GMm}{2r}$$

$$E = \frac{GMm}{2r} - \frac{GMm}{r}$$

$$E = -\frac{GMm}{2r}$$

(ii) A satellite of mass 470 kg leaves the space shuttle using power from its engine and boosts itself into a geostationary orbit, a distance of 4.23 × 10^7 m from the centre of the Earth.

Calculate the amount of energy the engine has to provide. [3]

$\Delta E = E_f - E_i = -GMm/2r_f - (-GMm/2r_i)$

$=(GMm)(1/2r_i - 1/2r_f)$

$=(6.67*10^{-11})(5.98*10^{24})(470)(1/2(6.72*10^6)-1/2(4.23*10^7))$

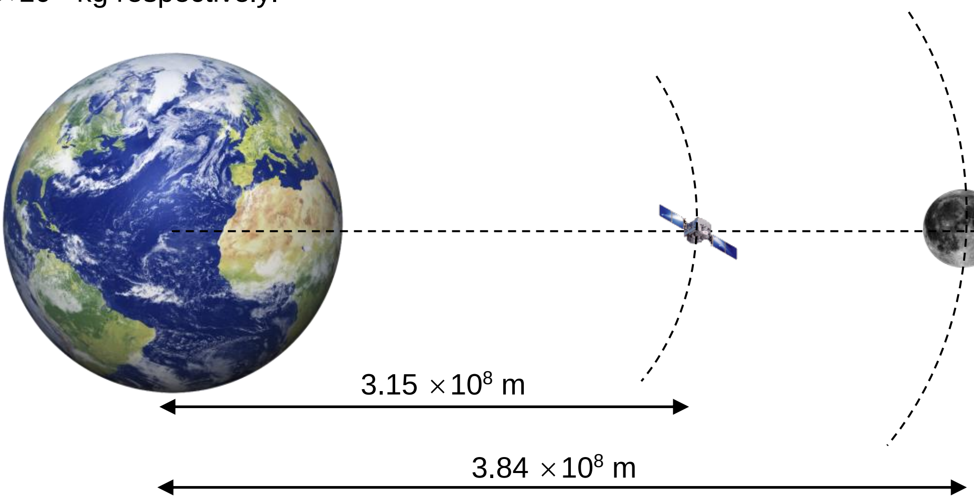
$=1.17*10^{10}$ J

(iii) Suggest one benefit of having a satellite in a **geostationary** orbit instead of a **polar** orbit. [1]

1. It is easier to transmit and receive from a geostationary satellite vs a polar satellite since the geostationary satellite is always the same spot above the equator. OR
2. Less energy required to maintain geostationary orbit vs polar orbit.

[TPJC 2012]

17 Singapore came up with a mission to put a satellite between the Earth and the moon in order to observe the moon at all times. The distance between the Earth and the moon is 3.84×10^8 m. The mass of the Earth and the moon can be taken to be 5.98×10^{24} kg and 7.35×10^{22} kg respectively.



The satellite is proposed to be at a distance of 3.15×10^8 m from the Earth.

(a) Calculate the net acceleration of the satellite. [3]

$$a = g_E - g_M$$

$$a = \frac{GM_E}{r_E^{\ 2}} - \frac{GM_M}{r_M^{\ 2}}$$

$$a = \frac{(6.67\times10^{-11})(5.98\times10^{24})}{\left(3.15\times10^8\right)^2} - \frac{(6.67\times10^{-11})(7.35\times10^{22})}{\left[\left(3.84\times10^8\right)-\left(3.15\times10^8\right)\right]^2}$$

$$a = 2.99\times10^{-3}\ \text{m s}^{-2}$$

(b) Assuming that the satellite undergoes a circular motion around the Earth, calculate the period of its orbit using your answer in **(a)**. [2]

$$a = r\omega^2$$

$$a = r\left(\frac{2\pi}{T}\right)^2$$

$$2.99\times10^{-3} = 3.15\times10^{8}\left(\frac{2\pi}{T}\right)^2$$

$$T = 2.04\times10^{6}\ \text{s}$$

(c) The moon's period is 27.3 days. Explain whether the proposed mission is feasible. [1]
The periods of the moon and satellite around the Earth are different. The satellite will not be able to observe the moon at all times, hence the mission is not feasible.

[SRJC 2012]

18 **(a)** State what is meant by angular velocity and write down its SI unit. [2]
Angular velocity is the rate of angular displacement. Unit: <u>rad s^{-1}</u>

(b) The acceleration of free fall on the surface of the Earth at the pole differs slightly from that at the equator. The Earth is assumed to be a uniform sphere. A man of mass 90 kg stands at the equator of the Earth. The radius of the Earth is 6.37×10^6 m and its period is 24 hours.
Using Newton's second law of motion, calculate the apparent weight of the man. [2]
$F = m a$
True Weight – Normal Reaction = Centripetal force needed

$$mg - N = mr\omega^2$$
$$N = mg - mr\omega^2$$
$$= 90\,(9.81) - 90\,(6.37 \times 10^6)\,(2\pi/24 \times 60 \times 60)^2$$
$$N = 883 - 3.03 = \underline{880\ \text{N}}$$

(c) **Fig. 18.1** shows how the total potential ϕ between the Moon and the Earth varies along the line of centres.

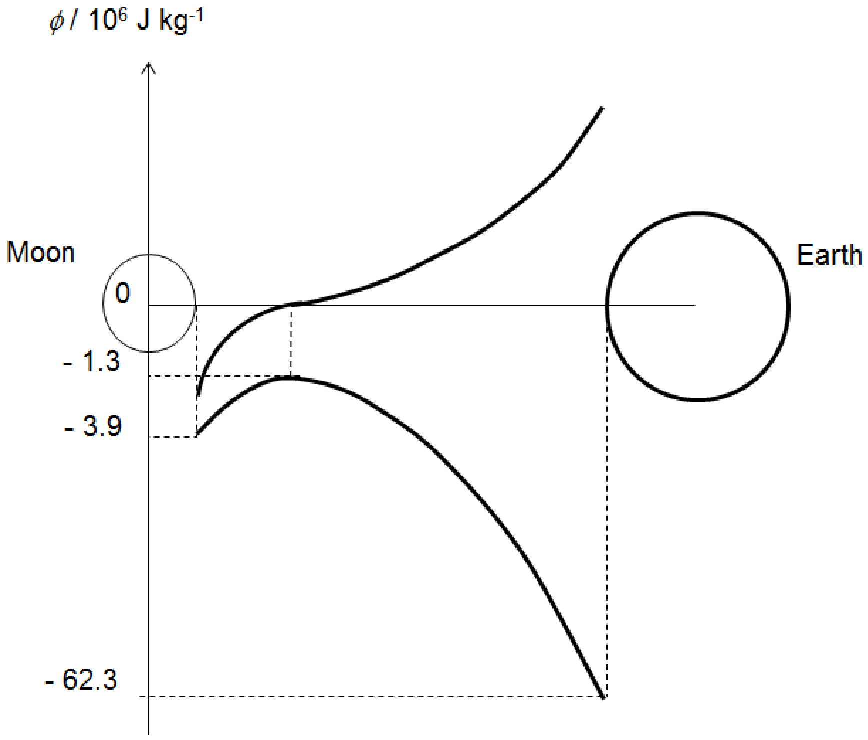


Fig. 18.1

(i) Explain qualitatively why the graph has a maximum. [2]
Maximum point corresponds to the point where the gravitational field strength (or force) due to the Moon is <u>equal and opposite</u> to that due to the Earth. Hence, g = 0. [No mark for merely stating that g = 0.]
Since gravitational field strength is proportional to the potential gradient, the gravitational potential has a maximum.

