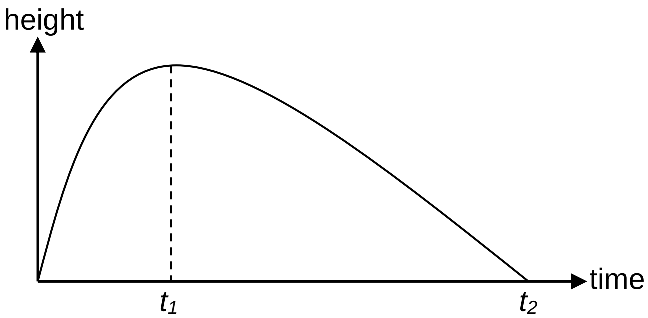
THE *PR:IME! PACKAGE* PART I

Mechanics 1 (Kinematics, Dynamics and Forces)

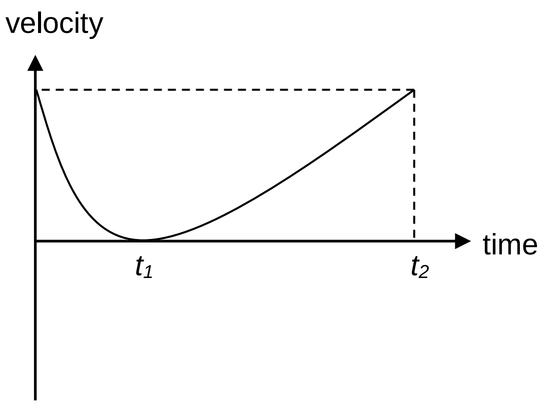
<u>MCQ</u>

1 A ball is thrown vertically upwards and returns along the same path. The graph shows how its height varies with time.

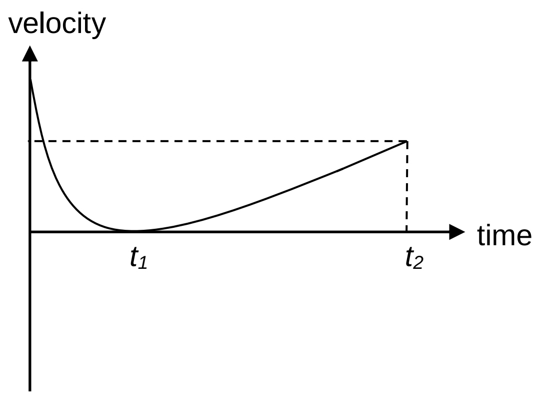


Which velocity-time graph describes this motion?

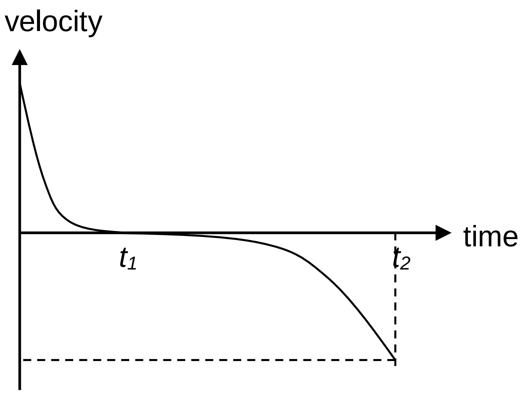
A



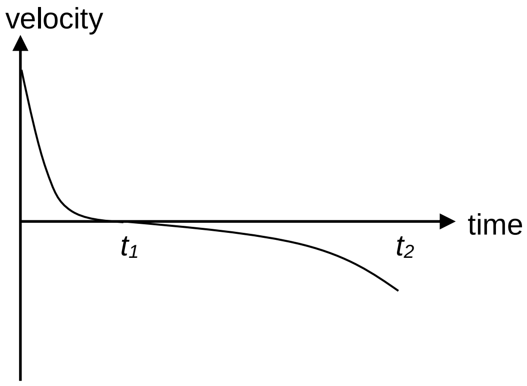
B



C



D

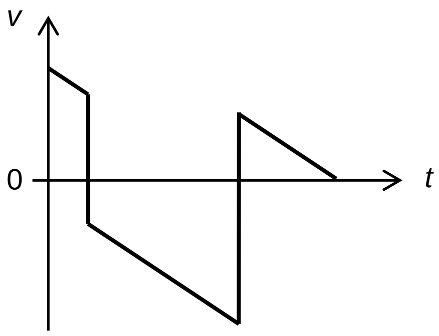


[AJC 2012]

Ans: D

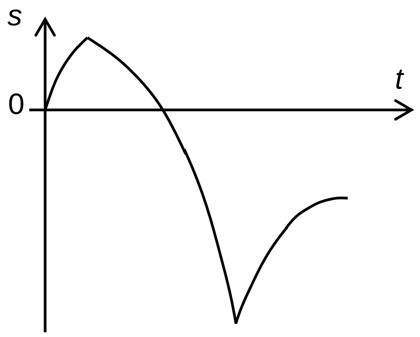
The velocity-time graph can be deduced by the gradient of the height-time graph. The upward velocity decreases to zero at an decreasing rate when the ball is thrown up, and the downward velocity increases from zero at a increasing rate when the ball falls to its initial position. The final speed is less than the initial speed.

2 The following graph shows the variation with time of the velocity v of a ball moving freely and vertically under gravity.

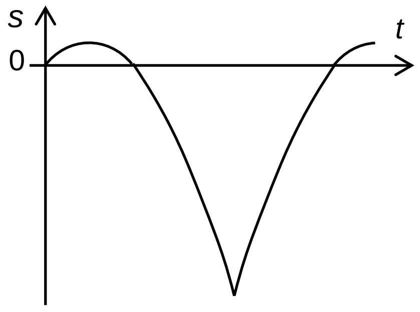


Which of the following graphs shows the variation with time of the displacement s of the ball from its initial position?

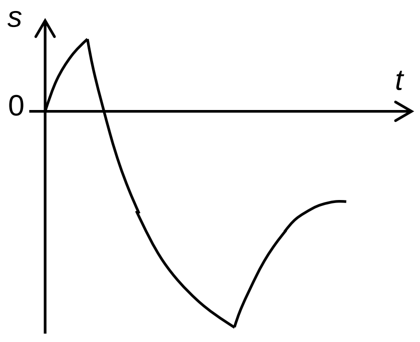
A



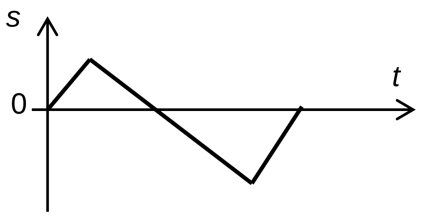
B



C



D



[VJC 2013]

Ans: A

$$v = \frac{ds}{dt}$$

$\therefore$ the value of the velocity v at any time on the given graph gives the gradient of the s-t graph at that time on the required graph.

For the first portion of the v-t graph, the value of v is positive but decreases, until it instantaneously decreases to a negative value. Hence, the gradient of the s-t graph starts off positive, but decreases slowly, until a certain point when it instantaneously changes to a negative value.

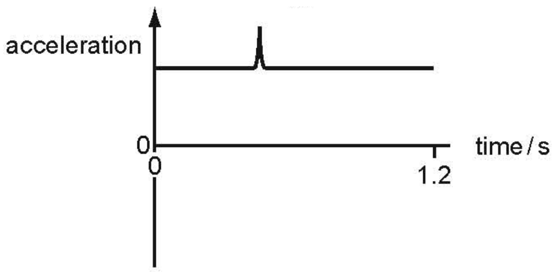
Continuing with the reasoning, we see that, in the second portion of the *v*-*t* graph, the value of *v* is negative and continues to decrease to even larger negative values. Hence, on the *s*-*t* graph, the curve will start off with a negative gradient in the second portion of the graph, and it will become even steeper with a more negative gradient with time.

At the start of the third portion of the *v*-*t* graph, the value of *v* changes instantaneously from a negative value to a positive value, before decreasing to zero. On the *s*-*t*, the gradient of the curve will also change instantaneously from a negative value to a positive value, before decreasing to zero.

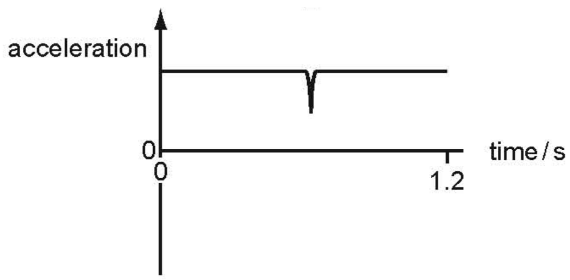
3 A student throws a ball vertically upwards. Upward velocities are taken as positive. The ball makes an elastic collision with the ceiling, rebounds and accelerates back to the student's hand in a time of 1.2 s.

Which graph best represents the acceleration of the ball from the moment it leaves the hand to the instant just before it returns to the hand?

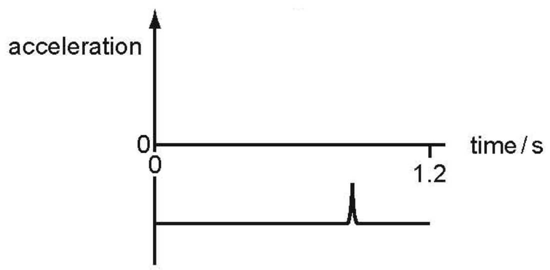
A



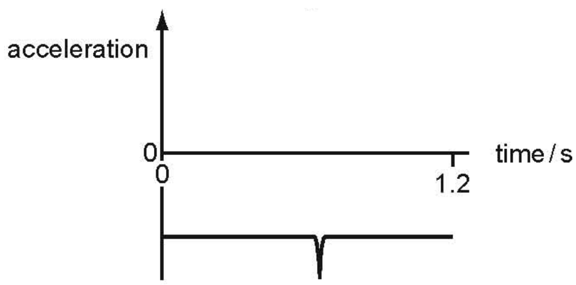
B



C



D



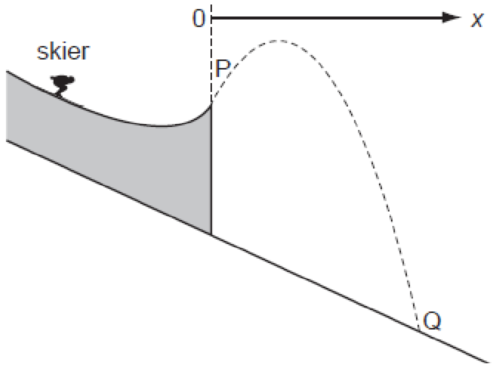
[MJC 2012]

Ans: D

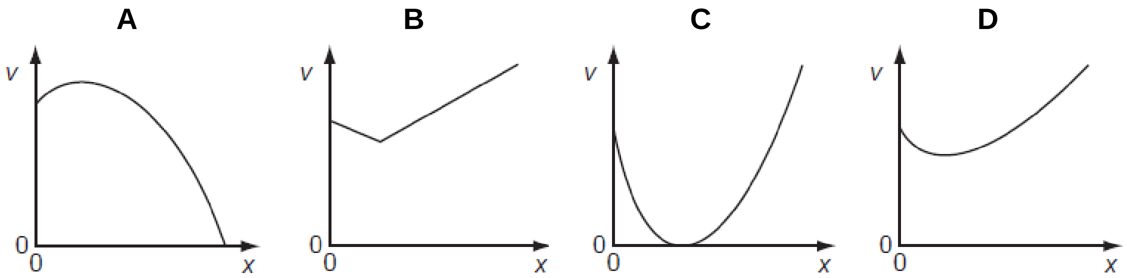
Take vertically upwards as positive direction, the acceleration (g) of the ball before and after the collision with the ceiling is negative (downwards).

During collision with the ceiling, the ball experienced an downward force from the ceiling and hence a larger negative (downwards) acceleration.

4 The dotted line shows the path of a competitor in a ski-jumping competition.



Ignoring air resistance, which graph best represents the variation of his speed *v* with the horizontal distance *x* covered from the start of his jump at P before landing at Q?

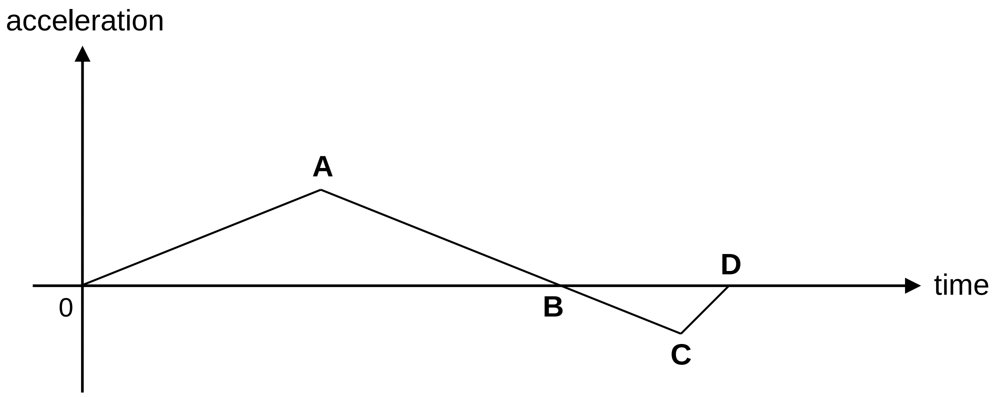


[IJC 2013]

Ans: D

At the highest point, the speed is at its minimum and is non-zero. The value of the speed at any point in the projectile motion will also be continuous.

5 The acceleration-time graph of an object moving in a straight line is as shown. The object started its motion from rest.



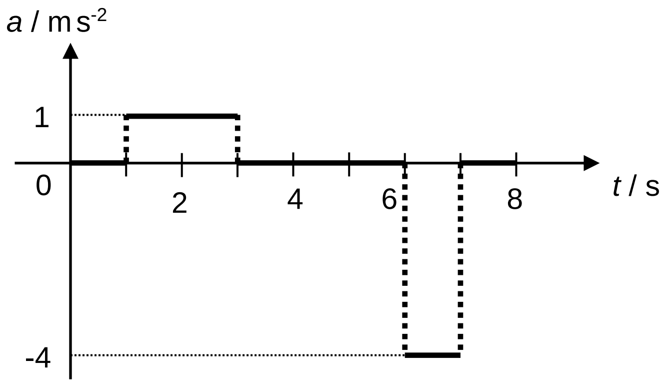
At which point is the body moving with the largest speed?

[MJC 2013]

Ans: B

Since the area under the acceleration time graph is the change in speed and the initial speed of the object is zero, the biggest area under the graph will correspond to the largest speed. At B, the area under the graph is the largest, following which the area is negative which means that the speed decreases from B.

6 An object, initially at rest, moves along a straight line path. The graph below shows the variation of its acceleration, a, with time, t.



What is the total displacement of the object until t = 8.0 s?

A 2.0 m **B** 3.0 m **C** 6.0 m **D** 11 m

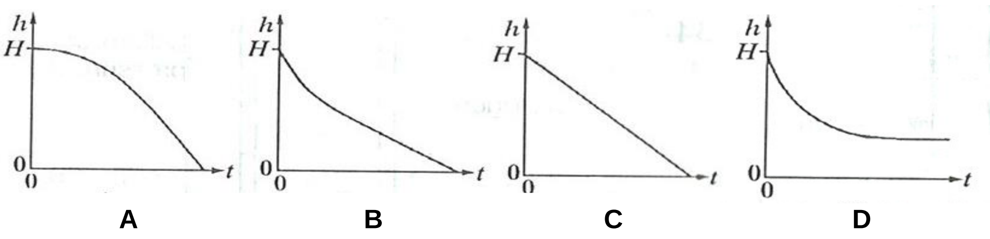
[MJC 2015]

Ans: C

Within the first 3 s, the object gains a speed of
$v = u + at = 0 + 1 \times 2 = 2$ m s^{-1}
For the next 3 s, the object maintains the same speed.
Finally, between 6 s and 7 s, it experiences deceleration.
$v = u + at = 2 + (-4 \times 1) = -2$ m s^{-1}
Total area under graph = 6 m

7 A steel ball-bearing is released at the surface of a viscous liquid contained in a tall, wide jar. In falling through the liquid, the ball-bearing experiences a retarding force proportional to the velocity.

If the depth of the liquid in the jar is H, which one of the following graphs best represents the variation of the height h of the ball-bearing above the base with time t?

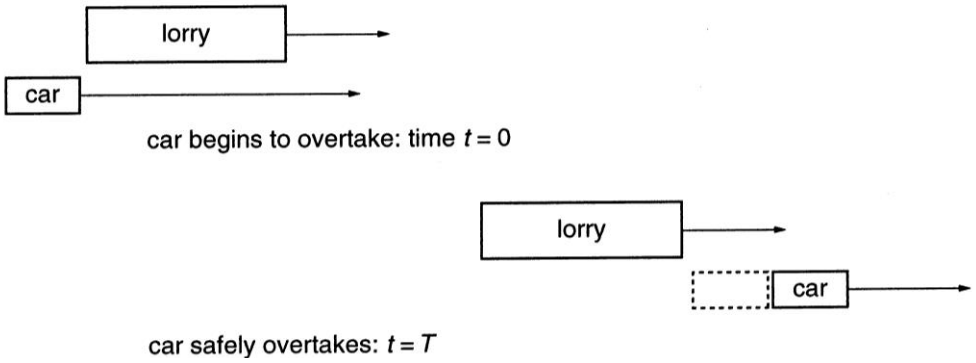


[TJC 2015]

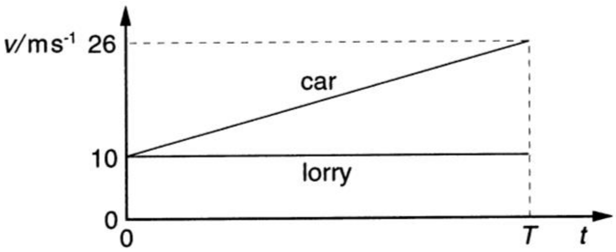
Ans: A

The ball bearing is released from rest and should accelerate until it reaches terminal velocity. This is represented by a steepening curve that eventually reaches constant gradient.

8 The minimum time T required for a car to safely overtake a lorry on the motorway is measured from the time the front of the car is level with the rear of the lorry, until the rear of the passing car is a full car-length ahead of the lorry.



The car is 3.5 m long and the lorry is 17.0 m long. The graph shows the variation with time t of the speeds v of the car and the lorry.



What is the value of T?

A 0.86 s **B** 1.2 s **C** 2.6 s **D** 3.0 s

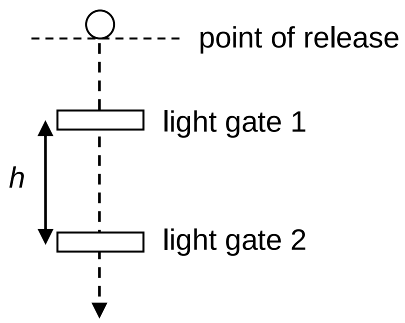
[ACJC 2015]

Ans: D

For the car to safely overtake, its displacement must be (17.0 + 2×3.5 = 24.0) m greater than the lorry.
This is represented by the area between the car's and lorry's graphs: 0.5(26–10)T = 24.0
T = 3.0 s

9 To determine the acceleration of free fall, a steel ball is dropped above two light gates as shown.

The ball passes light gate 1 and 2 at times t_1 and t_2 after release.