(ii) A cannon ball of mass 120 kg on the Moon is to be projected directly towards the Earth. Determine the minimum speed that the cannon ball must be projected in order to reach the Earth. Assume no energy loss due to resistive forces. [2]
Initial KE – Final KE = Change in Gravitational Potential Energy

$$\tfrac{1}{2} m V_{min}^2 - 0 = m \Delta\phi$$
$$\tfrac{1}{2} V_{min}^2 = (-1.3 - (-3.9)) \times 10^6$$
$$V_{min} = \underline{2280 \text{ m s}^{-1}}$$

(iii) On **Fig. 18.1**, sketch a graph to show the variation with distance of the gravitational force F, experienced by the cannon ball as it travels from the Moon to the Earth. A force towards the Earth is taken to be positive. [2]
(shown on Fig. 19.1)

[SAJC 2012]

19 (a) Define *gravitational field strength*. [1]
The gravitational field strength, g, at a point in a gravitational field is defined as the gravitational force per unit mass acting on a small mass placed at that point.

(b) State one way in which gravitational and electric fields are similar and one way in which they differ. [1]
The forces in both fields follow the inverse square law, that is the field strength is inversely proportional to the square of the distance away from the centre of the source mass/charge.
OR Both the forces involved are examples of non-contact forces.
Gravitational force is attractive in nature whereas electric force can be attractive or repulsive.

(c) The gravitational field strength at the surface of the Earth is 10 N kg^{-1}. The radius of Earth is 6.0 × 10^6 m.

(i) Sketch the graph of the variation of gravitational field strength, with height above the surface of the Earth on **Fig. 19.1**. [2]

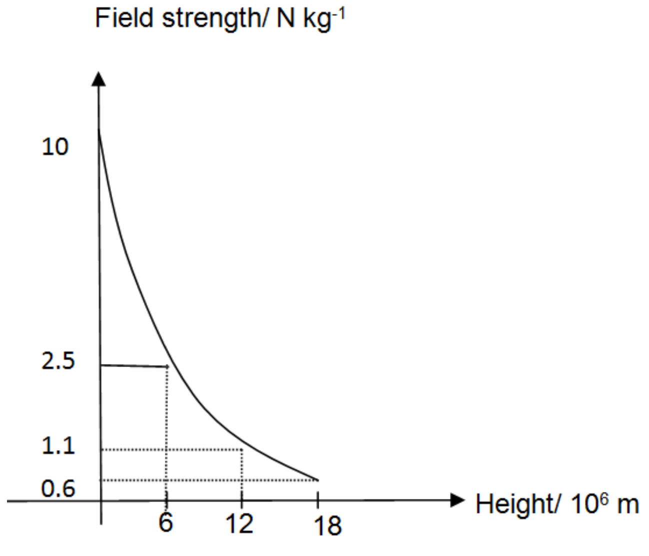


Fig. 19.1

(ii) From your sketch in **Fig. 19.1**, explain why the change in the potential per km when raising a spacecraft from the surface of the Earth decreases with height. [2]
The change in potential per km is numerically equal to the g-field strength of the Earth. From the graph, the field strength decreases with height and thus the change in potential per km decreases with height.
OR The area under the *g* vs *H* graph represent the change in the gravitational potential. Thus as *H* increase and *g* decreases, the area under the graph measured from infinity to that point decreases indicating a decreasing change in gravitational potential.

(iii) Calculate the *escape speed* of an object at the Earth's surface. [2]
Using conservation of energy:
Total energy on earth = Total energy at infinity
$\frac{1}{2} mv^2 + (-GMm/R) = 0$
$GMm/R = 1/2mv^2$
But $GM/R^2 = 10$,
Hence $v = 1.1 \times 10^6$ m s^{-1}

[NJC 2012]

20 **(a)** Define gravitational field strength. [1]
The gravitational field strength is defined as the gravitational **force per unit mass** acting on a small mass **placed at that point**

(b) The Earth may be assumed to be an isolated sphere of radius 6.37×10^3 km with its mass of 5.98×10^{24} kg concentrated at its centre.

(i) Determine the gravitational field strength at the North Pole. [2]

$$g = \frac{GM}{r^2} = \frac{(6.67\times10^{-11})(5.98\times10^{24})}{(6.37\times10^6)^2} = 9.83 \text{ N kg}^{-1}$$

(ii) The Earth spins on its axis with a period of 24.0 hours.

Compare and comment on the small differences between the calculated value in **(b)(i)** and the value of 9.79 m s^{-2} (which is the value of the acceleration of free fall obtained by making accurate measurements in Singapore, near the equator) [2]
The acceleration measured (9.79 m s^{-2}) is **not equal** to the calculated acceleration of free fall (9.83 m s^{-2}) because **part of the gravitational field strength is used to provide the centripetal acceleration to rotate around the axis of the Earth at the equator**

(c) **(i)** An object is projected vertically from the surface of the Earth so that it reaches a height of 10.0 km above the Earth's surface.
Calculate, for this object, the minimum speed of projection from the Earth's surface, assuming air resistance is negligible. [2]

$$\Delta\phi = \phi_f - \phi_i$$
$$= GM\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$
$$= (6.67\times10^{-11})(5.98\times10^{24})\left(\frac{1}{6.37\times10^6} - \frac{1}{6.38\times10^6}\right)$$
$$= 9.8145\times10^4 \approx 9.81\times10^4 \text{ J kg}^{-1}$$

By conservation of energy,

$$m\Delta\phi = \frac{1}{2}mv^2 - 0$$

$$v = \sqrt{2\Delta\phi} = \sqrt{2(9.8145\times10^4)} = 443 \text{ m s}^{-1}$$

(ii) Suggest why the equation $v^2 = u^2 + 2as$ is not appropriate for the calculation in **(c)(i)**. [1]
The gravitational field strength is **not constant** at from the surface to an altitude of 10 km.

[JJC 2012]

21 A satellite is set to orbit around the Earth. The mass of the Earth is M_E, radius of orbit of satellite is r and period of its orbit around Earth is T.

(a) Show that $T = 2\pi\sqrt{\frac{r^3}{GM_E}}$. [2]

Gravitational force provides centripetal force,

$$\frac{GM_E m}{r} = mr\left(\frac{2\pi}{T}\right)^2$$

$$T = 2\pi\sqrt{\frac{r^3}{GM_E}}$$

(b) Determine the radius of the orbit of a satellite around Earth when it has a period of 2 days. It is known that $M_E = 5.97 \times 10^{24}$ kg. [2]

$$T = 2\pi\sqrt{\frac{r^3}{GM_E}}$$

$$2(24)(3600) = 2\pi\sqrt{\frac{r^3}{G(5.97\times10^{24})}}$$

$$r = 6.70\times10^7 \text{ m}$$

(c) In the calculation in part **(b)**, the effect of the gravitational force of the moon has not been taken into account. Discuss any effect it has on the period of the orbit of the satellite if the gravitational force of the moon has been taken into consideration. The distance from the centre of Earth to the centre of moon is 384 000 km. [2]
As the gravitational force of the moon on the satellite is negligible compared to that of the Earth due to significant difference between the distance from Moon to satellite and Earth to satellite, the effect on the period is also negligible.

[SRJC 2013]

22 **(a)** Explain what is meant by *gravitational potential energy*. [1]
Gravitational PE of a mass at a point is the work done on a point mass in bringing it from infinity to that point.

(b) The minimum velocity required by a body to escape completely from the surface of the Earth is known as the escape velocity, v_e.

(i) Using energy considerations, show that $v_e = \sqrt{2gR}$ where R is the radius of the Earth. [3]

By energy considerations,
final energy of the body $E_f = 0$
initial energy of body E_i = KE + GPE

$$0 = \frac{1}{2}mv_e^2 + (-G\frac{mM_e}{R})$$

$$v_e = \sqrt{\frac{2GM_e}{R}}$$

Since $g = \frac{GM_e}{R^2}$, $v_e = \sqrt{2gR}$

(ii) Given that the radius of the Earth is 6400 km, determine the escape velocity for a rocket launched vertically upwards from the surface of the earth. [2]

$v_e = \sqrt{2gR} = \sqrt{2(9.81)(6400000)} = 1.12 \times 10^4$ m s^{-1}

(iii) If the rocket is launched off from the surface of the Earth with initial speed equal to half the escape speed, describe the possible motion of the rocket. [2]

With half the escape speed, the rocket will not reach infinity but can only rise to certain finite height (R/3) above the earth's surface before coming to a stop. It will then return to Earth

(iv) Suppose that an asteroid with zero speed at an infinitely far distance from the Earth now falls directly towards the Earth's surface. Suggest a value for the speed with which it will strike the Earth's surface. [1]

Same speed as the escape velocity = 1.12×10^4 m s^{-1}

(v) **Fig. 22.1** shows the variation of potential energy of the rocket, U with distance r, from the surface of the Earth. Sketch the variation of the kinetic energy, K of the rocket with distance r, from the surface of the Earth, given that the rocket has just sufficient kinetic energy to reach infinity. [1]

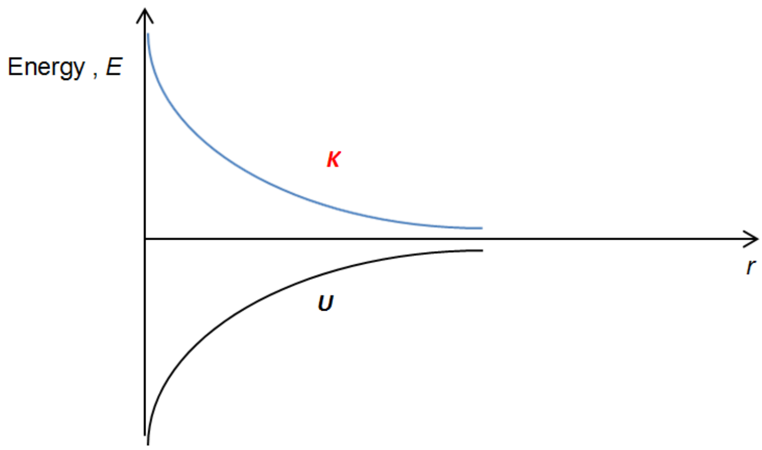


Fig. 22.1

(c) **(i)** One theory of atmospheric evolution suggests that the Earth originally had an atmosphere rich in hydrogen but that, as a result of a major thermal event in which the temperature rose to about 6000 K, the hydrogen concentration then fell to a very low level.

Making reference to the mean molecular speeds and escape velocity of Earth from **(b)(ii)**, explain how this increase of temperature could have led to a substantial loss of hydrogen. [1]
[The mean molecular speed of hydrogen atoms at 6000 K is about 1.2×10^4 m s^{-1}]
(The average kinetic energy of the hydrogen gas molecules is directly proportional to the thermodynamic temperature.) Since the mean molecular speed of hydrogen is greater than the escape speed, the hydrogen gas must have escaped from Earth.

(ii) Suggest why the moon has no atmosphere. [1]
The escape velocity of the moon is very low.

[YJC 2013]

23 **(a)** Fig. 23.1 shows the equipotential lines for Earth, where point **A** is at a potential of -4.0×10^{7} J kg^{-1} and points **B** and **C** are at a potential of -5.0×10^{7} J kg^{-1}.

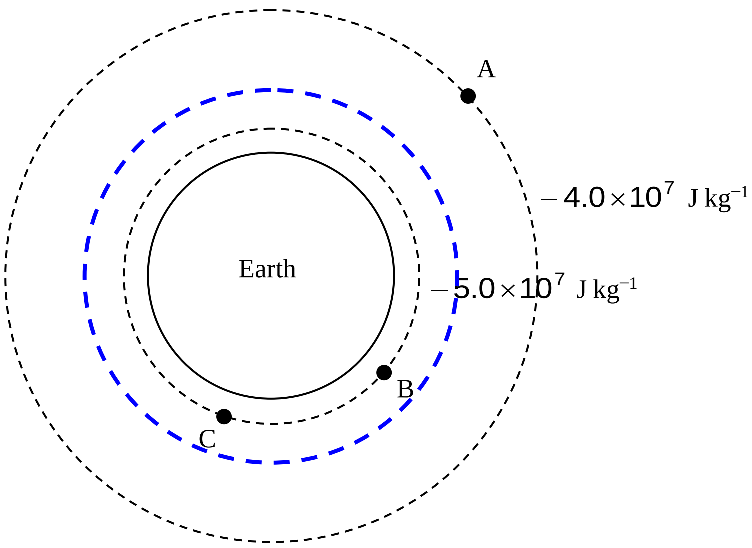


Fig. 23.1

(i) On Fig. 23.1, draw the equipotential line for the gravitational potential of -4.5×10^{7} J kg^{-1}. [2]
[1] for line drawn nearer to the -5.0×10^{7} J kg^{-1} equipotential line

(ii) Calculate the work done by the gravitational field in bringing a body of mass 2500 kg from A to B.
Work done by external force

$$= m(\phi_B - \phi_A)$$
$$= 2500 \times (-5.0 + 4.0) \times 10^{7}$$
$$= -2.5 \times 10^{10} \text{ J}$$

Work done by gravitational field

$$= 2.5 \times 10^{10} \text{ J}$$

work done = ... J [2]

(b) **(i)** Information related to the Earth and the Sun is given below.

$$\frac{\text{mass of Sun}}{\text{mass of Earth}} = 3.3 \times 10^5$$

$$\frac{\text{radius of Sun}}{\text{radius of Earth}} = 110$$

Given that the escape speed from the Earth is 1.1×10^4 m s^{-1}, show that the escape speed from the Sun is 6.0 × 10^5 m s^{-1}. [2]

Since $v_{\text{escape from Earth}} \geq \sqrt{\dfrac{2GM_E}{R_E}}$, we have

$$\frac{v_{\text{escape from Earth}}}{v_{\text{escape from Sun}}} = \sqrt{\frac{M_E R_S}{M_S R_E}}$$

$$\frac{1.1 \times 10^4}{v_{\text{escape from Sun}}} = \sqrt{\frac{110}{3.3 \times 10^5}}$$

$$v_{\text{escape from Sun}} \approx 6.0 \times 10^5 \text{ m s}^{-1}$$

(ii) The surface temperature of the Sun is about 6000 K and hydrogen is the most abundant element in the Sun's atmosphere. Using suitable calculations, explain why the hydrogen atom can be found in the Sun's atmosphere, assuming that the hydrogen atom behaves like an ideal gas and has a mass of 1.0 u.

Kinetic energy of an atom of mass m is given as

$$\frac{1}{2} m v_{r.m.s.}{}^2 = \frac{3}{2} kT$$

$$1 \times 1.66 \times 10^{-27} \times v_{r.m.s.}{}^2 = 3 \times 1.38 \times 10^{-23} \times 6000$$

$$v_{r.m.s.} \approx 1.2 \times 10^4 \text{ m s}^{-1}$$

Since the r.m.s. speed of the hydrogen atom is less than the escape speed, hydrogen atoms can be found in the Sun's atmosphere.

....

....

.... [3]

[MJC 2015]

24 **(a)** Fig. 24.1 shows a graph of the variation of the gravitational field strength *g* of the Earth with distance *r* from its centre.