What is the acceleration of free fall?

A $\dfrac{2h}{(t_2 - t_1)}$ **B** $\dfrac{2h}{(t_2 - t_1)^2}$ **C** $\dfrac{2h}{(t_2{}^2 - t_1{}^2)}$ **D** $\dfrac{2h}{\left(\dfrac{t_2 + t_1}{2}\right)^2}$

[PJC 2012]

Ans: C

Using simultaneous equations,

$$x = \frac{1}{2}at_1{}^2 \;---(1)$$
$$x + h = \frac{1}{2}at_2^2 \;---(2)$$

Solving, we have

$$h = \frac{1}{2}a\left(t_2^2 - t_1^2\right)$$
$$a = \frac{2h}{t_2^2 - t_1^2}$$

10 Ball **A** is dropped from the top of a building. One second later, ball **B** is dropped from the same building. Neglecting air resistance, as time progresses, the difference in their speeds

A remains constant.

B increases.

C decreases.

D Increases and then decreases.

[VJC 2012]

Ans: A

$v_A = gt$

$v_B = g(t - 1) = gt - g = v_A - g$

Hence $v_A - v_B = g$, the difference in speeds is always constant.

11 A photographer wishes to check the time for which the shutter on a camera stays open when a photograph is being taken. It is found that before the shutter opens, the ball falls 2.50 m from rest. During the time that the shutter remains open, the ball falls a further 0.12 m. What is the time that the shutter remains open?

A 0.017 s **B** 0.156 s **C** 0.714 s **D** 0.731 s

[AJC 2012]

Ans: A

Let the time taken for the shutter to fall 2.50 m be t.

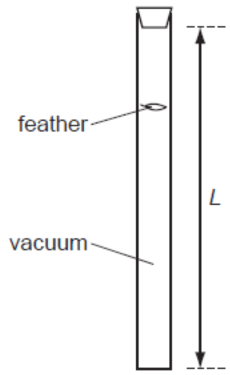
$$s = ut + \frac{1}{2}at^2$$
$$2.5 = 0 + \frac{1}{2}(9.81)t^2$$
$$t = 0.714 \text{ s}$$

Let the time taken for the ball to fall 2.62 m be t_1.

$$s = ut + \frac{1}{2}at^2$$
$$2.5 + 0.12 = 0 + \frac{1}{2}(9.81)t_1^2$$
$$t_1 = 0.731 \text{ s}$$

The time when the shutter remains open = $t_1 - t$ = 0.017 s

12 The diagram shows a laboratory experiment in which a feather falls from rest in a long evacuated vertical tube of length L.



The feather takes time T to fall from the top to the bottom of the tube. How far will the feather have fallen from the top of the tube in time $0.50T$?

A 0.13 L **B** 0.25 L **C** 0.38 L **D** 0.50 L

[IJC 2013]

Ans: B
Using $s = ut + ½ a t^2$, where $s = L$, $u = 0$, $t = T$, $a = g$, and taking downwards as positive.
(1): $L = ½ g T^2$,
Let the distance fallen be S at the time $t = 0.50 T$.
(2): $S = ½ g (0.5T)^2$,
(2)/(1): $S/L = ¼$
$S = ¼ L$ or $0.25 L$

13 In order that a train can stop safely, it passes a signal showing a yellow light before it reaches another signal showing a red light. The train driver applies the brake at the yellow light and this result in a uniform deceleration and the train stops exactly at the red light.

The distance between the red and yellow lights is x.

What must the minimum distance between the lights be, without changing the deceleration of the train, if the train speed is increased by 20 %?

A 1.20*x** **B** 1.25*x* **C** 1.44*x* **D** 1.56*x*

[MJC 2012]

Ans: C

$v^2 = u^2 - 2as,\ v = 0$

$(u_1)^2 = 2ax$ ………………. (1)

$(u_2)^2 = 2ax_2,\ and\ \ u_2 = 1.2\,u_1$

hence, $(1.2u_1)^2 = 2ax_2$ ………(2)

from $(1)\,\&\,(2)$,

$$\frac{1.44(u_1)^2}{(u_1)^2} = \frac{2ax_2}{2ax}$$

$x_2 = 1.44x$

14 Which of the following statements is true about an object undergoing projectile motion in a vacuum?

A The horizontal range of the projectile is fixed by virtue of the object's mass.

B The rate of change of the momentum of the object throughout the whole motion is equal to the object's weight.

C The maximum vertical height to which the object can reach can only be found when the time of flight is known.

D None of the above.

[CJC 2012]

Ans: B

Option A: The mass of the object plays no part in a projectile motion in a vacuum.

Option C: The vertical height can be calculated via the usage of the Law of Conservation of Energy. All we need is the speed of the object at the time of projection and at the maximum height, as seen in the following relationship:

$$\text{Loss in KE} = \text{Gain in GPE}$$

$$\frac{1}{2}mv^2_{\text{at projection}} - \frac{1}{2}mv^2_{\text{at the top}} = mgh$$

$$h = \frac{1}{2g}\left(v^2_{\text{at projection}} - v^2_{\text{at the top}}\right)$$

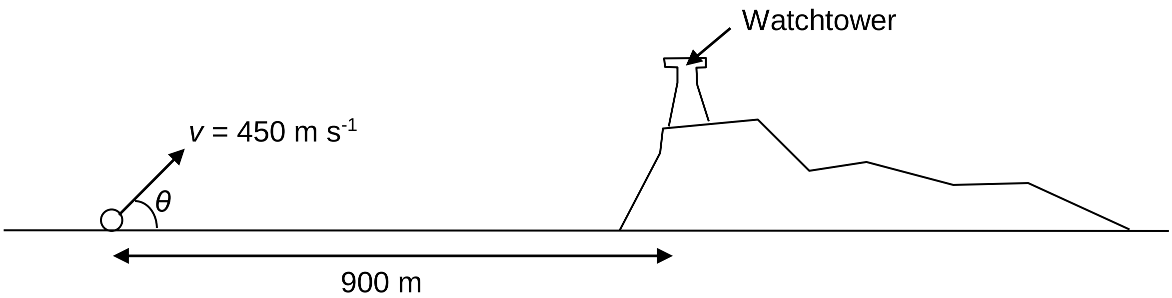
Or

It can also be found via the following equation:

$$v^2 = u^2 + 2as$$

Option B: The rate of change of momentum of the object is the force acting on the object. The only force acting on it is its weight.

15 A cannon ball is fired at a speed of 450 m s^{-1} at sea level at an angle of θ = 31.6° with respect to horizontal as shown below. The cannon ball hits the top of a watchtower located 900 m away.



How high is the top of the watchtower above sea level? Neglect air resistance.

A 526 m **B** 542 m **C** 580 m **D** 1390 m

[HCI 2012]

Ans: A

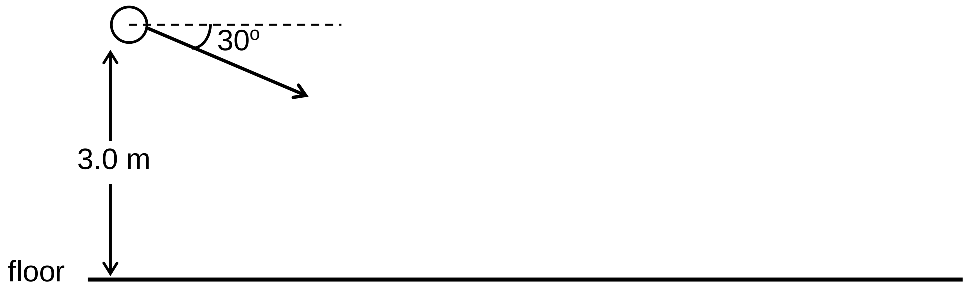
Horizontal component:

$$s_x = v_x t$$
$$900 = 450\cos(31.6°)t$$
$$t = 2.348 \text{ s}$$

Vertical component:

$$s_y = u_y t + \frac{1}{2}a_y t^2$$
$$s_y = 450\sin(31.6°)\,(2.348) + \frac{1}{2}(-9.81)(2.348)^2$$
$$s_y = 526 \text{ m}$$

16 A ball was thrown with a velocity 5.0 m s^{-1} at an angle of 30° below the horizontal from a height of 3.0 m as shown in the figure below.



What was the speed of the ball just when it reached the floor?

A 6.80 m s^{-1} **B** 7.67 m s^{-1} **C** 9.16 m s^{-1} **D** 10.0 m s^{-1}

[VJC 2012]

Ans: C

$$v_y^2 = u_y^2 + 2as_y$$
$$= (5.0\sin 30^o)^2 + 2(9.81)(3.0)$$
$$= 65.1$$
$$v = \sqrt{(v_y^2 + v_x^2)}$$
$$= \sqrt{65.1 + (5.0\cos 30^o)^2}$$
$$= 9.16 \text{ m s}^{-1}$$

17 Three identical rocks are thrown off the edge of a cliff with the same speed. Rock X is thrown vertically upward, rock Y is thrown horizontally, and rock Z is thrown vertically downward. If the base of the cliff is flat and air resistance is negligible, which rock will hit the ground with the highest speed?

A Rock X

B Rock Y

C Rock Z

D All three rocks hit the ground with the same speed.

[HCI 2013]

Ans: D

Since all 3 rocks experience the same vertical displacement (downward), they experience the same loss of gravitational potential energy and the same gain in kinetic energy (KE). Since they all started off with the same initial KE, they must hit the ground with the same final KE and same final speed.

18 A resupply aircraft is flying at 360 km h^{-1}, at an angle of 15° below the horizontal. The cargo door opens and a supply package drops out. Assuming negligible air resistance, how much time passes before the package's speed doubles?

A 7.9 s **B** 10.4 s **C** 15.2 s **D** 54.8 s

[RVHS 2013]

Ans: C

360 km h^{-1} → 100 m s^{-1}

Initial horizontal vel. = final horizontal vel. = 100 cos 15° = 96.59 m s^{-1}

Initial vertical vel. = 25.89 m s^{-1}

Final vel. = 2 × 100 m s^{-1}

Final vertical vel. = $(200^2 – 96.59^2)^{1/2}$ = 175.13 m s^{-1}

Using $v_y = u_y + a_y t$ → 175.13 = 25.89 + 9.81(t) → t = 15.2 s

19 Which of the following statements is true about the concept of force?

A If an object does not experience acceleration, there are no forces acting on it.

B If an object does not have a change in its kinetic energy, there are no forces acting on it.

C If an object experiences acceleration, there must be at least one external force acting on it.

D If an object does not have a change in its gravitational potential energy, there are no forces acting on it.

[CJC 2012]

Solution: C

Option A: The net force acting on the object is zero. But there might be forces acting on the system.

Option B: If the object's kinetic energy is constant, then the magnitude of its velocity is also constant, however the direction of velocity might change as velocity is a vector quantity. E.g. In the case of circular motion. Alternatively, if there is no change in KE means no work is being done, which could mean the resultant force acting on the object is zero.

Object D: Gravitational potential energy is dependent on height. Therefore, even though there might be a force acting on the body, the gravitational potential energy may not change.

20 Which of the following pairs of forces is not a valid example of action and reaction to which Newton's Third Law of motion applies?

A The weight of a satellite in orbit around the Earth and the attractive force on the Earth's centre of mass due to the satellite.

B The forces of repulsion experienced by two parallel wires carrying currents in opposite directions.

C The forces of attraction felt by two gas molecules passing near to each other.

D The weight of an object resting on the table and the force acting on the object due to the table supporting it.

[SRJC 2012]

Ans: D

Action reaction pair consists of forces where one is a Force **by B on A** and the other is a Force **by A on B**.

Option A: Force by Earth on satellite vs Force by satellite on Earth (valid action reaction pair)

Option B: Force by wire A on wire B vs Force by wire B on wire A (valid action reaction pair)

Option C: Force by molecule A on molecule B vs Force by molecule B on molecule B (valid action reaction pair)

Option D: Force by Earth on object vs Force by table on object (invalid action reaction pair)

21 A helicopter loaded with cargo has a total mass of M. It is descending with a downward acceleration of $\frac{g}{3}$, where g is the acceleration due to gravity. How much cargo must the helicopter offload so that it ascends with an upward acceleration of $\frac{g}{2}$? Assume that the lift force on the helicopter remains constant throughout.

A $\frac{M}{9}$ **B** $\frac{4M}{9}$ **C** $\frac{5M}{9}$ **D** $\frac{8M}{9}$

[MJC 2012]

Ans: C

<u>*before*</u>	<u>*after*</u>
$(Mg) - L = M\,(g/3)$	$L - (M - m)g = (M - m)\,(g/2)$
therefore $L = 2/3\ Mg$	therefore $L = (M - m)\ 3/2\ g$

combining:

$$2/3\ Mg = (M - m)\ 3/2\ g$$

$$m = M - 4/9\ M$$

$$= 5/\ 9\ M$$

22 A girl weighs herself in an elevator. The scale reads 65.0 kg when the elevator is moving at constant velocity. What does the scale read when the elevator is accelerating downwards at 2.00 m s^{-2}?

A 51.7 kg

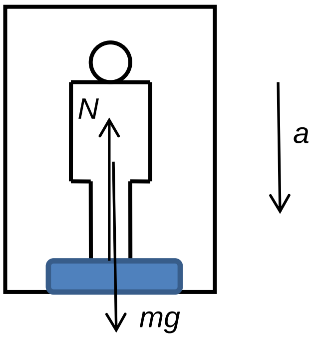
B 60.0 kg

C 65.0 kg

D 78.3 kg

[HCI 2013]

Ans: A



Let N be the force exerted by the balance on the girl.

By Newton's 2nd Law of Motion, the equation of Motion is given by

$$mg - N = ma \quad \text{.........(1)}$$

At constant speed, $a = 0$

$$m\,g = N = 65.0\,g \text{ N}$$

At $a = 2.0$ m s^{-2},

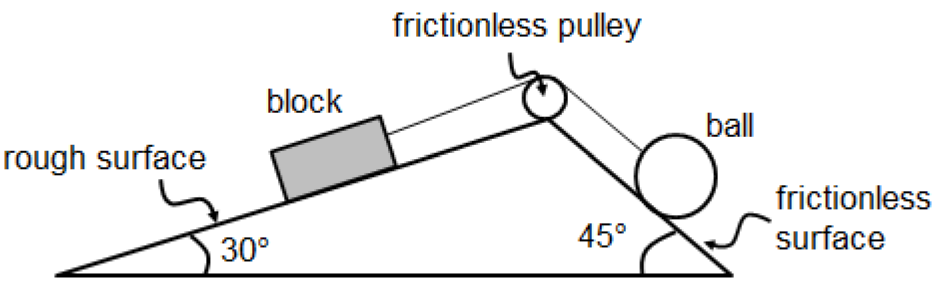
Substituting into (1)

$$65.0\,(9.81) - N' = 65.0\,(2)$$

$$N' = 507.65 \text{ N}$$

By Newton's 3rd Law of Motion, the girl exerts 507.65 N downwards which is registered by the balance as $\dfrac{507.65}{9.81} = 51.7$ kg

23 A block, resting on the rough surface of a ramp, is connected to a ball, resting on the frictionless surface of the same ramp, using a taut but inextensible string. The ball has a mass of 10.0 kg. The frictional force acting on the block is 30 N. If the ball accelerates uniformly at 1.0 m s^{-2} down the slope, what is the mass of the block?



A 3.8 kg **B** 5.0 kg **C** 6.7 kg **D** 7.4 kg

[RVHS 2013]

Ans: B

Considering the block:

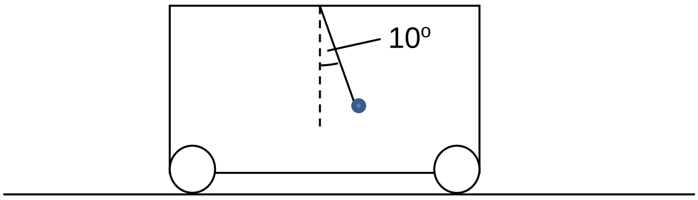
$$T - (m)(9.81)(\sin 30) - 30 = (m)(1)$$

Considering the ball:

$F_{net} = (10)(1) = (10)(9.81)(\cos 45) - T$

→ m = 4.9732 kg

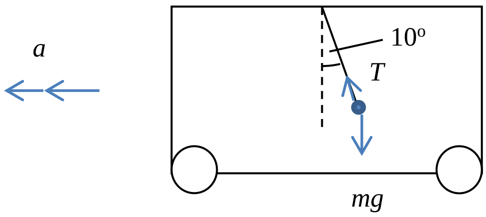
24 A pendulum bob was attached to a string and hung from the roof of a car. When the car accelerated on a flat road, the string of the bob made an angle 10° with the vertical. Find the acceleration of the car.



A 0.49 m s^{-2} **B** 1.73 m s^{-2} **C** 6.49 m s^{-2} **D** 9.67 m s^{-2}

[VJC 2013]

Ans: B



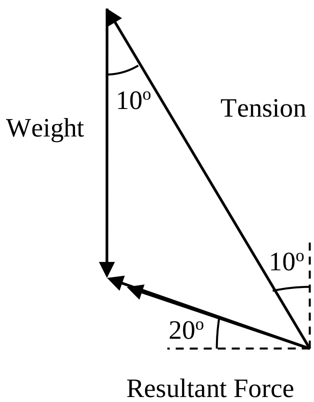
Consider the pendulum bob.

Vertical equilibrium: $T\cos 10^0 = mg$....(1)

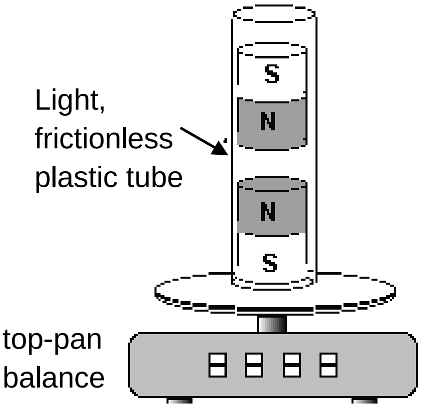
Horizontal acceleration: $T\sin 10^0 = ma$.....(2)

$\frac{(2)}{(1)}$ gives $\tan 10^0 = \frac{a}{g}$

$a = 9.81\tan 10^0 \approx 1.73\text{ m s}^{-2}$



25 Two identical cylindrical bar magnets are stored in a light plastic frictionless cylinder of negligible mass. When the magnets are arranged as shown in the figure below and weighed, the balance reads W. (The whole system is at rest.)



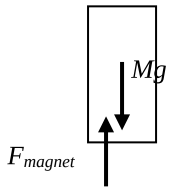
If the mass of each magnet is *M*, which of the following is correct?