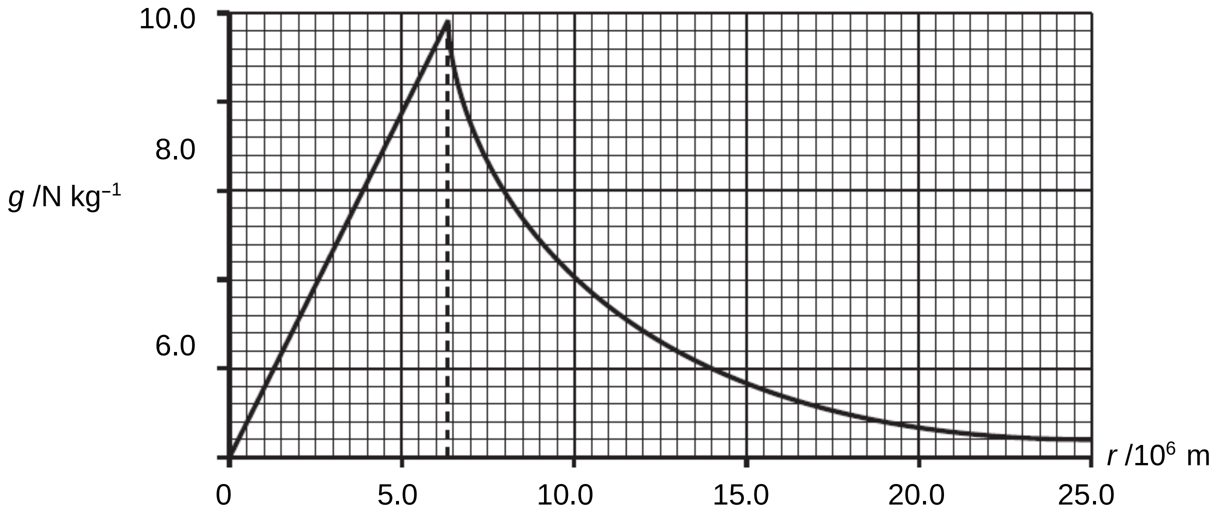


Fig. 24.1

(i) State the relationship between gravitational field strength *g* and gravitational potential *Φ*.

$g = -\frac{d\phi}{dr}$

.... [1]

(ii) Define *gravitational potential energy*.

The gravitational potential energy of a mass at a point within a gravitational field is the work done by an external force to move the mass from infinity to that point

....

.... [1]

(iii) Hence, estimate the work done in bringing a mass of 5.0 kg from $r = 25 \times 10^6$ m to $r = 10 \times 10^6$ m.

By estimation, area under graph = 2.2×10^7 J kg^{-1}

(Accepted range: 2.1×10^7 – 2.5×10^7)

Work done by external force = − m x area under graph

$= -5 \times 2.2 \times 10^7$

$= -1.1 \times 10^8$ J

work done = ... J [3]

(b) A star system consists of 3 stars X, Y and Z, each of mass 4.0 x 10^{22} kg, moving with the same speed and in the same direction in a circular orbit of radius 9.8 x 10^{13} m about a central star P of mass 7.0 x 10^{24} kg. Stars X, Y and Z are always positioned at a distance 1.7 x 10^{14} m from each other as shown in Fig. 24.2.

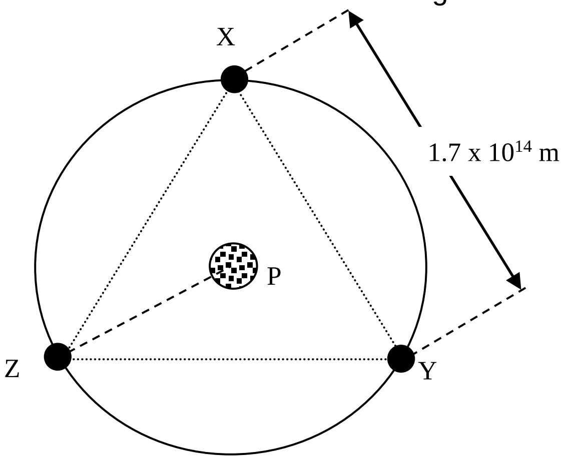


Fig. 24.2

(i) By considering all the forces acting on X, show that the acceleration of X is 4.9 x 10^{-14} m s^{-2}. [3]

The sum of gravitational forces due to Z, Y and P provide the centripetal force for circular motion.

F_{net} = ma

$F_Z + F_Y + F_P$ = ma

$2F_Z \cos 30° + F_P$ = ma

$$2\frac{Gm^2}{\left(1.7\times10^{14}\right)^2}\cos 30° + \frac{GmM}{\left(9.8\times10^{13}\right)^2} = ma$$

$$a = 2\frac{Gm}{\left(1.7\times10^{14}\right)^2}\cos 30° + \frac{GM}{\left(9.8\times10^{13}\right)^2}$$

$$= 2\frac{6.67\times10^{-11}\times4.0\times10^{22}}{\left(1.7\times10^{14}\right)^2}\cos 30° + \frac{6.67\times10^{-11}\times7.0\times10^{24}}{\left(9.8\times10^{13}\right)^2} \qquad [M1]$$

$$= 4.9\times10^{-14}\,ms^{-2}$$

(ii) Comment on the work done in bringing a unit mass from X to Y.
There is no change in potential from X to Y, hence zero work done.

....

....

.... [1]

[SRJC P3 2015]

Long structured questions

25 **(a)** State the Principle of Conservation of Energy. [1]

The Principle of Conservation of Energy states that energy is a quantity that can be converted from one form to another but cannot be created or destroyed. The total energy of an isolated system is constant.

(b) Two students, Maria and Peter, want to imitate the acrobatic performances often seen in circus events. However, due to a lack of equipment, they decide to make do with a simple pulley system which they found at the school gymnasium. The rope (assumed light and inextensible) is just taut when the two students are at their initial positions as shown in **Fig. 25.1**. The radius of the pulley is negligible.

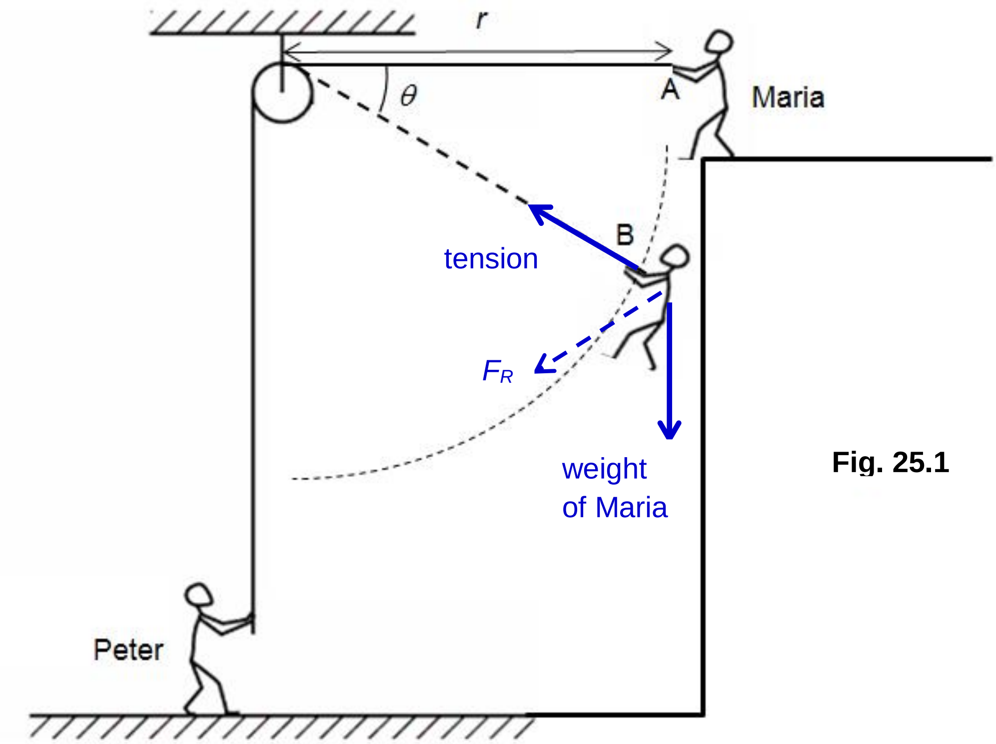


Fig. 25.1

(i) Maria takes a step forward, falls from rest and swings in a circular manner towards Peter, passing point B along her way. On **Fig. 25.1** draw the forces acting on Maria at point B. Label the forces clearly. [2]

(ii) Hence, draw a dotted arrow indicating the direction of the resultant force acting on Maria at point B on **Fig. 25.1**. Label it F_R. [1]

(F_R should not be tangential to circular motion.)

(c) **(i)** Maria's speed v changes with θ, the angle the rope makes with the horizontal. The radius of Maria's circular path is r. Derive an expression for v in terms of g, r and θ. [2]

Using Conservation of Energy:

Decrease in GPE = Increase in KE

$mg(r\sin\theta) = \frac{1}{2}mv^2$

$v = \sqrt{2gr\sin\theta}$

(ii) Hence, show that the tension T in the rope can be expressed as $T = 3mg\sin\theta$, where m is Maria's mass. [2]

Consider the forces on Maria:

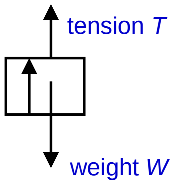
$$T - mg\sin\theta = mv^2 / r$$

$$T = m\left(\frac{2gr\sin\theta}{r} + g\sin\theta\right)$$

$$T = 3mg\sin\theta$$

(d) **(i)** Explain, with the aid of a free body diagram of Peter, why the normal contact force exerted by the ground on Peter decreases as Maria swings in a circular manner towards Peter. [3]

As Maria swings downwards, the tension in the rope, given as $T = 3mg\sin\theta$, increases as θ increases.



As a result, tension in the rope gets larger.
From ($R + T = W$), we can see that the normal contact force will get smaller.

(ii) Given that Maria's mass is 40.0 kg and Peter's mass is 75.0 kg, calculate θ_{max}, the value of θ at which Peter would be lifted off the ground. [3]

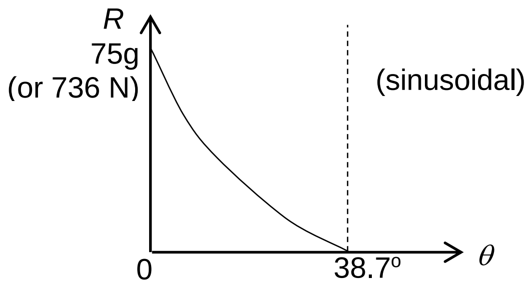
When Peter is about to lift off, $R = 0$

$$m_P g = T = 3m_M g\sin\theta$$

$$75.0 = 3(40.0)\sin\theta$$

$$\theta = 38.7^\circ$$

(iii) On the axes below, sketch a labelled graph to show how the normal contact force exerted by the ground on Peter, *R*, varies with θ, from the time Maria falls to the time at which Peter is lifted off the ground. Mark the axes with appropriate values. [3]



(iv) Peter comes up with the idea that as Maria starts her swing, he shall pull his end of the rope downwards so that the radius of the path which Maria takes becomes smaller as she swings downwards. He hypothesizes that by doing so, he will be able to maintain contact with the ground throughout her swing.
State whether his hypothesis is sound and briefly give a reason for your answer. [3]

When Peter exerts a downward force on the rope, the tension in the rope T gets larger compared to the previous case where Maria is travelling in a circular path.
This will lead to the normal contact force R getting smaller at each corresponding value of θ (compared to previous scenario).
As such, Peter will actually be lifted off at a smaller angle. Hypothesis is not true.

[HCI 2013]

26 **(a)** The Republic of Singapore Air Force operates the F16 Fighting Falcon Fighter Jet as part of its air combat strategy, as shown in **Fig. 26.1** below.

Fig. 26.1

The aircraft has a mass of 12000 kg and can execute circular turns at a speed of 450 km h^{-1} and fly in a horizontal circle of radius 175 m.

(i) **1** Calculate the centripetal force required for the 12000 kg aircraft to make a horizontal circular loop of 175 m with a speed of 450 km h^{-1}. [2]

$$F_c = \frac{mv^2}{r} = \frac{12000\left(\frac{450}{3.6}\right)^2}{175}$$

$$F_c = 1.07 \times 10^6 \text{ N}$$

2 Hence show that the centripetal acceleration acting on the aircraft is approximately equivalent to 9G (9 times that of acceleration due to free fall on Earth). [1]

$$a = \frac{F_c}{m} = \frac{1.07 \times 10^6}{12000}$$

$$a = 89.3 = 9.1g$$

(ii) Explain clearly why centripetal force in circular motion is not an actual force, and is instead a net force. [2]

In order for circular motion to occur, <u>external forces must act on the body.</u>
This results in a <u>net force that causes centripetal acceleration</u>, and the net force is termed as centripetal force.

(b) In combat situations, it is sometimes important for the F16 to make a vertical circular loop. This tactic is useful for the F16 to get behind an enemy jet, allowing it to fire at the enemy. A diagram of such a circular loop is shown in **Fig. 26.2** below.

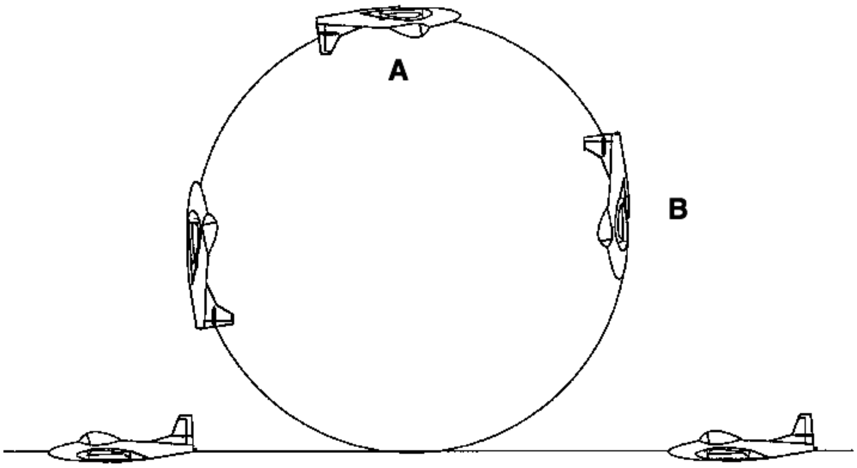


Fig. 26.2

(i) Taking into account downward acceleration due to freefall, show that the apparent weight experienced by the pilot at **A** is smaller than that at **B**. [2]

Point A: $N_A + W = m\dfrac{v^2}{r}$

$$N_A = m\frac{v^2}{r} - W$$

Point B: $N_B - W = m\dfrac{v^2}{r}$

$$N_B = m\frac{v^2}{r} + W$$

Hence $N_B > N_A$

(ii) Explain why it is structurally dangerous for the aircraft to continue making a circular turn of radius 175 m with a speed of 450 km h^{-1} at the bottom of the circular loop. [2]

In order for the jet to make the loop at the same speed with the same radius at the bottom, normal force acting on the jet is equal to the sum of centripetal force required and weight of plane.

$F_C = N - W$

$N = F_C + W$

Wings might be damaged by the extremely large force.

(iii) During ground training, pilots are trained to prevent themselves from blacking out during high speed turns. Blacking out occurs when they make a high speed turn as seen in position **B** in **Fig. 26.2**, and the blood in their bodies tend to gather at the lower half of the bodies. This reduces the blood supply to their brains and causes them to lose consciousness.