A $W = Mg$ **B** $Mg < W < 2Mg$ **C** $W = 2Mg$ **D** $W > 2Mg$

[SRJC 2013]

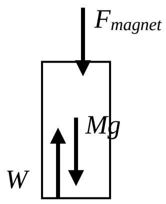
Ans: C.

FBD for *top* magnet



$F_{magnet} = Mg$ --(1)

FBD for *bottom* magnet



$F_{magnet} + Mg = W$ --(2)

Subst (1) into (2),

$W = Mg + Mg = 2Mg$

26 Two objects of different mass, m_1 and m_2, are pushed across a frictionless inclined surface. The same amount of force *F* is applied in the following two scenarios, Fig. 23.1 and Fig. 23.2.

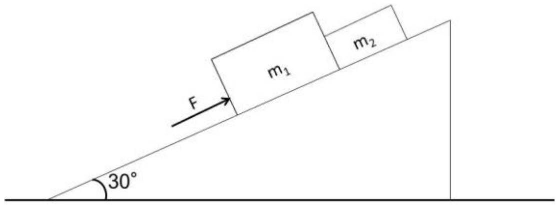


Fig. 23.1

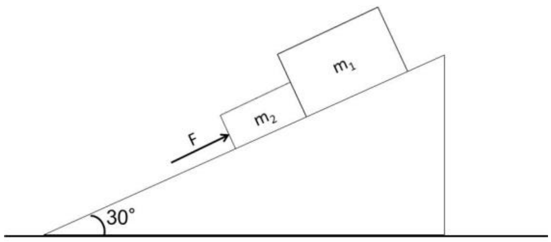


Fig. 23.2

Which of the following statements is true if m_1 is greater than m_2?

A There are no forces acting between the two masses.

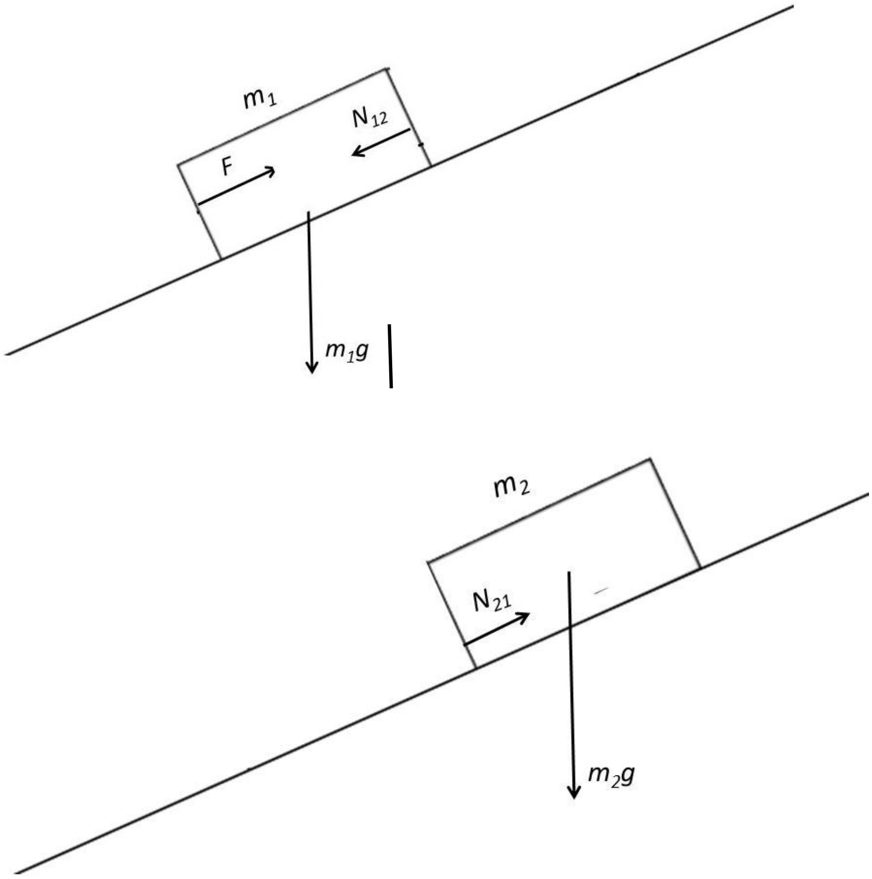
B The force that m_1 exerts on m_2 in Fig. 23.1 is equal to that which m_2 exerts on m_1 in Fig. 23.2.

C The force that m_1 exerts on m_2 in Fig. 23.1 is smaller than that which m_2 exerts on m_1 in Fig. 23.2.

D The force that m_1 exerts on m_2 in Fig. 23.1 is greater than that which m_2 exerts on m_1 in Fig. 23.2.

[CJC 2012]

Ans: C



For Fig 23.1

Consider m_1:

$F - m_1 g \sin 30° - N_{12} = m_1 a$

$N_{12} = F - m_1 g \sin 30° - m_1 a$

Consider m_2:

$N_{21} - m_2 g \sin 30° = m_2 a$

$N_{21} = m_2 g \sin 30° + m_2 a$ - - - - (1)

For Fig 23.2 (Drawing similar FBDs)

Consider m_2:

$F - m_2 g \sin 30° - N_{21} = m_2 a$

$N_{21} = F - m_2 g \sin 30° - m_2 a$

Consider m_1:

$N_{12} - m_1 g \sin 30° = m_1 a$

$N_{12} = m_1 g \sin 30° + m_1 a$- - - - (2)

Comparing equation 1 and equation 2, since m_2 *is* smaller than m_1, N_{21} is smaller than N_{12}.

27 A 4.0 kg block is pushed up a 36° incline by a force of magnitude *F* applied parallel to the incline. When *F* is 31 N, it is observed that the block moves up the incline with a constant speed.

Determine the magnitude of *F* that is required to lower the block down the incline at the same constant speed.

A 31 N **B** 17 N **C** 15 N **D** 13 N

[IJC 2015]

Ans: C

When pushing the object up,
F - mg sin θ - resistive force = 0
31 – (4.0)(9.81)(sin 36°) + resistive force = 0
resistive force = 7.93
When pushing object down,
F + mg sin θ - resistive force = 0
F = 7.93 - (4.0)(9.81)(sin 36°) = -15 N

28 A mass of 2.0 kg is at rest on a smooth floor. A horizontal stream of water, travelling at speed 8.0 m s^{-1}, strikes it at a rate of 1.0 kg s^{-1} for a duration of 50 s without splashing.

What is the initial acceleration of the mass?

A 0.080 m s^{-2}

B 0.16 m s^{-2}

C 4.0 m s^{-2}

D 8.0 m s^{-2}

[PJC 2013]

Ans: C

Rate of change of momentum of the water = $\frac{m}{t}\Delta v$ =1.0 × 8.0 = 8.0 kg m s^{-1}.

From Newton's second law of motion, force on the water = 8.0 N

From Newton's third law, force on the notice-board = –8.0 N.

Hence magnitude of acceleration of the notice-board = $\frac{F}{m} = \frac{8.0}{2.0}$ = 4.0 m s^{-2}.

29 An empty conveyor belt requires a constant force of 17.0 N to be driven horizontally at 1.50 m s^{-1}. Sand is then vertically dropped at a rate of 3.00 kg s^{-1} onto the conveyor belt.

What is the total average force required to be exerted on the conveyor belt In order to maintain the conveyor belt at the speed of 1.50 m s^{-1} while the sand is being poured?

A 4.50 N **B** 17.0 N **C** 21.5 N **D** 23.5 N

[HCI 2012]

Ans: C

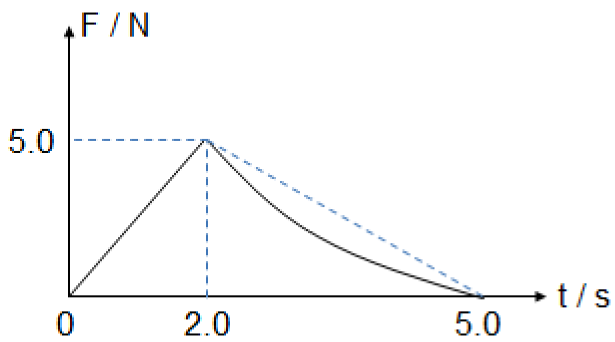
Consider the mass of sand accelerated from 0 m s^{-1} to 1.50 m s^{-1} in 1 second.

The mass if 3.00 kg.

Thus the rate of change of momentum is (mv-mu)/t = (3.00 x 1.50 - 0)/1 = 4.50 N.

The total force required is 17 + 4.5 = 21.5 N.

30 A body of mass 1.0 kg is moving at 10 m s^{-1}. A force now acts in its direction of motion, varying with time as shown below.



What is most likely the momentum of the body after 5.0 s?

A 15.0 kg m s^{-1} **B** 20.5 kg m s^{-1} **C** 22.5 kg m s^{-1} **D** 25.0 kg m s^{-1}

[RVHS 2013]

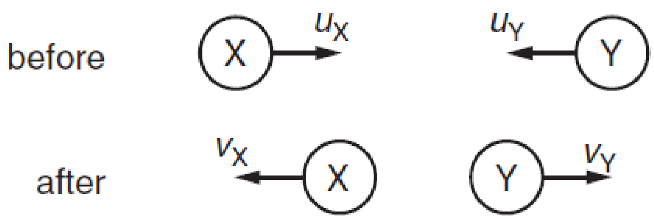
Ans: B

$$F_{avg} = \frac{\Delta mv}{\Delta t} = \left[\frac{5\times 2}{2} + \frac{5\times 3}{2}\right] = (1)(v_f - 10) \rightarrow v_f = 22.5$$

Final momentum = 22.5 kg m s^{-1}

However since 2.0 → 5.0 s is non-linear decrease of force, final momentum will be less than 22.5 ks m s^{-1}, but greater than 15.0 ks m s^{-1}, which is given by when t is until 2.0s.

31 Two balls **X** and **Y** approach each other along the same straight line and collide elastically. Their speeds are u_x and u_y respectively. After the collision they move apart with speeds v_x and v_y respectively. Their directions are shown on the diagram.



Which of the following equation is correct?

A $u_x - u_y = v_y - v_x$

B $u_x + u_y = v_x - v_y$

C $u_x + u_y = v_x + v_y$

D $u_x - u_y = v_x - v_y$

[MJC 2013]

Ans: C

For elastic collision,

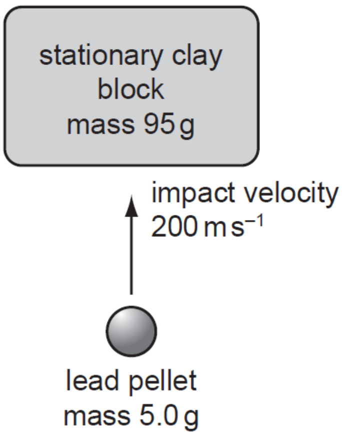
Relative speed of approach = relative speed of separation

$v_2 - v_1 = u_1 - u_2$ (*where* the sign conventions of u_1, u_2, v_1, v_2 are to the right)

Hence $u_x - (-u_y) = v_y - (-v_x)$

$u_x + u_y = v_x + v_y$

32 A lead pellet is shot vertically upwards into a clay block that is stationary at the moment of impact but is able to rise freely after impact.



The pellet hits the block with an initial velocity of 200 m s^{-1}. It embeds itself in the block and does not emerge.

How high above its initial position will the block rise?

A 5.1 m **B** 5.6 m **C** 10 m **D** 102 m

[DHS 2015]

Ans: A

For a completely inelastic collision, $m_1u_1 + m_2u_2 = (m_1+m_2)v$
Velocity of pellet & block immediately after collision, v = 10 m s^{-1}
By conservation of energy, as pellet & block travel upward, loss in KE = gain in GPE
$0.5(0.100)(10)^2 = (0.100)(9.81)h$
h = 5.1 m

28 A 5.00 kg object moves at 15.0 m s^{-1}. It collides perfectly inelastically with a 10.0 kg object which was at rest. How much kinetic energy is lost in the collision?

A 188 J **B** 375 J **C** 563 J **D** 702 J

[SRJC 2013]

Ans: B

For perfectly inelastic collision, the two objects moved off as one with a common v.

By conservation of momentum, 5.00(15.0) + 10.0(0) = (5.00 + 10.0) v

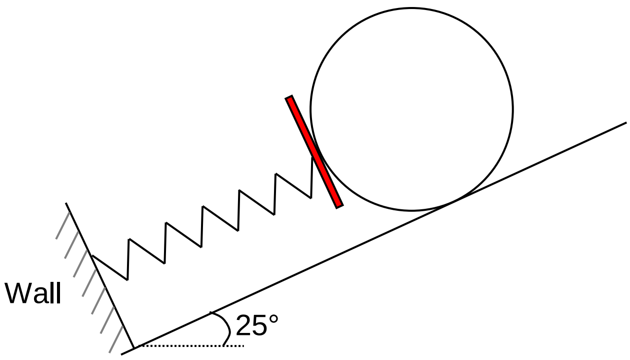
v = 5.0 m s^{-1}

Initial k.e.=$1/2(5.00)(15.0)^2$ = 562.5 J

Finall k.e.=$1/2(5.00 +10.0)(5.0)^2$ = 187.5 J

k.e. lost in collision = 562.5 – 187.5 = 375 J

29 A sphere of mass 3.00 kg rests on a rough slope as shown. The frictional force between the sphere and the slope is given as 2.0 N. The spring obeys Hooke's Law. The spring constant is 500 N m^{-1}.



What is the compression of the spring?

A 0.0208 m **B** 0.0248 m

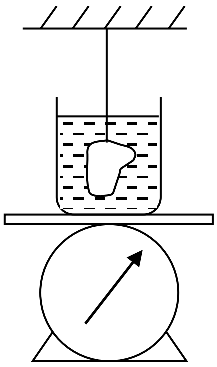
C 0.0493 m **D** 0.0548 m

[MJC 2013]

Ans: A
In equilibrium,

$$mg\sin\theta - f = ke$$
$$e = \frac{3.00(9.81)(\sin 25) - 2.0}{500}$$
$$= 0.0208 \text{ m}$$

30 When a beaker of water rests on a balance, the weight indicated is *X*. A solid object of weight *Y* in air displaces weight *Z* of water when immersed.



What will be the balance reading when the object is suspended in the beaker of water so that it is totally immersed?

A $X + Z$

B $X + Y$

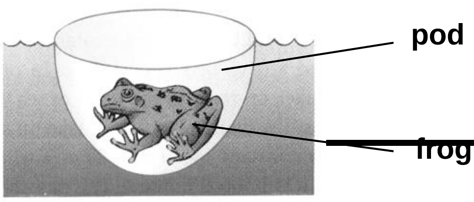
C $X + Y - Z$

D $X + Z - Y$

[PJC 2012]

Ans: A
When the solid object is placed in water, it exerts a force Z on the water. The total downward force exerted on the beaker of water will be its weight X and this additional force Z, which is balanced by the upward force exerted by the balance. The upward force exerted by the balance will hence be X + Z. By Newton's third law, the force by the beaker of water on the balance will also be X + Z, which is the balance reading.

31 A frog is trapped in an empty hemispherical pod of density 1.2×10^3 kg m^{-3} in a pond. The pod is submerged to a depth of 0.060 m such that the water level is at the brim of the pod.



Given that the pond water has a density of 1.35×10^3 kg m^{-3} and the pod has an internal radius of 0.055 m, find the mass of the frog.

A 0.360 kg **B** 0.490 kg **C** 0.610 kg **D** 0.120 kg

[MJC 2012]

Ans: B

Wt of frog + Wt of pod = Upthrust

Wt of frog + Wt of pod = Wt of fluid displaced by hemispherical pod

$m_{frog}\, g + \rho_{pod}\, V_{pod}\, g = m_{fluid\ displaced}\, g$

$m_{frog} + 1200\,(1/2 \times (4/3\,\pi\, 0.060^3 - 4/3\,\pi\, 0.055^3) = (1.35 \times 10^3)\,(1/2 \times 4/3\,\pi\, 0.060^3)$

m_{frog} = **0.49 kg**

32 A small steel ball bearing is held just below the surface of a deep tank of water and released. Which one of the following statements best describes the acceleration of the sphere during its descent?

A The acceleration is constant at 9.81 m s^{-2}.

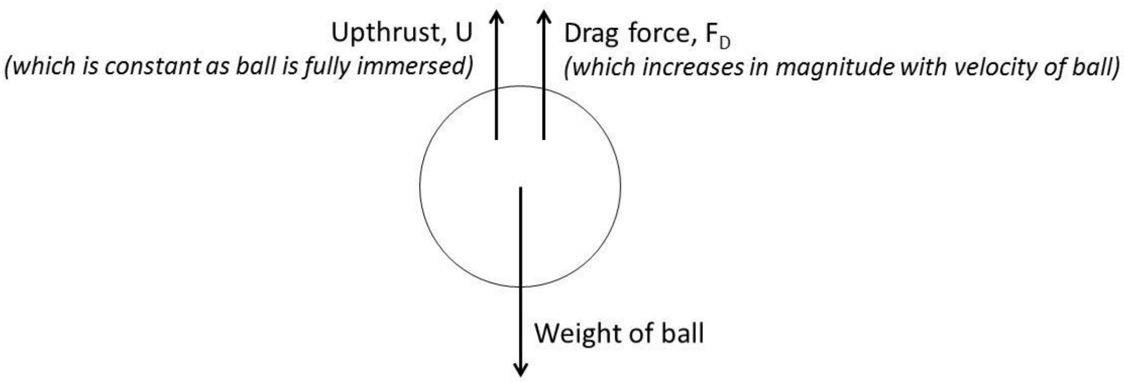
B The acceleration starts with a value of 9.81 m s^{-2} and decreases with time until it reaches zero.

C The acceleration starts with a value of 9.81 m s^{-2} and decreases with time but it will never reach zero.

D The acceleration starts with a value lesser than 9.81 m s^{-2} and decreases with time until it reaches zero.

[CJC 2012]

Ans: D



As the ball's motion is downwards, let us consider the following equation:

$W - U - F_D = ma$

Just before descending

As F_D is proportional to v^2, the ball's motion will be described as

$W - U = ma_1$

since the ball does not move – just at the point of release.

Here, U is a small number since the ball's volume is small – hence the volume of the water displaced is small. Thus a_1 has a magnitude which is slightly smaller than gravitational acceleration, g.

During the descent

As the ball accelerates downwards, F_D will be significant and thus the describing equation will be

$W - U - F_D = ma_2$

a_2 will be decreasing as F_D increases in size as the velocity of the ball increases.

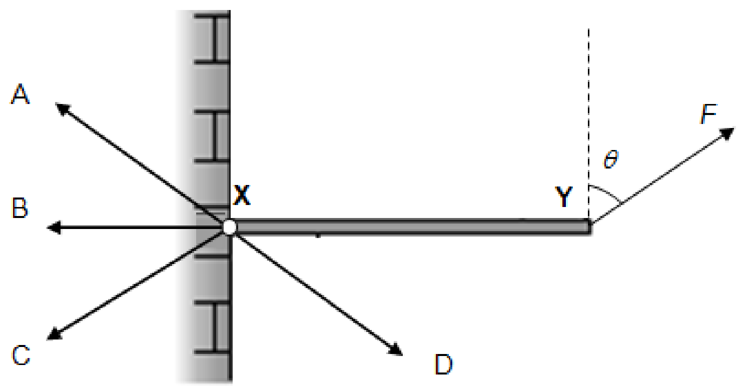
When terminal velocity is reached (if the tank is deep enough),

$W = U + F_D$

and the ball would travel at a constant speed.