Making use of the concepts of inertia, explain why blood and other fluids in our body will tend to gather at the lower half of the body during a sharp high speed turn. [2]

Fluids in body are free to flow and not subjected to normal force, until they reach the lower limbs, which will provide a normal force
Hence by centrifugal effect/ inertia, they will remain at bottom of body.

(c) **(i)** Define *gravitational potential* at a point. [1]
Gravitational potential energy lost per unit mass for an object in moving from infinity to the point.

(ii) A satellite of mass m is orbiting Earth at a radius of R, measured from the centre of Earth and R >> radius of Earth.

1 Given that the orbit is a circular one, show that the tangential speed v of the satellite is given by

$$v = \sqrt{\frac{GM}{R}}$$

where M is the mass of Earth, and G is the Universal Gravitational Constant. [2]

$$\frac{mv^2}{R} = \frac{GMm}{R^2}$$

$$v^2 = \frac{GM}{R}$$

$$v = \sqrt{\frac{GM}{R}}$$

2 Hence derive an expression of the kinetic energy of the satellite. [1]

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{GM}{R} = \frac{GMm}{2R}$$

(iii) On the same axes below in **Fig. 26.3**, sketch graphs to show for the satellite how its:

1 gravitational potential energy (GPE) varies with respect to distance R from the centre of Earth, and

2 kinetic energy (KE) varies with respect to distance R from the centre of Earth, and

3 total energy (TE) varies with respect to distance R from the centre of Earth. [3]

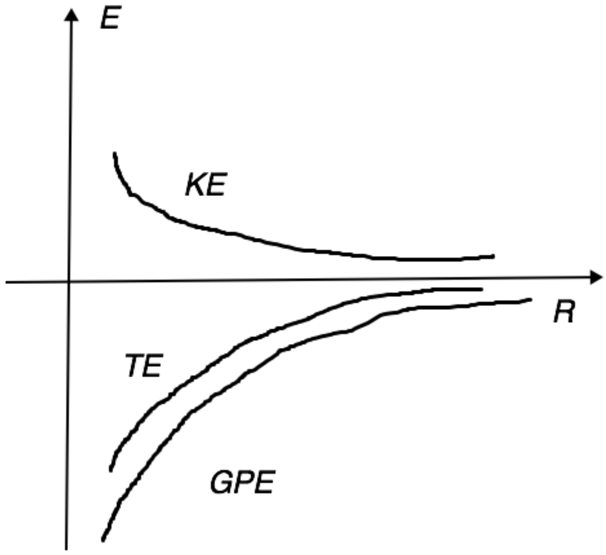


Fig. 26.3

(iv) The gravitational force acting on the satellite due to Earth is directed towards the centre of Earth. However, the satellite does not fall back towards Earth and instead continues its orbit around Earth.
State and explain how this is possible. [2]
Satellite has tangential velocity, and resultant acceleration acts towards centre of Earth, causing it to change direction.
However curvature of path matches that of the earth's curvature, hence it remains at same height above earth's surface.

[MI 2013]

27 (a) (i) Define gravitational potential energy of a mass at a point. [1]
Gravitational potential energy of a mass at a point is defined as the work done on the mass in moving it from infinity to that point.

(ii) Explain why gravitational potential energy is always negative. [2]
By convention, the gravitational potential energy is taken to be zero at infinity.
Since all gravitational forces are attractive, work is done by the mass as it moves from infinity and thus the work done on the mass is negative.

(b) A spherical planet has mass *M* and radius *R*. The planet may be considered to have all its mass concentrated at its centre.

A rocket is launched from the surface of the planet such that the rocket moves radially away from the planet. The rocket engines are stopped when the rocket is at a height *R* above the surface of the planet as shown in **Fig. 27.1**.

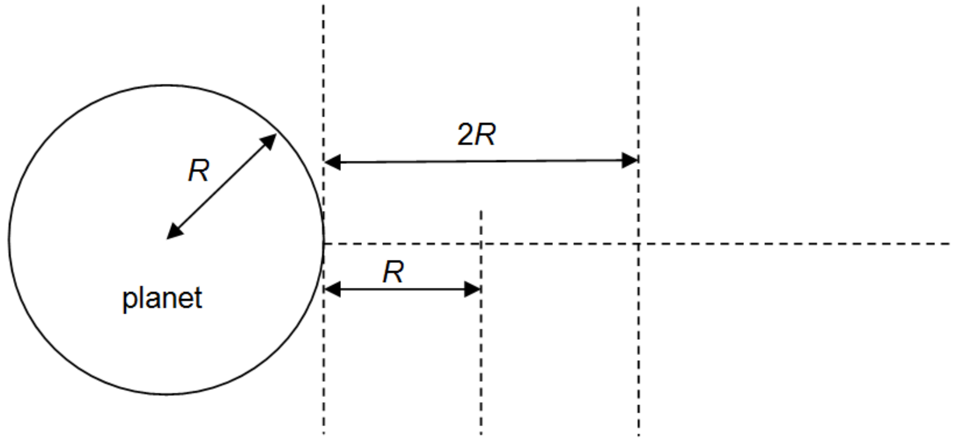


Fig. 27.1

The mass of the rocket, after its engines have been stopped, is m.

(i) Show that, for the rocket to travel from a height R to a height $2R$ above the planet's surface, the change ΔU in magnitude of the gravitational potential energy of the rocket is given by the expression

$$\Delta U = \frac{GMm}{6R}$$

[2]

Change in gravitational potential

$\Delta\Phi = -\frac{GM}{3R} - (-\frac{GM}{2R}) = \frac{GM}{6R}$

Hence the change in gravitational potential energy

$\Delta U = m\,\Delta\Phi = \frac{GMm}{6R}$

(ii) During the ascent from a height R to a height $2R$, the speed of the rocket changes from 7600 m s^{-1} to 7320 m s^{-1}. The planet has a radius of 3.40×10^6 m. Use the expression in **(b)(i)** to determine a value for the mass M of the planet. [3]

Gain in gravitational potential energy = Loss in kinetic energy

$\Delta U = ½\, mu^2 - ½\, mv^2$

$\frac{GMm}{6R} = ½\, m\,(7600^2 - 7320^2)$

$\frac{GM}{6R} = (2.09 \times 10^6)$

$(6.67 \times 10^{-11}\, M)/(6 \times 3.4 \times 10^6) = 2.09 \times 10^6$

$M = 6.39 \times 10^{23}$ kg

(iii) State two assumptions made in the determination in **(b)(ii)**. [2]

Ignore friction with atmosphere.
Rocket is outside atmosphere.
Not influenced by another planet.
Any two correct responses.

(c) A satellite is orbiting in a geostationary orbit around the Earth of mass M. Explain

(i) what is meant by the term *geostationary orbit*, [1]

Geostationary orbit refers to a circular orbit around the Earth in which a satellite would appear to be stationary to an observer on the Earth's surface.

(ii) why the satellite must move from West to East, and [1]
The Earth's surface moves from West to East.
In order for the satellite to appear stationary for an observer on the Earth's surface, the satellite must move in the same direction.

(iii) why the satellite must be above the equator. [2]
The gravitational force acting on the satellite provides the centripetal force for the circular motion. Since gravitational force acts towards the centre of the earth, the circular orbit must be centred on the Earth's centre.
If the orbit is not vertically above the equator, the satellite would have varying latitude and so cannot be geostationary.