33 A uniform rod **XY** is freely hinged to the wall at **X.** It is held horizontal by a force F acting from **Y** at an angle θ to the vertical as shown in the diagram.

Which of the four options (A, B, C and D) best shows the direction of the force exerted by the rod **on the wall?**



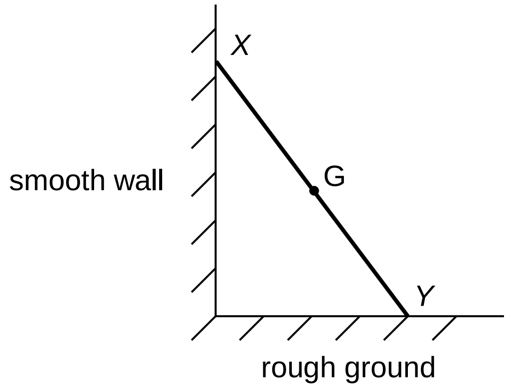
[JJC 2013]

Ans: D

The 3 forces acting on the rod must all act through the same point. Since there is an rightward component provided by F, there should be a leftward component provided by the unknown force. Thus the force on the rod by the wall should be pointing in the option A direction. By Newton's 3rd

Law, the force acting on the wall by the rod should be equal in magnitude and opposite in direction, therefore the ans is option D.

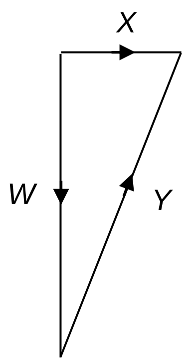
34 A ladder rests on a rough ground and leans against a smooth wall.



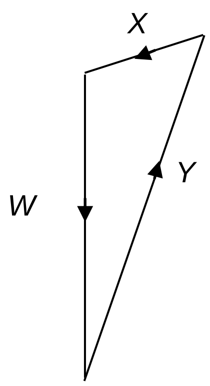
Its weight *W* acts through the centre of gravity G. Forces also act on the ladder at *X* and *Y*. These forces are *X* and *Y* respectively.

Which vector triangle represents the forces on the ladder?

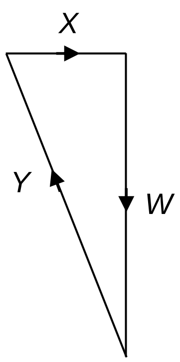
A



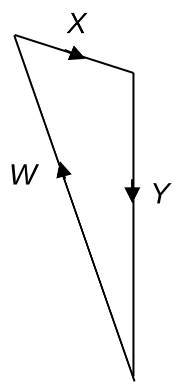
B



C



D



[PJC 2012]

Ans: C
Since the wall is smooth, there is only a horizontal normal contact force acting at X. The three forces form a closed polygon.

35 The diagrams show two ways of hanging the same picture.

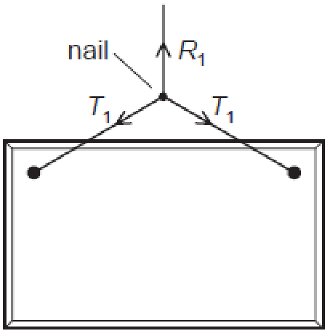


diagram 1

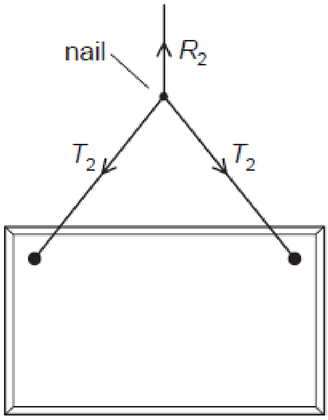


diagram 2

In both cases, a string is attached to the same points on the picture and looped symmetrically over a nail in a wall. The forces shown are those that act on the nail.

In diagram 1, the string loop is shorter than in diagram 2.

Which information about the magnitude of the forces is correct?

A $R_1 = R_2$ $T_1 < T_2$

B $R_1 = R_2$ $T_1 > T_2$

C $R_1 > R_2$ $T_1 < T_2$

D $R_1 < R_2$ $T_1 = T_2$

[AJC 2012]

Ans: B

$R_1 = R_2 = W$, weight of picture

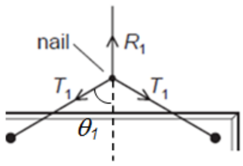
$R_1 = 2T_1 \cos \theta_1$

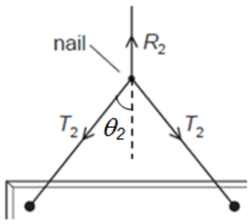
$R_2 = 2T_2 \cos \theta_2$

$\therefore\ 2T_1 \cos \theta_1 = 2T_2 \cos \theta_2$

Since $\theta_1 > \theta_2 \Rightarrow \cos \theta_1 < \cos \theta_2$

$\Rightarrow T_1 > T_2$



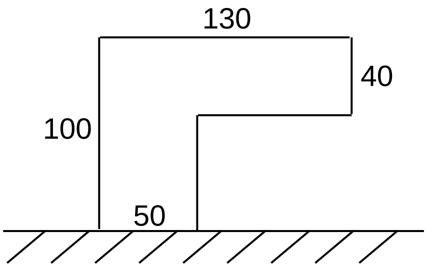


OR Draw and compare the closed triangle force diagram

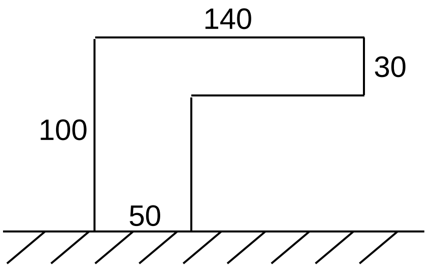
36 On a building site, uniform L-shaped girders, of dimensions in centimetres as shown, are placed on the ground on one side.

Which girder will fall over?

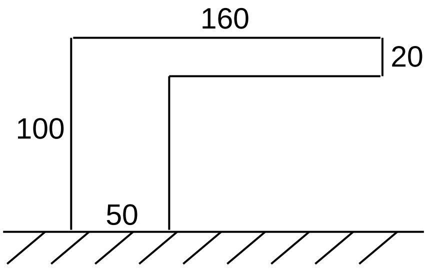
A



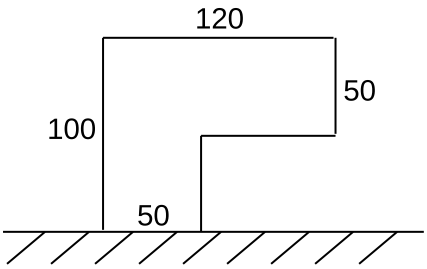
B



C



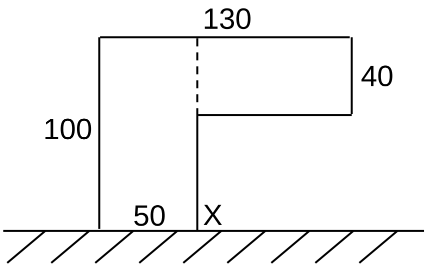
D



[PJC 2012]

Ans: A

Consider separate blocks and take moments of its individual weight about point X where it is likely to topple.

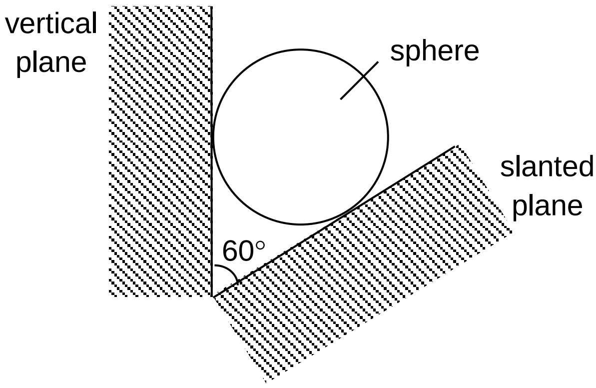


Clockwise moment $\propto 80 \times 40 \times 40 = 128000$

Anti-clockwise moment $\propto 100 \times 50 \times 25 = 125000$

Since clockwise moment is greater than anti-clockwise moment, the girder will topple.

37 A uniform sphere of mass 1.5 kg is placed in static equilibrium in between two smooth planes as shown in the figure below.

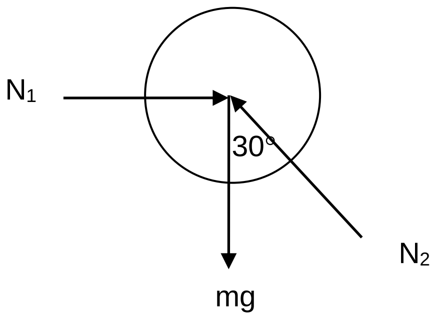


What is the magnitude of the force exerted by the vertical plane on the sphere?

A 0 **B** 8.5 N **C** 15 N **D** 17 N

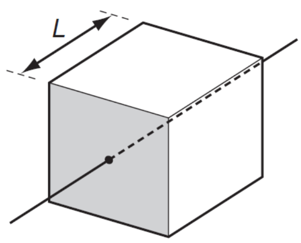
[SRJC 2012]

Ans: B

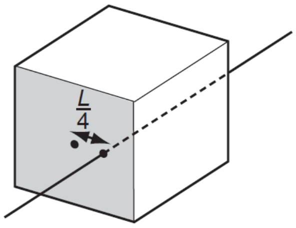


$$N_2 \sin 30 = N_1 \quad (1)$$
$$N_2 \cos 30 = mg \quad (2)$$
$$\frac{(1)}{(2)}: \quad \tan 30 = \frac{N_1}{mg}$$
$$N_1 = mg \tan 30 = 1.5(9.81)\tan 30 = 8.5$$

38 The diagram shows a solid cube with weight W and sides of length L. It is supported by a frictionless spindle that passes through the centres of two opposite vertical faces. One of these faces is shaded.



The spindle is now removed and replaced at a distance $\frac{L}{4}$ to the right of its original position.



When viewing the shaded face, what is the torque of the couple that will now be needed to stop the cube from toppling?

A $\dfrac{WL}{2}$ anticlockwise

B $\dfrac{WL}{2}$ clockwise

C $\dfrac{WL}{4}$ anticlockwise

D $\dfrac{WL}{4}$ clockwise

[AJC 2013]

Ans: D

At the new spindle position, the weight of the cube causes a anticlockwise torque of $\dfrac{WL}{4}$. Hence the torque of the couple required to prevent the cube from toppling must be $\dfrac{WL}{4}$ clockwise.

<u>Short structured questions</u>

1 The graph of Fig. 1 shows the variation with time t of velocity v of a ball that is released from rest a distance h above a rigid horizontal surface, and the ball is allowed to bounce.

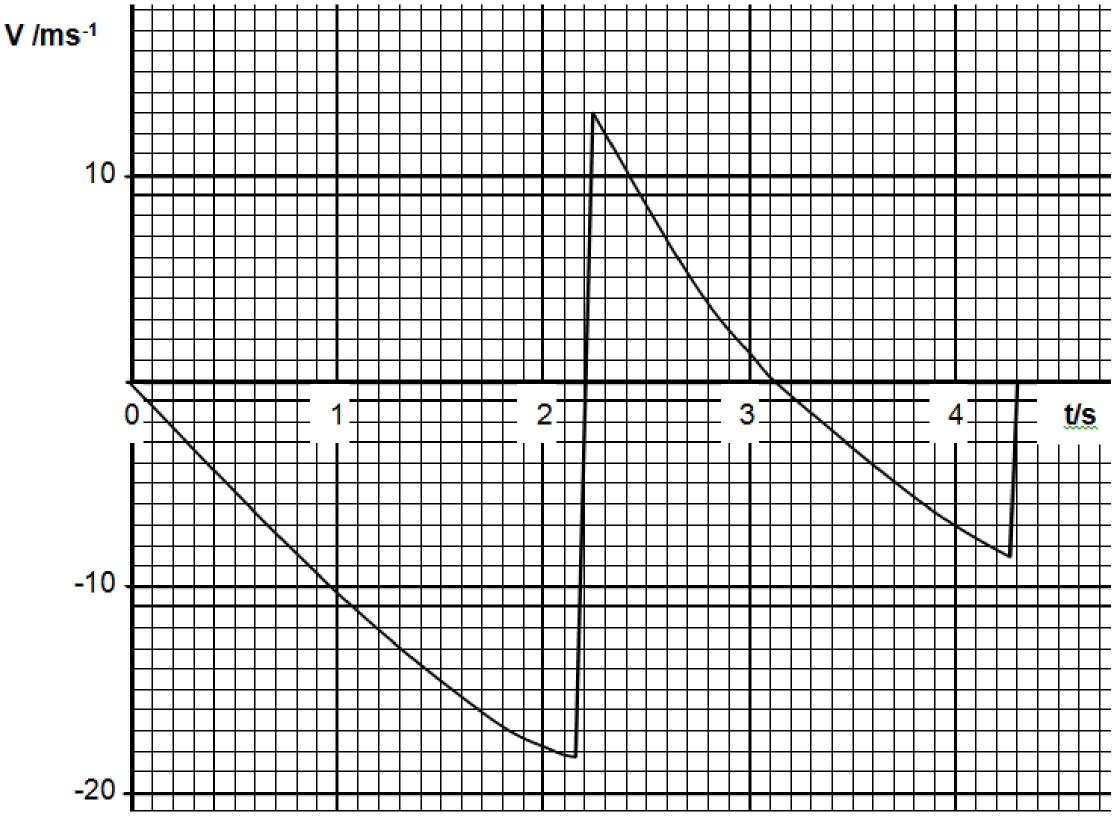
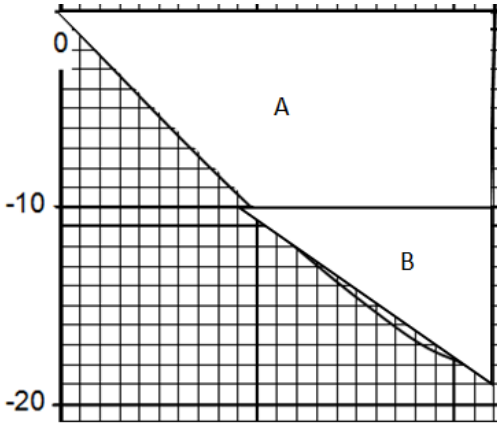


Fig. 1

(a) Using Fig. 1, determine the distance *h*.

Consider area under the v-t graph from t = 0 s to t = 2.2 s.

Divide the area into A and B.



h = 0.5(2.2 + 1.2)(10) + 0.5(1.3)(9)
= 22.9 m

Accept answers from the range of 20.0 m to 25.0 m.

h = ... m [2]

(b) Explain whether the collision with the surface is elastic.

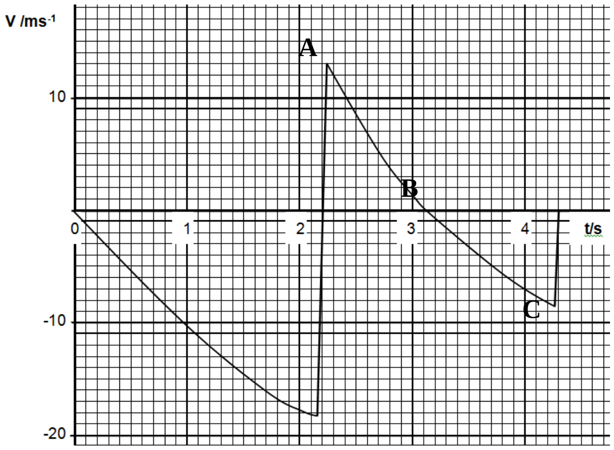
As there is a **change in speed of the ball** before it collides and after it collides with the wall, **kinetic energy is not conserved**, hence the collision is inelastic.

....

.... [1]

(c) Explain the difference between the time the ball takes to reach the maximum height after the first bounce and the time the ball falls back to the ground.

Label the graph as A (ball is about to move away from the horizontal surface after the first bounce), B (ball is at maximum height) and C (ball is about to reach the horizontal surface after it falls from the maximum height).



The magnitude of the ball's acceleration from A to B is <u>greater</u> than that of ball from B to C due to a <u>greater net force</u> (weight + air resistance) experienced by the ball. Thus, the change in velocity from A to B is greater than the change in velocity from B to C. Hence, the time taken from A to B is shorter than the time taken from B to C because the ball travelled the same distance when it moves from A to B and from B to C.

....

....

....

.... [1]

(d) Determine the average force exerted by the horizontal surface on the ball during the first bounce. The mass of the ball is 0.050 kg.

$$F = \frac{mv - mu}{t} = \frac{13 - \left[(-18.3)\right] \times 0.050}{0.10} = 15.7 \text{ N} \quad [A1]$$

Accept the range of $18.0 \le v \le 18.5$ and $0.05 \le t \le 0.20$ for the calculation of the force.

force = ... N [2]

(e) Use energy considerations to suggest how the energy of the ball changes between the time that it is released and the time that the ball reaches the maximum height after the first bounce.

The gravitation potential energy of the ball changes to kinetic energy and heat (due to work done against air resistance) when the ball approaches the floor.

Some of the kinetic energy is changed to sound and heat when the ball collides with the floor.

The kinetic energy of the ball changes to gravitation potential energy and heat (due to work done against air resistance) after the ball bounces and moves to its maximum height.

[3]

[AJC 2012 P2 Q1]

2 **(a)** Soft drink cans are being packed into a cubic crate of volume 1.0 m^3.

(i) Estimate the volume of a soft drink can.

Actual dimensions of 330 ml soft drink can

radius r = 3.2 cm, height h= 12.2 cm

(*the volume of the can is greater than the 330 ml of soft drink contained*)

Acceptable range

$3.5 \text{ cm} \geq r \geq 2.5 \text{ cm}$, $14 \text{ cm} \geq r \geq 10 \text{ cm}$

Volume of a can, $V = \pi r^2 \times h$

Acceptable range for volume of can, $(550 > V > 196)\,\text{cm}^3$

Accept if student quotes 330 cm^3

volume of a soft drink can = ... cm^3 [1]

(ii) Estimate the volume of empty space in a 1.0 m^3 cubic crate filled with the maximum number of soft drink cans.

Consider cans as being blocks of dimensions 2r x 2r x h. These blocks are stacked in the crate

No of blocks in 1 m^3, $N = \dfrac{1}{2r \times 2r \times h}$

Empty space per block, $V_{empty} = V_{block} - V_{can}$

$= 2r \times 2r \times h - \pi \times r \times r \times h$

$= (4 - \pi) r^2 h$

volume of empty space = ... m^3 [3]

(b) As part of an advertising campaign, a sports car and a jet fighter will race over a straight 2.0 km track. The velocity-time graphs of both the car and the aircraft are shown in Fig. 2.1.

The car maintains a constant acceleration of 9.0 m s^{-2} throughout the race while the jet fighter experiences an acceleration of 5.0 m s^{-2} for the first 10 seconds of the race, after which its acceleration is 15.0 m s^{-2}.

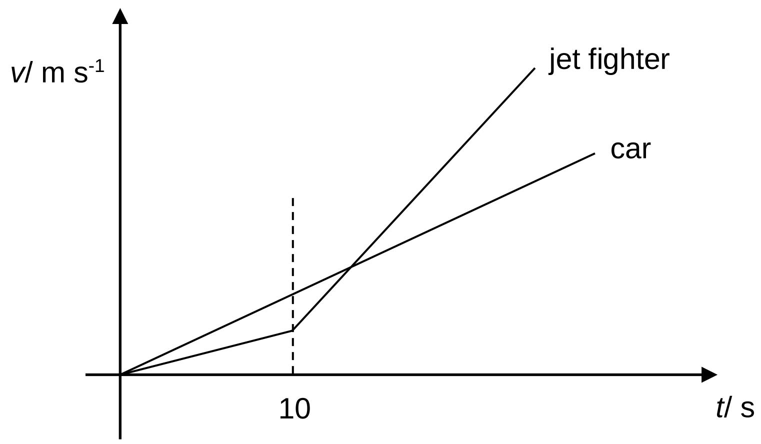


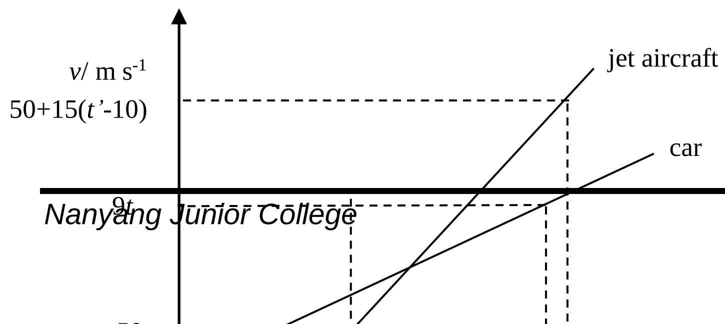
Fig. 2.1

(i) Determine the time taken for the sports car to reach a speed of 100 km h^{-1}.

$$v = u + at$$
$$\frac{100 \times 1000}{3600} = 0 + (9)t$$
$$t = 3.1 \text{ s}$$

time taken = ... s [2]

(ii) The ence in finish times of the car and the jet fighter, Δt .



time for car to move 2 km = t

$$\frac{1}{2} \times t \times 9t = 2000$$
$$t = 21.1 \text{ s}$$

time for jet to move 2 km = t'

$$\frac{1}{2} \times 10 \times 50 + \frac{1}{2}\left(50 + 50 + 15(t' - 10)\right) \times (t' - 10) = 2000$$
$$250 + \frac{1}{2}(100 + 15t' - 150)(t' - 10) = 2000$$
$$15t'^2 - 200t' - 3000 = 0$$
$$t' = 22.3 \text{ s}$$

time difference = 1.2 s

Alternative Solution

time difference = 1.2 s

time for car to move 2 km = t

$$s = ut + \frac{1}{2}at^2$$
$$2000 = \frac{1}{2}(9)t^2$$
$$t = 21.1 \text{ s}$$

time for jet to move 2 km = t'

s_1 distance travelled by jet in first 10 s

s_2 distance travelled by jet from 10 s to finish

$$s_1 = 0 \times 10 + \frac{1}{2} \times 5 \times (10)^2 = 250 \text{ m}$$
$$s_2 = 50(t' - 10) + \frac{1}{2} \times 15 \times (t' - 10)^2$$
$$s_1 + s_2 = 2000$$
$$250 + 50(t' - 10) + \frac{1}{2} \times 15 \times (t' - 10)^2 = 2000$$
$$t' = 22.3 \text{ s}$$

time difference = 1.2 s

Δt = ... s [3]

(iii) Timmy claims that if the race track was longer, the jet fighter would have won the race. With reference to Fig. 2.1 or otherwise, explain why Timmy's statement is correct.

There will be a time where the distance travelled by the jet fighter will exceed that travelled by the car. If the track is at least that distance, the jet fighter would win the race.

This can be seen from Fig 1.1, after a certain time t, the area under the graph (which represents the distance covered) for the fighter will exceed that of the car.

OR

Acceleration of jet fighter is larger than the car. If given enough distance for the jet fighter to accelerate, it can overtake the car.

[1]

1]

3 In a skiing competition, skier A and B starts off from rest at the top of the hill and race in straight parallel routes 700 m downhill. In order to win the competition, the winner has to be 3 full ski lengths ahead when crossing the finishing line.

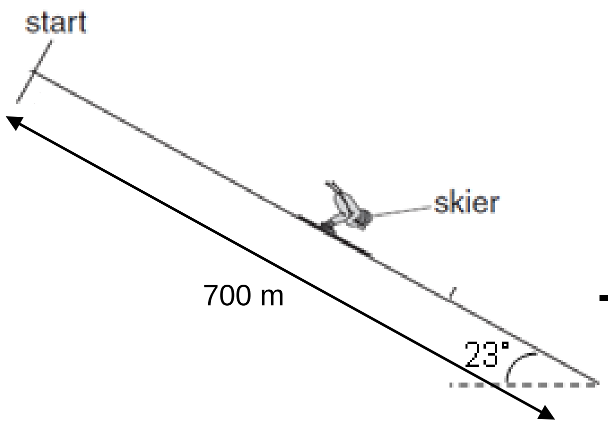
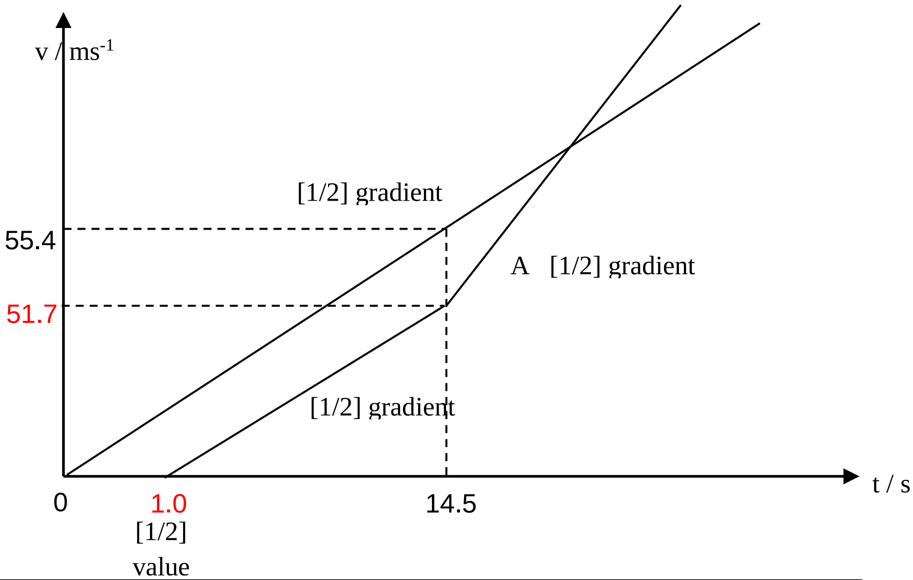


Fig 3.1

As shown on Fig 3.1, the slope used in the competition is inclined at an angle of 23˚ to the horizontal and can be assumed to be frictionless. Any resistive forces on the skiers can be ignored. The length of the ski used by each skier is 2.0 m.

Due to a longer reaction time, skier A starts down the hill 1.0 s later than skier B. When skier B just reached the 400m mark, skier A doubles his acceleration in an attempt to overtake skier B before they reach the finishing line.

(a) Sketch the variation with time of the velocity for skier A and skier B on the same axes. Label the graphs clearly and include any relevant values on your graph. [4]

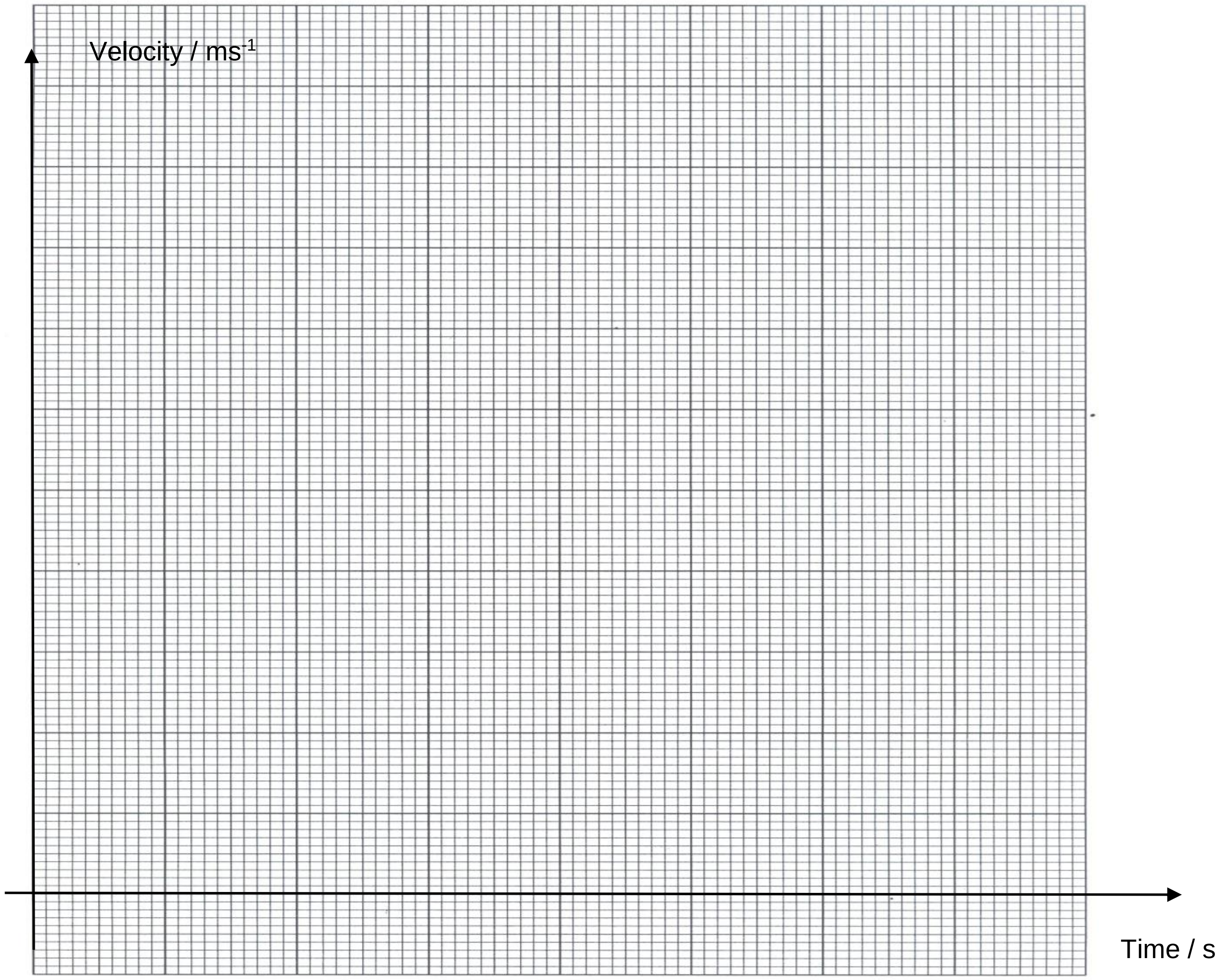


Acceleration of B = initial acceleration of A = g sin23˚ = 3.83 ms^{-2}

Time taken for B to reach 400 m = √{2(400) / 3.83} = 14.5s

Velocity of B at time t=14.5s = 14.5(3.83) = 55.4 ms^{-1}

Velocity of A at time t = 14.5s = (14.5 – 1.0) (3.83) = 51.7 ms^{-1}



(b) Hence or otherwise, determine if skier A can win the competition. [3]

Time taken for B to cover (700 – (3*2.0) = 694m) = √{ 2(694) / 3.83 } = 19.0 s

Distance covered by A in 19.0 s

= ½ (51.7)(13.5) + (51.7 *(19.0 – 14.5) + ½ (2*3.83)(19.0 – 14.5)^2)

= 348.98 +310.21

= 659 m < 694 m

Hence, skier A will not win the race.

Or

Time taken for B to cover 700m = √{ 2(700) / 3.83 } = 19.1 s

Distance covered by A in 19.1 s

= ½ (51.7)(13.5) + (51.7 *(19.1 – 14.5) + ½ (2*3.83)(19.1 – 14.5)^2)

= 348.98 +318.21

= 666 m < 700 m

Hence, skier A will not win the race as not possible for A to be 3 ski lengths ahead of B.

Or

Time taken for B to cover 700m = √{ 2(700) / 3.83 } = 19.1 s

Time taken for A to cover 700m: 14.5 seconds to cover 348.98 m

5.0 seconds to cover (700-348.98)= 351m

Total time for A to cover 700m = 14.5 + 5.0 = 19.5 seconds

Hence skier A will not win as he takes longer to cover 700m

Q2]

4 **(a)** Define *acceleration*.

It is rate of change of velocity with respect to time.

[1]

(b) The graph in Fig. 4.1 shows the variation of the acceleration of a ball bearing being released into a beaker filled with an unknown fluid.

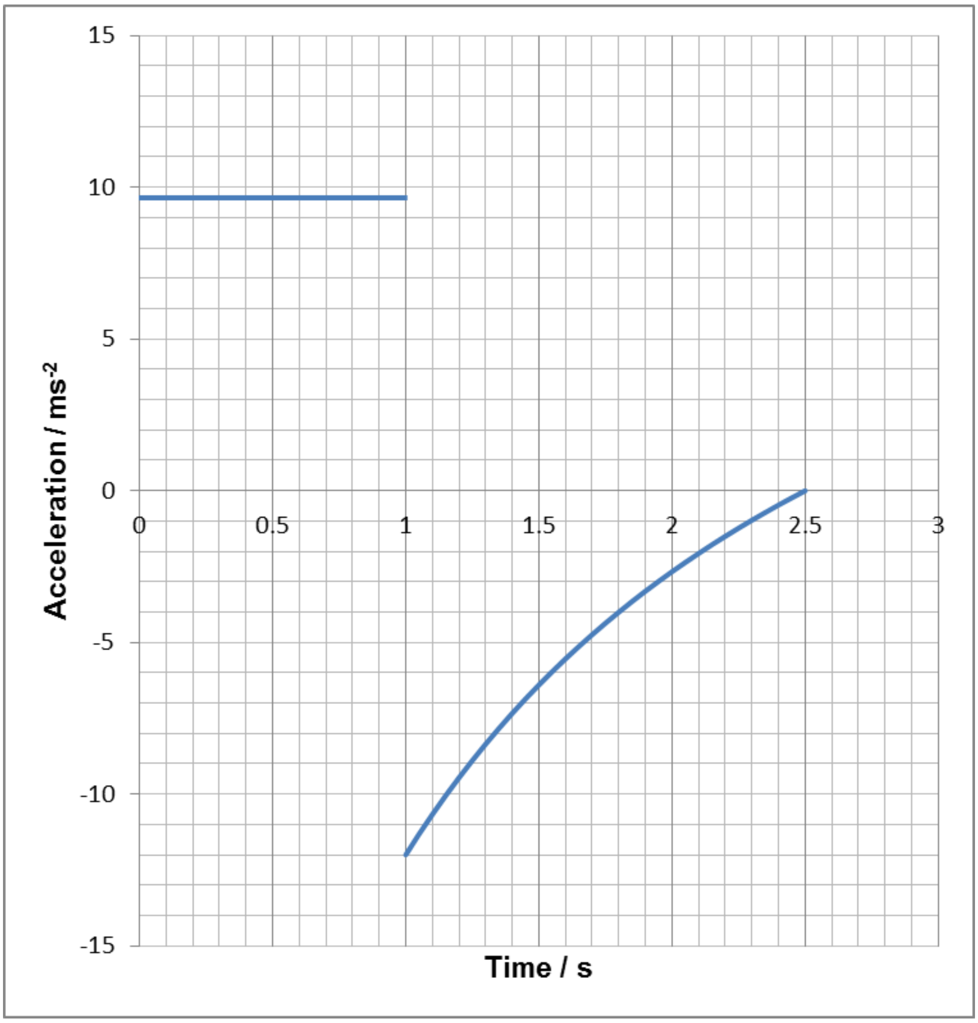


Fig. 4.1

(i) Explain whether air resistance is negligible in Fig. 4.1.

Air resistance is negligible as the acceleration of the ball is 9.8 ms^{-2}, which shows that the net force acting on the ball is weight of ball only. (or since acceleration is 9.8 ms^{-2} which is equal to the gravitational acceleration, the net force acting on ball is weight only.)

[1]

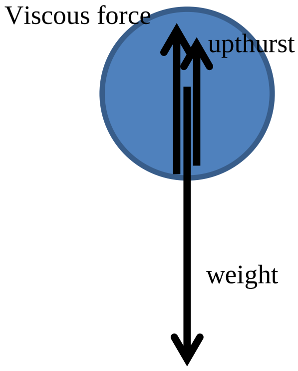
(ii) At the time of 2.5 s, the acceleration of the ball is zero. Explain whether this means that the ball has reached the bottom of the beaker.

No. Area under acceleration time graph gives us the change of velocity. Since area under the graph of acceleration time graph from t=0 to t=2.5s is not zero, it means that the ball has not reached the bottom.

Note: If the ball reach the bottom, change of velocity $=v_f-v_i = 0$(at t=0s, v=0, when ball reach bottom. v=0)

[1]

(iii) Draw a free body diagram indicating all forces acting on the ball bearing at the time of 2.5 s.



[1]

(iv) Determine the magnitude of the highest velocity of the ball bearing.

Highest velocity = area under acceleration-time graph for t between 0 to 1s,
$= (9.81)(1) = 9.81\ ms^{-1}$.
Or using $v= u + at$ where $u = 0$
$v = 0 + 9.81(1) = 9.81\ ms^{-1}$

velocity = ... m s^{-1} [2]

(v) Determine the terminal velocity of the ball bearing.