(iv) Show that the distance r of the geostationary satellite from the centre of the Earth is given by the expression

$$r^3 = 0.0126\,M.$$

Explain your working clearly. [2]
The gravitational force provides the centripetal force

$$mr\omega^2 = \frac{GMm}{r^2}$$

$$r^3 = \frac{GM}{\omega^2} = \frac{GMT^2}{4\pi^2}$$

$$= \frac{6.67 \times 10^{-11} \times (24 \times 60 \times 60)^2\, M}{4\pi^2} = 0.0126\, M$$

(v) Given that the mass of Earth, M, is 5.98×10^{24} kg, calculate the Earth's gravitational field strength at this distance r of the satellite from the centre of the Earth. [2]

$r^3 = 0.0126 \times 5.98 \times 10^{24}$
$r = 4.22 \times 10^7$ m

gravitational field strength $= \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(4.22 \times 10^7)^2} = 0.224$ N kg^{-1}

(d) Explain why an astronaut in a satellite orbiting the Earth may be described as weightless. [2]
The gravitational force is just enough to provide the centripetal force for orbiting.
Both satellite and astronaut are falling freely with the same acceleration
OR
Contact force between satellite and astronaut is zero and hence the astronaut appears to be weightless.

[AJC 2013]

28 **(a)** **Fig. 28.1** shows an aircraft flying in a horizontal circular path at constant speed.

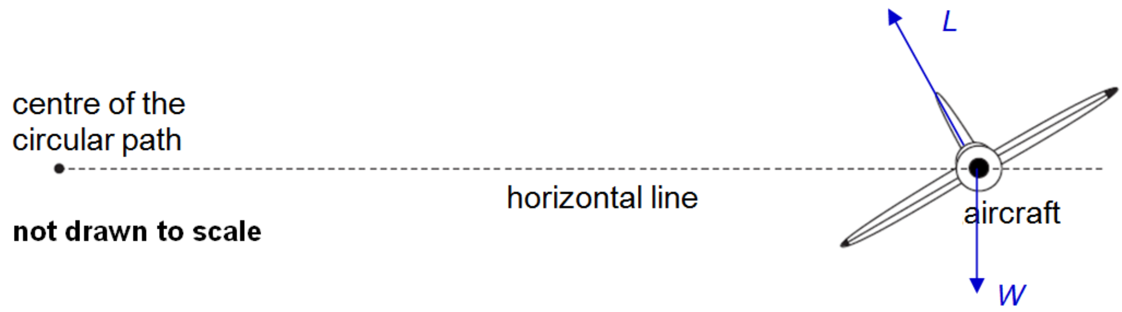


Fig. 28.1

(i) Explain why the aircraft has a *constant speed* but not a *constant velocity*. [2]
The plane is constantly changing direction. Since velocity is a vector, it is not constant.
The speed is constant because the net force is perpendicular to the direction of motion at all times.

(ii) To turn along the circular path, the aircraft needs to be banked at an angle, as shown in **Fig. 28.1**. State how this helps to produce the centripetal force necessary for the circular motion. [1]
The horizontal component of the lift force acts as the centripetal force.

(iii) On **Fig. 28.1**, draw and label the forces acting on the plane. [2]

(iv) The aircraft flies at a speed of 450 km h^{-1} in a circular path of radius 3.5 km. Calculate the horizontal acceleration experienced by the pilot as the aircraft turns. [2]

$$a = \frac{v^2}{r} = \frac{(450/3.6)^2}{3500} = 4.5 \text{ m s}^{-2}$$

(b) Some satellites are used to monitor weather conditions on Earth, for surveillance and for communications. Such satellites may be placed in a *geostationary orbit*.

(i) Explain what is meant by the term '*geostationary orbit*' and why a satellite in this orbit is used for communications. [2]
Orbit where the satellite is constantly above the same position above the Earth's surface.
It maintains a fixed position relative to surface of Earth and hence can provide uninterrupted communication between transmitter and receiver.

(ii) State one disadvantage of this orbit. [1]
Signal can be weak as the satellite will be at a large distance above the Earth.

(c) A satellite of mass *m* travels at angular speed *ω* in a circular orbit at a height *h* above the surface of a planet of mass *M* and radius *R*.

(i) Using only the symbols stated above and *G* as the gravitational constant, give an equation that relates the gravitational force on the satellite to the centripetal force. [1]

$$G\frac{Mm}{(R+h)^2} = m\omega^2(R+h)$$

(ii) Use your equation from **(i)** to derive an expression for the orbital period, *T*, of the satellite. [2]

Use of $\omega = \dfrac{2\pi}{T}$

$$\frac{GM}{(R+h)^3} = \frac{4\pi^2}{T^2}$$

$$T^2 = \frac{4\pi^2(R+h)^3}{GM}$$

(iii) Explain why the period of a satellite in orbit around the Earth cannot be less than 84 minutes. Your answer should include quantitative justification.

Given: mass of the Earth = 6.00×10^{24} kg
radius of the Earth = 6.40×10^{6} m [3]

Students must show calculations that for this orbital period, *h* will be negative
Hence they need to discuss that it is not possible for *T* to be less than 84 min as it will imply that the orbital radius is less than the radius of the Earth.

(iv) **1** Describe and explain what happens to the speed of a satellite when it moves to an orbit that is closer to the Earth. [2]
Speed increases because it loses potential energy but gains kinetic energy.

2 Explain why such low Earth orbits are harder to maintain, with reference to your answer in **(c)(iv)1**. [2]
With a higher speed, the atmospheric drag force will be larger, hence satellite loses more energy and will "decay" to a lower altitude.
Any logical answers will be accepted, provided that student must make reference to (c)(iv)1.

[YJC 2012]

29 **(a)** A satellite of mass 200 kg is placed between the Earth and Sun. The satellite is at a distance of 1.51×10^{9} m from the centre of the Earth and a distance of 148.1×10^{9} m from the centre of the Sun as shown in **Fig. 29.1**.

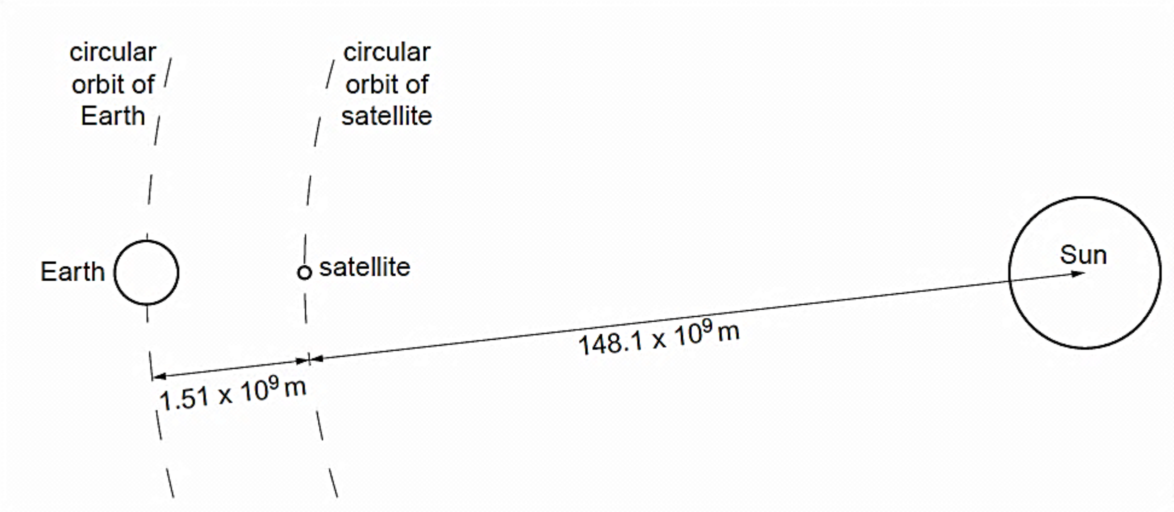


Fig. 29.1 (not to scale)

The speed of the satellite is adjusted so that it orbits the Sun with a period of 1 year (3.1536×10^{7} s). The rocket motor of the satellite is then switched off. The satellite then travels round the Sun in a circle, keeping constant the distances between the satellite, the Earth and the Sun.