Terminal velocity occurs at t = 2.5 s.
Terminal velocity = area under acceleration-time graph for t between 0 to 3 s,
= 9.81 - area under acceleration-time graph for t between 1 to 3s
= 9.81 - 73(1)(0.1)
$= 2.51\ ms^{-1}$

velocity = ... m s^{-1} [2]

(vi) Sketch a velocity time graph for the motion of the ball in Fig. 4.2. Indicate all relevant values on your graph. [2]

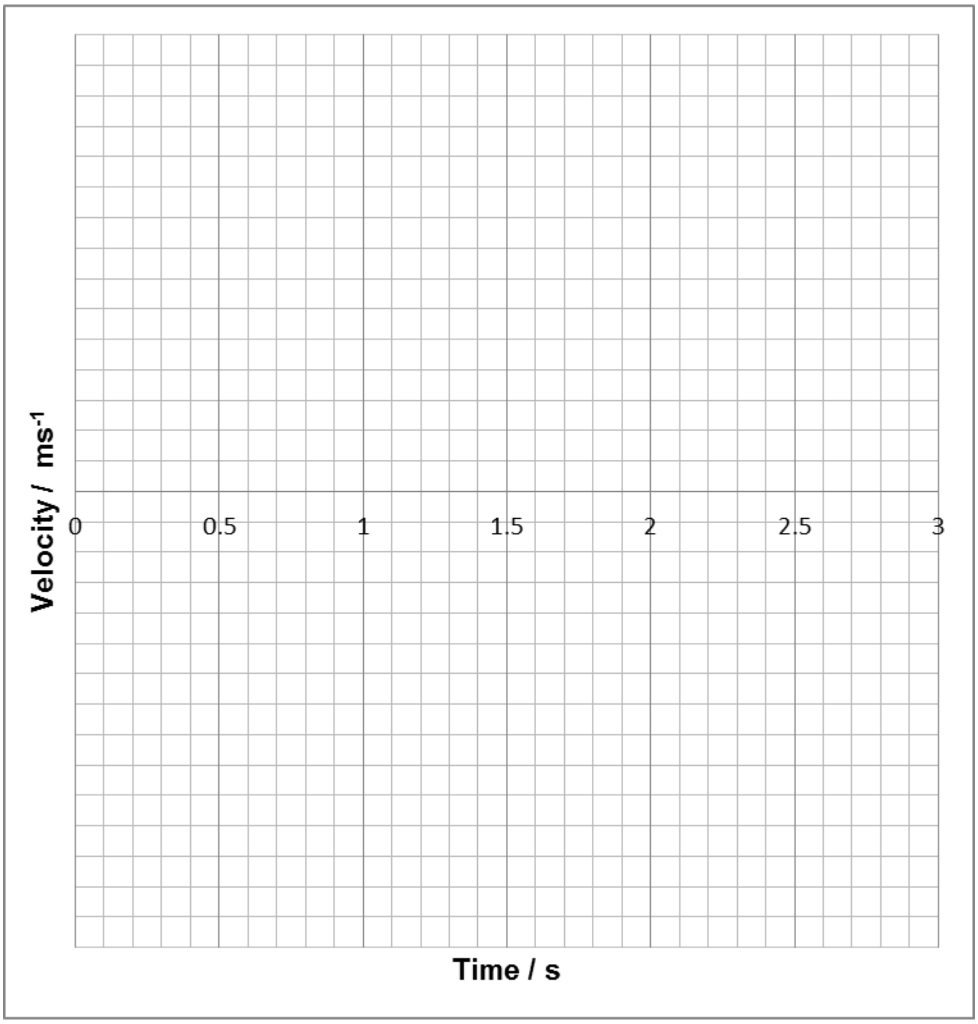
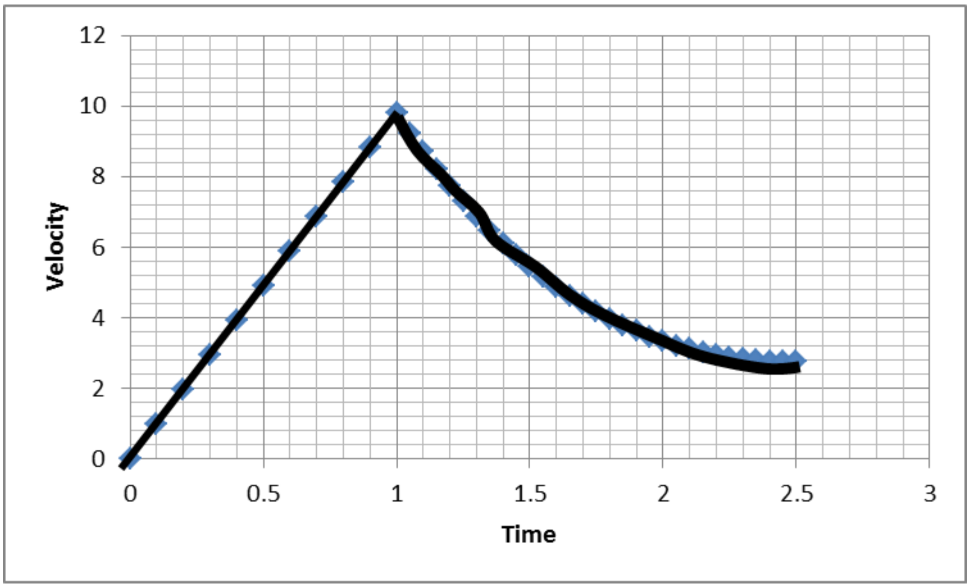


Fig. 4.2



[NJC 2013 P2 Q1]

5 A hot air balloon was rising steadily at a speed of 10.0 m s^{-1} when weather conditions turned windy. A constant breeze of 3.0 m s^{-1} blew horizontally across the sky, which caused the hot air balloon to travel with a resultant velocity of v_R at an angle θ to the horizontal, as shown in Fig. 5.1 below.

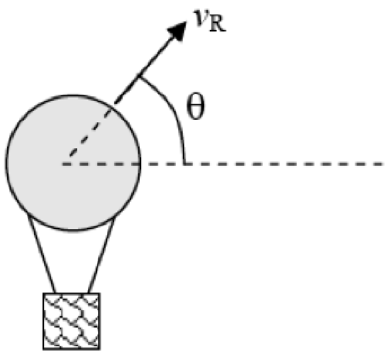


Fig 5.1

(a) Calculate the resultant velocity v_R.

Resultant velocity = $\sqrt{(3.0^2 + 10.0^2)}$ = 10.4 m s^{-1}

Direction $\theta = \tan^{-1}(10/3)$

= 73.3° above the horizontal

velocity = ... m s^{-1}

angle θ = ... ° [2]

(b) A sandbag was dropped from the balloon. Determine how far below the balloon the sandbag will be after 4.0 s, assuming that it had not landed on the ground. (Assume that the dropping of sandbags did not affect the velocity of the hot air balloon and that effects of air resistance on the sandbags were negligible.)

For sandbag,

$s_y = u_y t - \frac{1}{2} g t^2$

$= 10.0(4.0) - \frac{1}{2}(9.81)(4.0^2)$

$= -38.5$ m

(-ve sign means it was displaced downwards from point of release)

For hot air balloon, it continues to move upwards at constant velocity = 10 m s^{-1}. Thus displacement s = ut

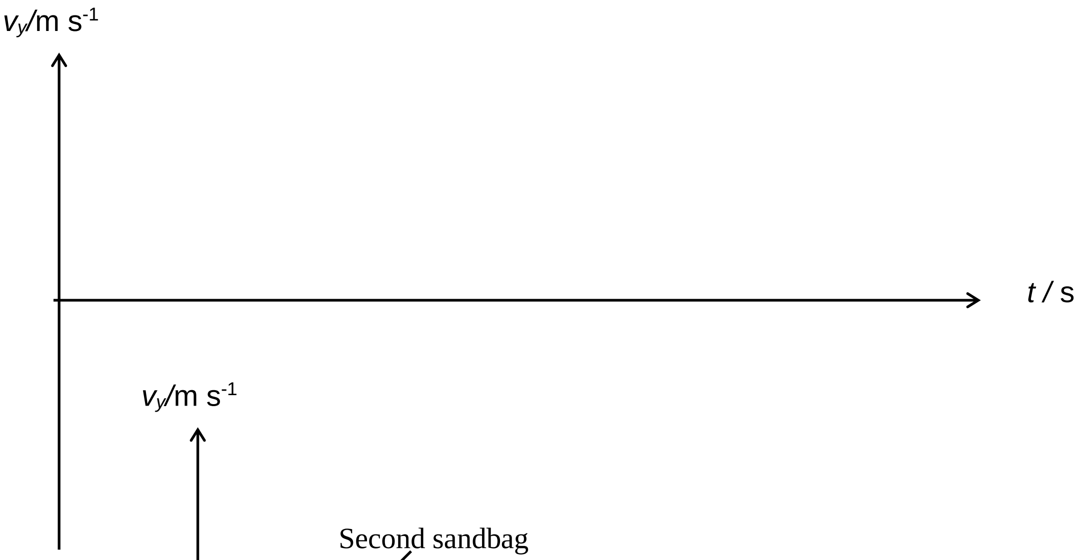
= 10.0 x 4.0

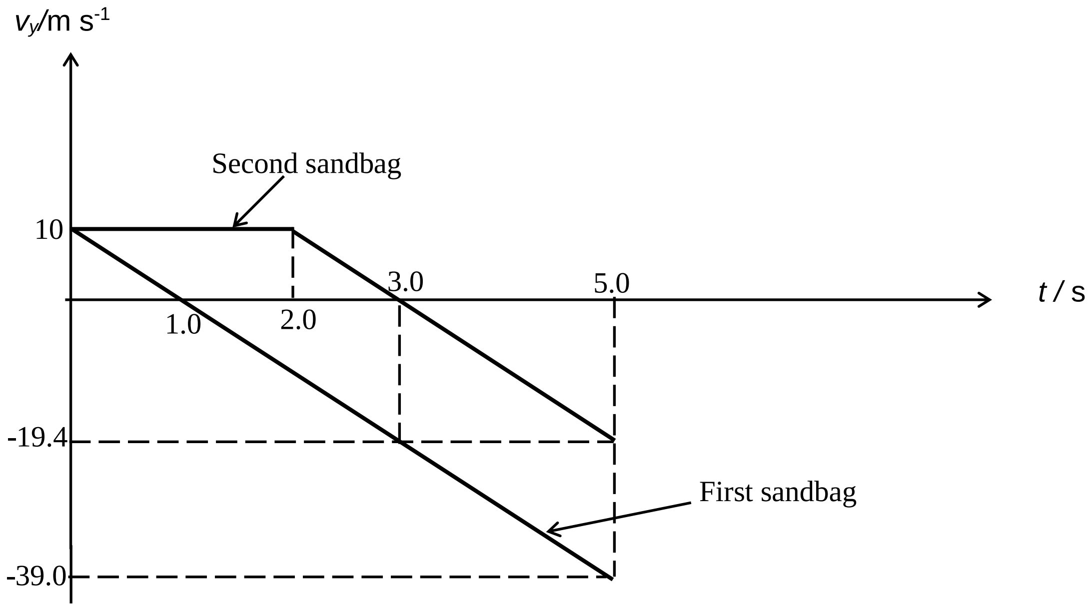
= 40.0 m.

Thus total distance between balloon and sandbag = 40.0 + 38.5 = 78.5 m

distance = ... m [3]

(c) Another sandbag was dropped 2.0 s after the first. Considering only the vertical velocities v_y, sketch the v_y against time t graph of the two sandbags from the time the first sandbag was released to 3.0 s after the second sandbag was released on the axes below. Label your graphs clearly. Appropriate values should be indicated. [4]





(d) Using the graphs or otherwise, calculate the vertical distance of the second sandbag above the first sandbag 3.0 s after the second sandbag was released.

For first sandbag after t = 5.0 s,

Displacement, $s_1 = 10 \times 5.0 - \frac{1}{2}(9.81)(5.0^2)$
$= -72.6$ m

For 2nd sandbag, displacement, s_2 = area under graph
$= 10 \times 2.0 + \frac{1}{2}(10)(1.0) - \frac{1}{2}(19.4)(2.0)$
$= 5.6$ m

(alternative: $s_2 = 10 \times 2.0 + (10 \times 3.0 - \frac{1}{2}(9.81)(3.0^2)) = 5.6$ m)

Hence total distance between them = 72.6 + 5.6 = 78.2 m.

distance = ... m [2]

[VJC 2012 P3 Q1]

6 **(a)** Define *acceleration*.

Rate of change of velocity with respect to time. [1]

(b) A stone is projected from a cliff on a faraway planet. It travels from point A, through point B and to point D as shown in Fig. 6.1. The horizontal dotted line AB is at a vertical distance of 1.1 m above the horizontal dotted line CD. The initial velocity of the stone is 5.0 m s^{-1} at 30^{o} to the horizontal. Assume air resistance is negligible.

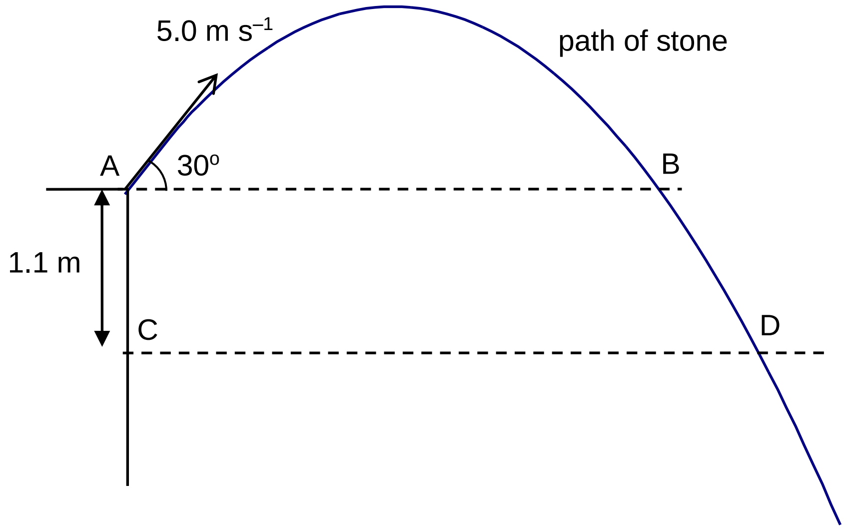


Fig. 6.1

Fig. 6.2 shows the variation of the stone's vertical velocity with time.

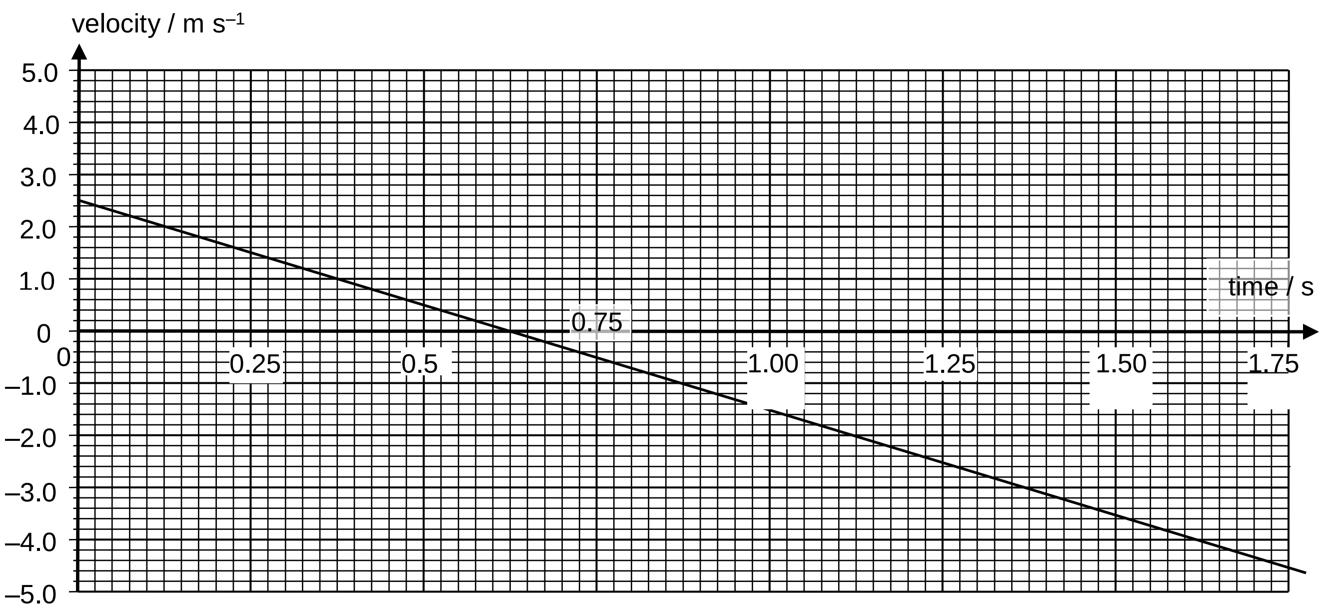


Fig. 6.2

(i) Determine the acceleration in the vertical direction.

Vertical acceleration $= -5.0 / 1.25$

$= -4.0 \text{ m s}^{-2}$

acceleration = ... m s^{-2} [2]

(ii) Calculate the vertical velocity of the stone at point D.

Taking upward as positive,

$v^2 = u^2 + 2as$

$v^2 = (5.0 \sin 30^o)^2 + 2(-4.0)(-1.1)$

$v = -3.9 \text{ ms}^{-1}$

velocity = ... m s^{-1} [2]

(iii) Calculate the time of flight from point A to point D.

Taking upward as positive,

$v = u + at$

$-3.9 = 5.0 \sin 30^o + (-4.0)\, t$

$t = 1.6$ s

time = ... s [2]

(iv) Shade on Fig. 6.2 the area corresponding to the vertical displacement between point B and D. [1]

Area bounded by the line between 1.25 s and 1.60 s.

(v) Mark with an 'X' on the line in Fig. 6.2 when the direction of velocity is 45º with respect to the vertical. Briefly explain your answer.

'X' marked at velocity of -4.3 m s^{-1}.

When the velocity is 45º with respect to the vertical, both vertical and horizontal velocity are of the same magnitude,

i.e. $v_x = v_y = 5.0 \cos 30^o = 4.3$ m s^{-1}.

[2]

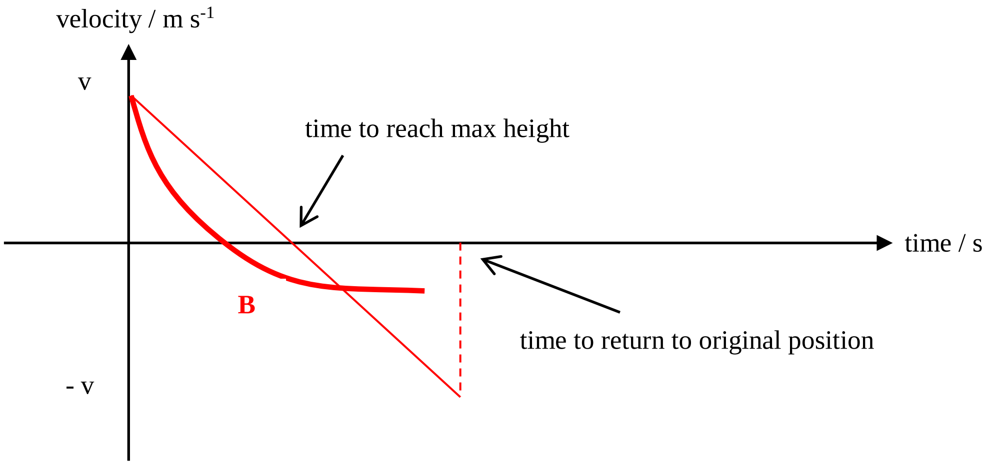
[HCI 2013 P2 Q2]

7 **(a)** An object is thrown vertically upwards with initial velocity *v* near the surface of Earth. Air resistance can be neglected.

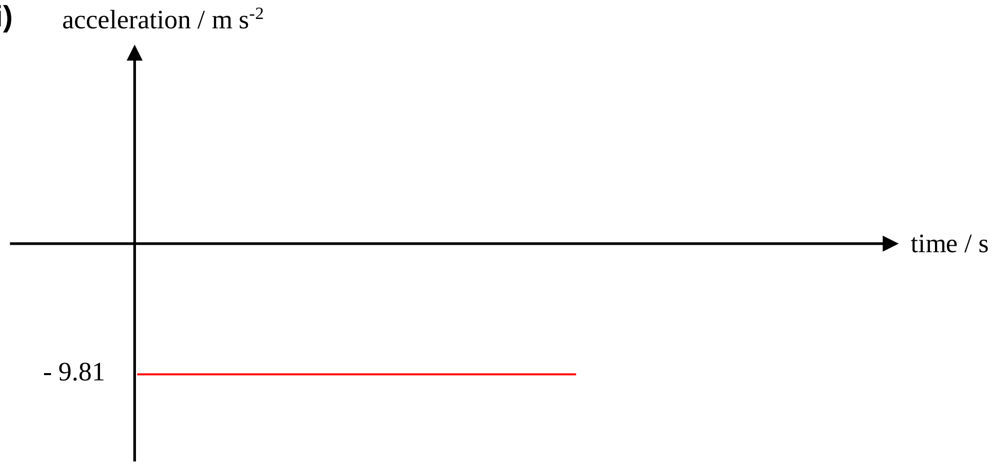
Sketch labelled graphs on the axes below to show how **(i)** the velocity, and **(ii)** the acceleration of the object, varies with time.

Mark on the graphs the time, t_1, at which the object reaches maximum height and the time, t_2, at which it returns to its original position. [3]

(i)



(ii)



(b) When air resistance is not negligible, the object will experience drag force in motion. Sketch on the velocity – time graph in **(a)(i)** above, how the velocity of the object varies with time when air resistance is present. Label this sketch **B**. [2]

(c) A water fountain shoots a jet of water at 5.0 m s^{-1} towards a target a distance away as shown in **Fig 7.1**.

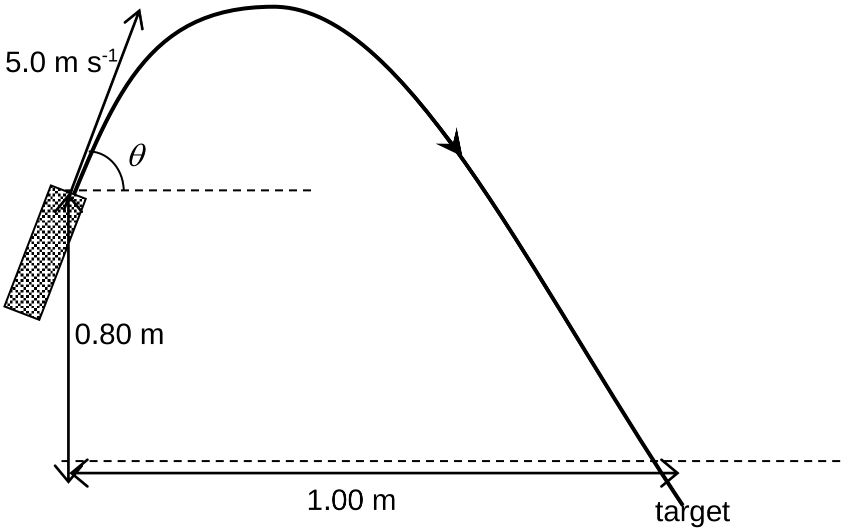


Fig 7.1

(i) State the relationship of vertical displacement, s_y, and horizontal displacement, s_x, in terms of t and θ. [2]

$s_y = u_y t + ½ a_y t^2$

$-0.80 = 5 \sin\theta\ t - ½ \times 9.81 \times t^2$ --- (1)

$s_x = u_x t + ½ a_x t^2$

$1.00 = 5.0 \cos\theta\ t$ --- (2)

(ii) Hence, determine the required projection angle θ of the jet of water for it to hit the target.

You may wish to use the following relationship: $\dfrac{1}{\cos^2\theta} = 1 + \tan^2\theta$

sub (2) in terms of t in (1)

$-0.80 = \tan\theta - 0.1962(1/\cos\theta)^2$

Using the relationship given($\frac{1}{cos^2\theta} = 1 + tan^2\theta$)

$-0.80 = \tan\theta - 0.1962 (1 + \tan^2\theta)$

$-0.80 = \tan\theta - 0.1962 - 0.1962 \tan^2\theta$

$0.1962 \tan^2\theta - \tan\theta - 0.6038 = 0$

$\tan\theta = 5.64227$ or -0.545431 (rej)

$\theta = 79.9^o$

θ = ... ° [3]

[MJC 2013 P3 Q1]

8 The acceleration – time graph for a toy rocket from launch is as shown in Fig. 8.1. On launch, highly pressurised gas is ejected from the rocket.

The rocket leaves the ground at time t = 0 and the remains of the rocket reach the ground at time t = 6.0 s.

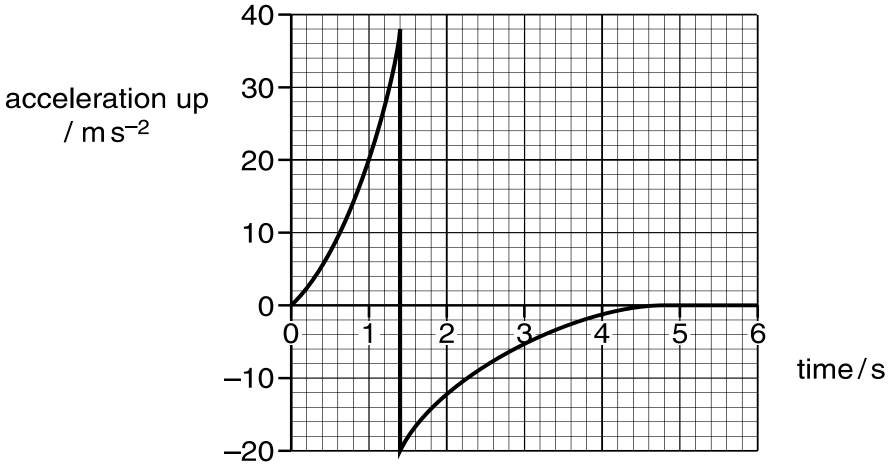


Fig. 8.1

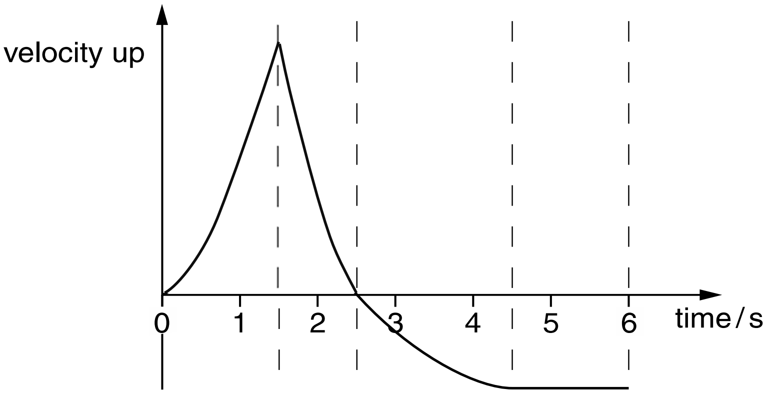


Fig. 8.2

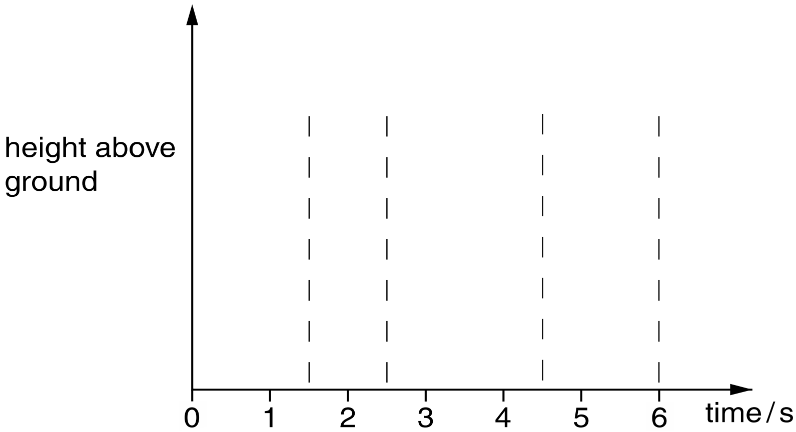


Fig. 8.3

(a) **(i)** State *Newton's second law of motion*.

Newton's second law states that <u>rate of change of linear momentum</u> of a system is (directly)<u> proportional to the resultant force </u>acting on the system and is <u>in the same direction as the force.</u> .... [1]

(ii) Explain how the ejection of gas from the rocket provides the thrust upwards on the rocket.

The <u>rocket exerts a downward force on the gas</u> to eject it out of the rocket. By Newton's third law, <u>the gas exerts an equal and opposite force on the rocket</u>, launching it upwards. .... [1]

(iii) Explain why the acceleration suddenly changes from positive to negative.

The rocket <u>has used up all its fuel</u> <u>and there is no longer an upward force</u> acting on the rocket. The <u>net force now acts downwards, </u>hence the acceleration switches from positive to negative. .... [1]

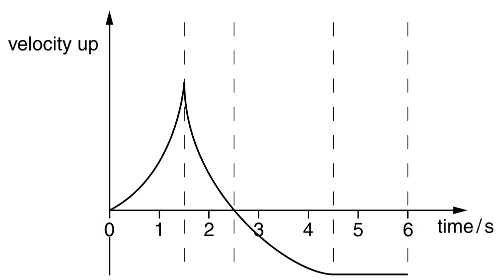
(iv) State the **two** forces acting downwards which cause the negative acceleration of magnitude 20 m s^{-2}.

1 weight of the rocket or gravitational force

2 air resistance/drag .... [1]

(b) The corresponding velocity-time graph is shown in Fig. 8.2.

(i) Mark with a **P** on Fig. 8.2 the point at which the rocket reaches maximum height. [1]



(ii) Use the acceleration-time graph (Fig. 8.1) to find an approximate value of the maximum velocity of the rocket.

Maximum velocity = <u>Area under the acceleration-time graph from t = 0 to t = 1.4 s</u>

= ½ (0.6)(10) + ½ (10+20)(0.4) + ½ (20 + 38)

= 20.6 = 21 m s^{-1}

<u>Alternatively</u>: find area using counting of squares

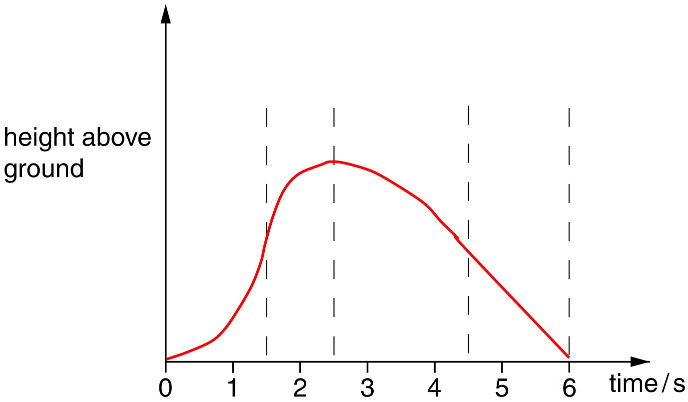
Number of small squares = 45 to 50

Area of each small square = 0.4 m s^{-1}

Accepted range = 18 to 22 m s^{-1}

2]

(c) Use the information from Fig. 8.2 to sketch the shape of the height-time graph on Fig. 8.3. [3]



height is increasing with increasing gradient from t = 0 to t = 1.4 s

height continues to increase from t = 1.4 to 2.5 s but with decreasing gradient

height decreases from t = 2.5 to 4.5 s with increasing gradient (negative)

height decrease constantly from t = 4.5 to t = 6 s (straight line)

Final height = 0

[HCI 2012 P2 Q1]

9 A howitzer shown in Fig. 9.1 has a mass of 5600 kg.

Fig. 9.1

(a) A military truck is towing an unloaded howitzer at a uniform speed of 30.0 km h^{-1}. The howitzer is connected to the rear of the truck by a horizontal tow-bar, which can sustain a maximum force of 40.0 kN. Total resistive forces acting on the howitzer are 10.0 kN. Calculate the shortest time the howitzer can be brought to rest safely.

$v = 8.33$ m s^{-1}

$F_{net} = F_{max} = F - \text{friction}$

$$F_{max} = 40.0\,\text{kN} = \frac{\Delta\rho}{\Delta t} = \frac{5600(8.33)}{\Delta t} - 10.0\,\text{kN}$$

$\Delta t = 0.933$ s

time = ... s [2]

(b) The howitzer is now loaded with an ammunition round of mass 43.0 kg. Before firing, the two rear extensible legs of the howitzer are secured to the ground. When fired, the round has an exit velocity of 563 m s^{-1}.

(i) Explain why there is a need to secure the rear legs of the howitzer to the ground.

To minimize the distance the howitzer moves backwards, as a result of the recoil.

.... [1]

(ii) The barrel of the howitzer can be elevated to an angle above the horizontal. Calculate the horizontal recoil speed of the howitzer if it is not secured to the ground as it fires a round at an angle of 25° to the horizontal.

By conservation of momentum,

$m_{howitzer}v_{howitzer} + m_{round}v_{round} = 0$

$v_{round,horizontal} = 563\cos 25° = 510.25 \text{ m s}^{-1}$

$(5600)(v_{howitzer}) = (43)(510.25)$

$v_{howitzer} = 3.92 \text{ m s}^{-1}$ (backwards)

speed = ... m s^{-1} [3]

(iii) The two rear extensible legs can be assumed to undergo only compressive force whenever the artillery is fired. Given that the recoil force acting on each leg is 110 kN, as shown in Fig. 10.2, determine the spring constant of each of the extensible legs, if each leg is compressed by 0.50 cm.

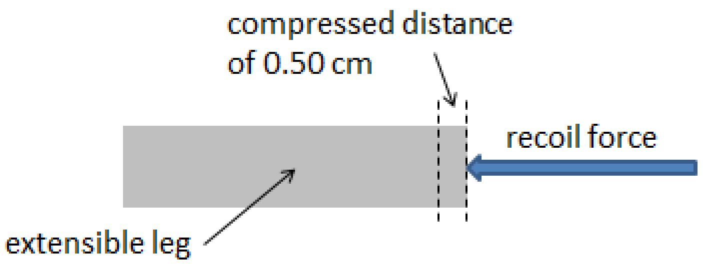


Fig. 10.2

$220 \text{ kN} = 2F = 2kx = 2(k)(0.00500)$

$k = 2.2 \times 10^{7} \text{ N m}^{-1}$

spring constant = ... N m^{-1} [2]

[RVHS 2013 P3 Q1]

10 Two rubber balls are moving toward each other in an elastic head-on collision as shown in Fig 10.1. The mass of ball **A** and **B** are 1.7 kg and 1.6 kg respectively.

Fig. 10.1

(a) Given that the initial speed of ball **A** and ball **B** are 10.0 m s^{-1} and 3.0 m s^{-1} respectively before collision, determine the final speed and direction of the balls after collision.

Let v_A = final velocity of ball A

v_B = final velocity of ball B

Taking rightwards to be positive

By conservation of momentum,

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$1.7(10.0) - 1.6(3.0) = 1.7v_A + 1.6v_B \quad --(1)$$

velocity of approach=velocity of separation

$$10.0 - (-3.0) = v_B - v_A$$

$$v_B = 13.0 + v_A \quad --(2)$$

Solving both equations (1) and (2)

V_A = - 2.61 ms^{-1}

V_B = 10.4 ms^{-1}

Final speed of ball A = 2.61 ms^{-1}

Direction of travel of ball A = towards the left

Final speed of ball B = 10.4 ms^{-1}

Direction of travel of ball B = towards the right

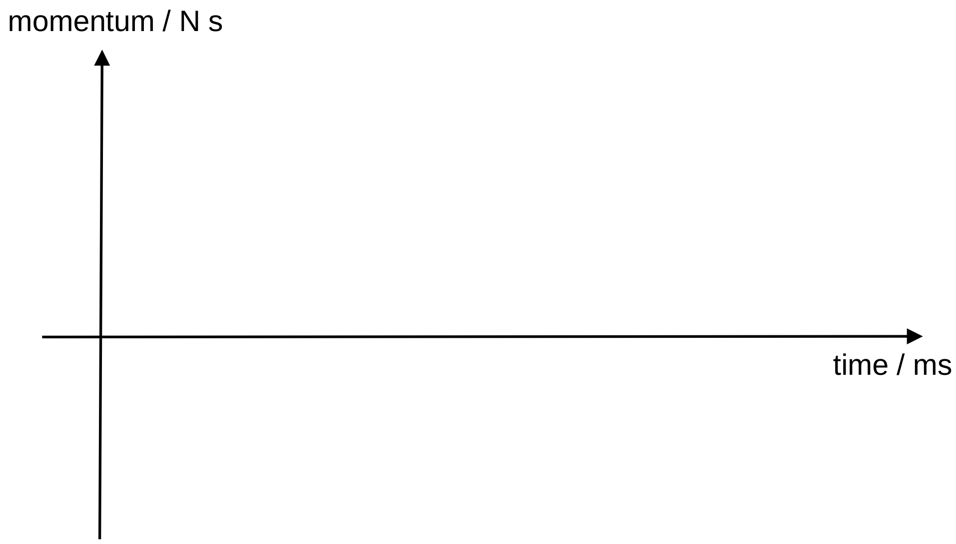
final speed of ball **A** = ... m s^{-1}

direction of travel of ball **A** = ...

final speed of ball **B** = ... m s^{-1}

direction of travel of ball **B** = ... [3]

(b) **(i)** Given that the two balls are in contact for 10.0 ms during collision, draw on the same axes below, the momentum-time graph for ball **A** and ball **B**, before, during and after the collision. [2]



(ii) Sketch on **b(i)** to show how the total momentum varies with time. [1]

A horizontal straight line drawn on the above graph with vertical intercept at 12.2 N s.

(iii) State and explain whether the total kinetic energy remains constant throughout the entire duration of impact.

Some of the kinetic energy is converted to elastic potential energy as the balls are deformed during the impact.

Hence the total kinetic energy does not remain constant throughout the entire duration of impact.

N.B – the total kinetic energy of the two balls is still conserved because the elastic potential energy is converted back to kinetic energy when the balls separate.

[2]

(iv) If the collision is an inelastic one, state and explain how the speeds of the balls after the collision will differ from your answers in **(a)**.