(i) Calculate

1 the speed of the satellite, [2]

$$\text{Speed} = \frac{2\pi r}{t} = \frac{2\pi(148.1\times10^{9}\ \text{m})}{(3.1526\times10^{7}\ \text{s})}$$
$$= 2.95 \times 10^{4}\ \text{m s}^{-1}$$

2 the centripetal acceleration of the satellite. [2]

$$\text{Acceleration} = \frac{v^2}{r} = \frac{(2.95\times10^4\ \text{m s}^{-1})^2}{(148.1\times10^9\ \text{m})}$$
$$= 5.87 \times 10^{-3}\ \text{m s}^{-2}$$

(ii) The mass of the Sun is 1.99×10^{30} kg and the mass of the Earth is 5.98×10^{24} kg. Use this information to calculate the resultant force on the satellite. [3]

Gravitational force due to the Earth

$$\Rightarrow\ F_e = \frac{GM_{earth}m_{satellite}}{r_{earth-satellite}^{\ 2}} = \frac{(6.67\times10^{-11})(5.98\times10^{24}\,\text{kg})(200\ \text{kg})}{(1.51\times10^9\ \text{m})^2}$$
$$= 3.499 \times 10^{-2}\ \text{N}$$

Gravitational force due to the Sun,

$$\Rightarrow\ F_s = \frac{GM_{sun}m_{satellite}}{r_{sun-satellite}^{\ 2}} = \frac{(6.67\times10^{-11})(1.99\times10^{30}\ \text{kg})(200\ \text{kg})}{(148.1\times10^9\ \text{m})^2}$$
$$= 1.210\ \text{N}$$

Resultant force = $F_s - F_e$ = 1.210 N – 3.499×10^{-2} N = <u>1.175 N (towards the Sun)</u>

(iii) Show that the centripetal acceleration of the satellite is caused by this resultant force. [1]

Acceleration due to resultant force, $a = \dfrac{F}{m_{satellite}} = \dfrac{1.175\ \text{N}}{200\ \text{kg}} = 5.87\ \text{m/s}^2$

This is <u>in agreement with the centripetal acceleration calculated in (a)(i)2</u> hence the resultant force must the force causing the centripetal acceleration.

(iv) If the satellite's mass were 400 kg instead, how should its position and/or speed be adjusted to achieve the same orbital period of 1 year around the Sun? [1]

No adjustment is needed. Gravitational acceleration <u>is independent of mass</u> of the satellite.

(v) For such a satellite, suggest why

1 the satellite has an advantage over a geostationary satellite for observing the Sun. [1]

the <u>Sun is always visible to it</u> because it <u>does not go into the shadow of the Earth</u> (as a geostationary satellite would).

2 the satellite requires frequent small corrections of position and/or speed, [1]

it is in unstable equilibrium / the orbit is not really circular/ may be affected by other planets etc

Not Accepted:

Burning of the sun causing a depreciation in mass. The loss in mass is not so significant to warrant small corrections continually over the lifetime of the satellite.

3 the satellite is considerably more expensive to put into orbit than a geostationary satellite circling the Earth. [2]

It has greater potential energy/is located further from earth and has larger speed/k.e hence a greater amount of energy is required launch it into orbit so rocket and fuel costs are greater.

(b) A space rocket is launched with high speed from the Earth. When it just leaves the Earth surface, its engine is shut off. The kinetic energy E_K and the distance r of the space rocket from the Earth's centre are monitored by the ground station. **Fig. 29.2** shows how the values of E_K varies with $\frac{1}{r}$.

[Note: You may treat the rocket and Earth as an isolated system in subsequent parts of the question.]

(i) By drawing an appropriate line on **Fig. 29.2**, determine the vertical-intercept (y-intercept) E_o of the graph. [2]

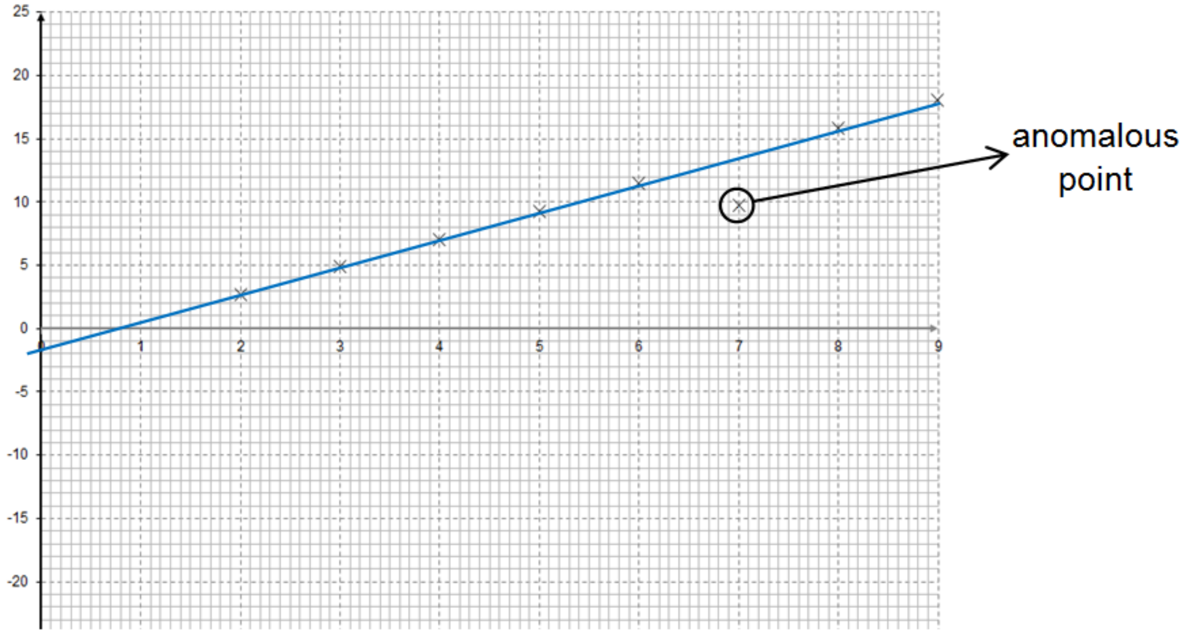


Fig. 29.2

Identifying the anomalous point (circled and labelled) **and** sketching an appropriate BFL
Reading correctly the *y*-intercept: E_o = -2.0 x 10^{11} J

(ii) Explain the physical significance of the value of E_o found in **(b)(i).** [2]

At $\frac{1}{r} = 0 \Rightarrow r \to \infty$. The vertical intercept represents the kinetic energy of the rocket when it is at an infinite distance from the Earth.
Since E_o is negative, it means that the rocket cannot escape from the Earth's gravitational field / it requires additional k.e. to escape from the Earth's gravitational field.

(iii) Assuming that total energy of the rocket remains constant during its flight, sketch labelled graphs on the axes in **Fig. 29.2** to represent

1 the variation of the total energy E_T . [1]
E_T – a horizontal straight line passing through E_o.

2 the variation of the gravitational potential energy E_P. [2]
Treating the satellite and Earth as an isolated system, the gravitational potential energy of the system $E_p = -G\frac{M_{Earth}m_{satellite}}{r} \propto -\frac{1}{r}$

When $r \to \infty \Rightarrow \frac{1}{r} = 0$, $E_p = 0$.

When $\frac{1}{r} = 9 \times 10^{-8}$ m^{-1}, $E_K = 18.0 \times 10^{11}$ J,

$\Rightarrow$ Therefore $E_P = E_T - E_K = (-2-18) \times 10^{11}$ J $= -20.0 \times 10^{11}$ J

Hence, E_p graph is a straight line graph with negative gradient, passing through (0,0) and (9, -20).

[HCI 2012]