If collision is inelastic, there would be loss of energy and the total kinetic energy of the system is not conserved.

Hence the final speed of ball A and ball B would be lesser than calculated in (a).

[2]

[MJC 2013 P3 Q2]

11 Two blocks, P and Q, of masses 0.30 kg and 1.50 kg respectively, are connected by a string that passes over a pulley as shown in Fig. 11.1. The pulley is frictionless and the string is inelastic. The system is released from rest. Block Q falls vertically before it strikes a spring that is firmly attached to the floor. The spring constant is 500 N m^{-1}.

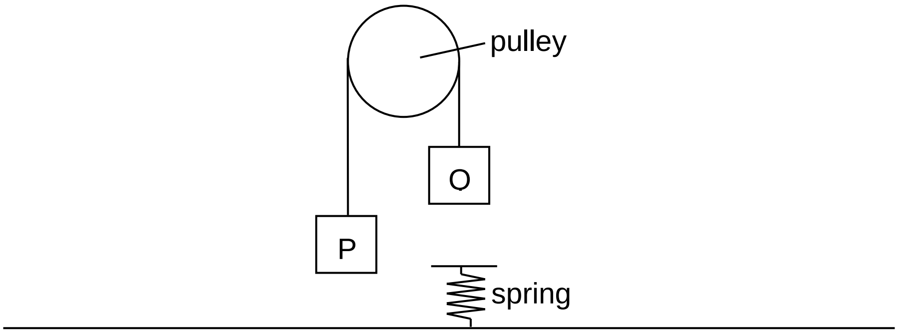
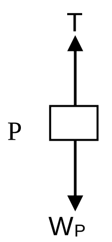
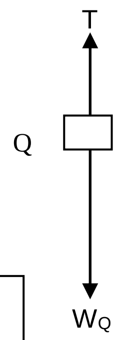


Fig. 11.1

(a) **(i)** Draw the free-body diagram of Blocks P and Q at the instant when the system is released from rest.





Note:

- $T > W_P$
- $W_Q > T$
- T are of equal length

[2]

(ii) Determine the magnitude of acceleration of Block Q before striking the spring.

<u>Method 1</u>

$T - W_P = m_P a$ ------------------ (1)

$W_Q - T = m_Q a$ ------------------ (2)

Solving the simultaneous equations,

$$(1.50)(9.81) - \left[(0.30)(9.81) + 0.30a\right] = 1.50a$$

$$a = \frac{11.772}{1.80} = 6.54 \text{ m s}^{-2}$$

acceleration = ... m s^{-2} [3]

(iii) Hence, determine the tension in the string before Block Q strikes the spring.

From equation (2) in 1(a)(ii),

T = 0.30(9.81) + 0.30(6.54) = 4.905 = 4.91 N

tension = ... N [1]

(b) The acceleration of Block Q decreases after it touches the spring. Block Q comes to a stop after some time and the spring is observed to be compressed. Calculate the compression of the spring.

After Block Q comes to a stop, the forces acting on P and Q are in equilibrium.

$$T' = W_P \text{ ------------------ (1)}$$

$$T' + kx = W_Q \text{ ------------------ (2)}$$

Substitute (1) into (2)

$$0.30g + (500)x = 1.50g$$

$$x = \frac{1.20 \times 9.81}{500} = 0.0235 \text{ m}$$

compression = ... m [2]

(c) The spring in Fig. 11.1 is replaced by a volume of fluid with density ρ as shown in Fig. 11.2. When Block Q is in the fluid, it floats horizontally in the liquid. Block Q floats when its lower face is at a depth d in the fluid. Block Q has a volume of V, and a cross sectional area of A.

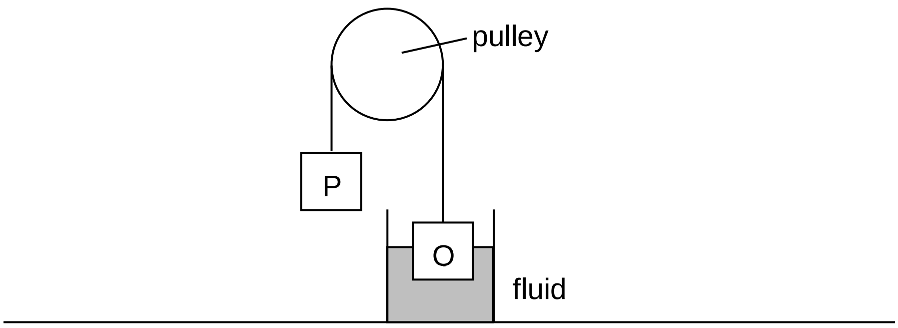


Fig. 11.2

(i) Find the expression for the depth *d*.

At equilibrium, $T'' = W_P$ ------------------ (1)

$T'' + U = W_Q$ ------------------ (2)

Solving (1) and (2),

$0.30g + Ad\rho g = 1.50g$

$d = \dfrac{1.2}{A\rho}$

d = ... [2]

(ii) Explain how the forces acting on Block Q cause Block Q to be in translational equilibrium.

There are 2 **upward forces** (upthrust and tension) and 1 **downward** force (weight) acting on Block Q.

As the upward forces are **equal** to the weight of Block Q, there will be no net force that acts on Q.

.... [2]

[AJC 2012 P3 Q1]

12 An object is situated in a field of force.

(a) **(i)** Explain what is meant by the term *field of force*.

region of space where a force will be felt by the particle when placed in the field or the force which is experienced by two bodies(of the same type) which are not in contact. [1]

(ii) State two similarities for gravitational, electric and magnetic fields.

1 All explain action at distance

All define as forces per unit something

2 Field lines never cross

Density of lines indicates relative strength of field [2]

(b) **Deduce** the nature of the force-field (magnetic, electric or gravitational) when the force on the object

(i) is along the direction of the field regardless of its velocity and charge.

Since indep on vel and charge and along direction of field

Ans: gravitational field [1]

(ii) is independent of the magnitude of its velocity but depends on its charge.

Since indep on velocity, E or g field and dep on charge, E or B-field

Or since dep on charge, E or B –field but indep on vel [1]

Ans: E-field

(iii) depends on both its velocity and charge of the particle.

Since dependent on charge, E and B-field

Also dependent on vel, [1]

Ans: B-field

(c) State the property of the object such that it experiences a force opposite to the direction of the field when placed in a **magnetic field**.

S-pole of the object [1]

(d) Fig. 12.1 shows a beam of particles is directed horizontally into vertical gravitational, electric and magnetic field in turn

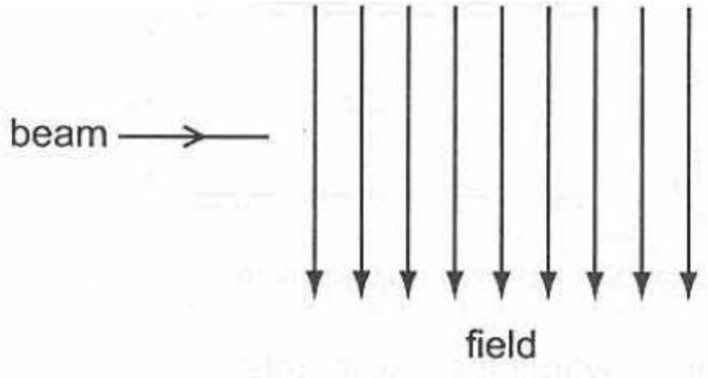


Fig. 12.1

If the beam of particles are beam of protons, complete the table below which shows feature of the force on the beam and the shape of the beam in each case.

	Gravitational field	**Electric field**	**Magnetic field**
Force	negligible	Downwards	Into the page
Shape of beam	a horizontal line in plane of page	a downward (parabolic) curve in plane of page	A horizontal circle perpendicular to plane of page.

[3]

[ACJC 2012 P3 Q1]

13 **(a)** A lump of pure ice floats on pure water in a beaker, as shown in Fig. 13.1.

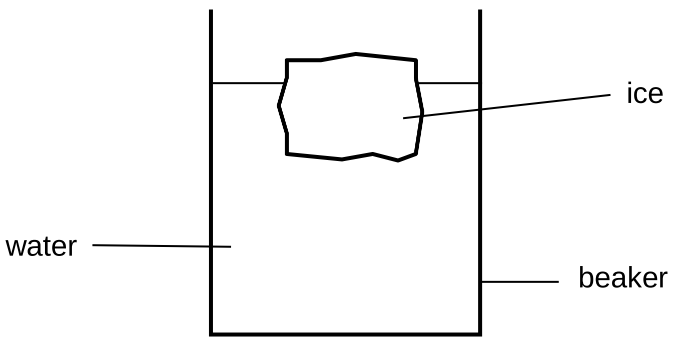


Fig. 13.1

(i) State, qualitatively, the relation between

1 the mass of the ice and the mass of the displaced water,

.... equal .... [1]

2 the density of ice and the density of water.

density of ice is less

.... [1]

(ii) A student marks the level of water surface in the beaker and then observes the level as the ice melts. State and explain qualitatively the change, if any, in this level during the melting.

mass of ice becomes equal mass of water after melted/mass of melted ice equal mass of displaced water

density of water from melted ice equals density of displaced water/
volume of melted ice equals volume of water displaced /
melted ice fills the space of water displaced by ice

so level does not change

.... [3]

(b) A heavy anchor of volume 0.50 m^3 and density 7800 kg m^{-3} lies at the bottom of the seabed. A fisherman intends to use a lifting bag to raise the anchor from the seabed. Take density of sea water to be 1030 kg m^{-3}.

(i) Determine the upthrust acting on the anchor.

Upthrust = weight of water displaced by the anchor
= density of water x volume of anchor x g
= $1030 \times 0.50 \times 9.81$
= 5050 N

upthrust = ... N [2]

(ii) Estimate the volume of air that needs to be released into the lifting bag suddenly in order that the initial acceleration of the anchor is 2.50 m s^{-2}.

Let volume of air be V
Upthrust on lifting bag + upthrust on anchor – weight of anchor = ma

$$(1030)(V)(9.81) + 5052 - (7800)(0.50)(9.81) = (7800)(0.50)(2.50)$$

V = 4.25 m^3

volume of air = ... m^3 [2]

[DHS 2013 P2 Q1]

14 **(a)** Fig 14.1 shows three beakers. Similar objects are placed inside the beaker for Case B and Case C. In all three cases, the water level in the beaker is always filled to the brim. The three beakers are each placed on a weighing scale. Arrange the three cases in ascending order of their weighing scale reading. Explain how you have reached your conclusion.

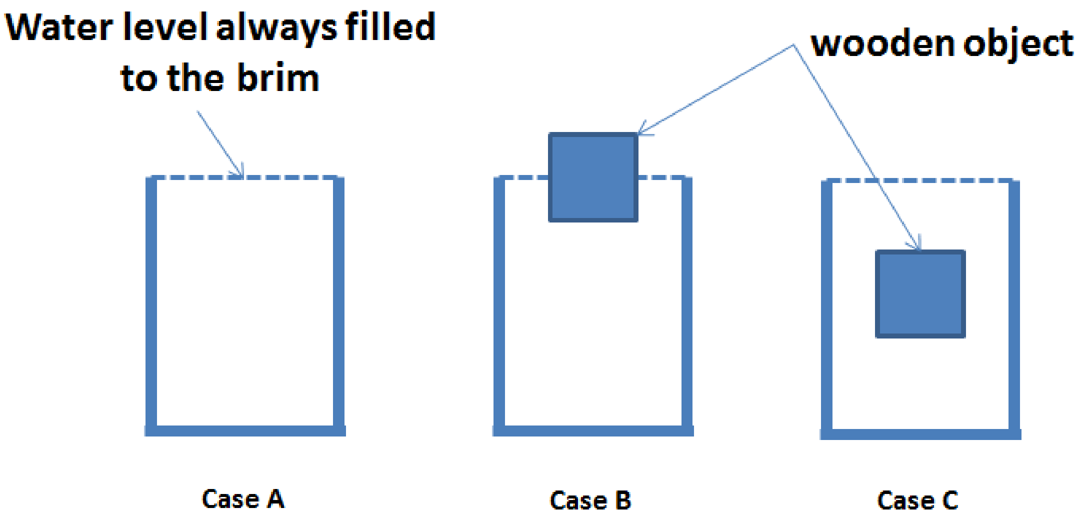


Fig 14.1

Their weights will all be the same. [no marks]

The reading on the scales of Case B and C are:

Weight of water in the Beaker + Reaction Force to Upthrust on the wooden object.

Or

the combined weight of water/beaker and block.

The weight of the wooden object = Upthrust on the wooden object = to the weight of the fluid displaced.

As the fluid displaced + the fluid in the beaker gives back the original Case for A, all cases give the same reading.

[2]

(b) A beaker and weighing scale are placed in an elevator as shown in Fig 14.2. The solid object in the beaker is suspended from a string hung from the elevator's ceiling.

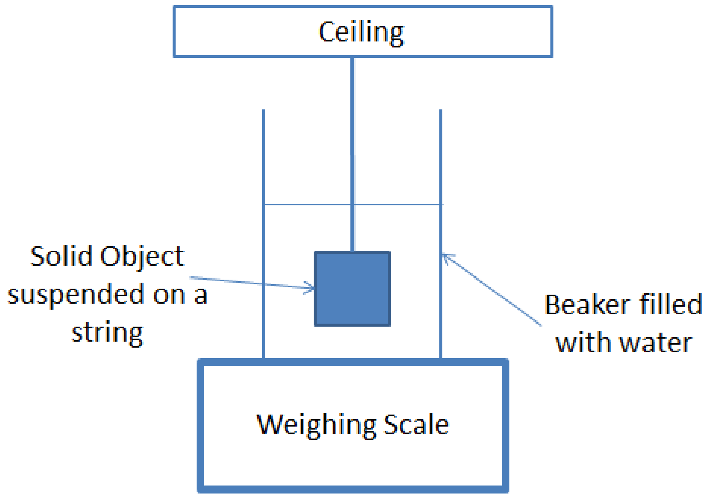


Fig 14.2

Draw a free body diagram showing all the forces

(i) on the solid object in Fig. 14.3a [1]

(ii) on the beaker in Fig. 14.3b [1]

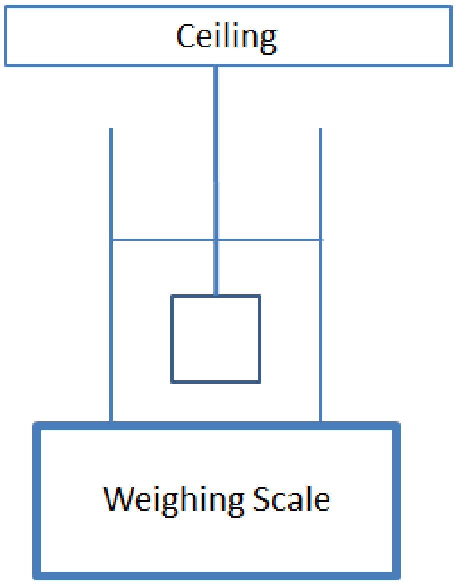


Fig 14.3a

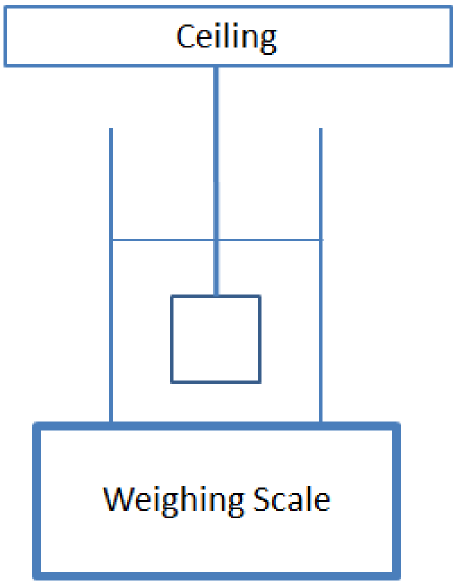
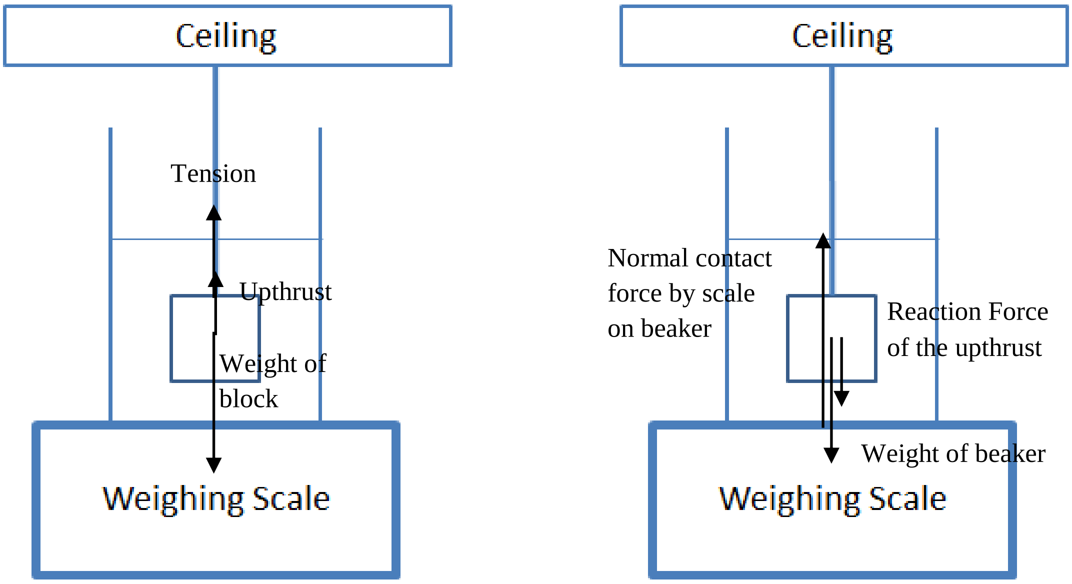


Fig 14.3b



(c) (i) The beaker of water is placed on a balance and the weight indicated is **X**. A solid object has weight **Y** in air and it displaces weight **Z** of water when immersed. Express, in terms of **X**, **Y** and **Z,** the **change** in the reading of the weighing scale after the string is cut and the object falls to the bottom of the weighing scale.

Considering the system (beaker and block) as a whole,

Initial case (before cutting string):

Normal contact force = weight of beaker + weight of block – tension

Final case (after cutting string):

Normal contact force = weight of beaker + weight of block

Difference is tension which is Y – Z (can be deduced from (b)(i) that tension = weight of block – upthrust)

[1]

(ii) The solid object of mass *m* is now resting on the bottom of the beaker. The elevator malfunctions and falls downwards. Express, in terms of **X,Y,Z** and ***m***, the minimum acceleration of the elevator such that the solid object loses contact with the beaker.

Balancing Forces on the solid object, we have

W – N – U = ma

Solving for a when N = 0, we have a = (Y-Z) / m

[2]

[NJC 2012 P2 Q3]

15 **(a)** State the principle of moments.

The principle of moments states that, for a body to be in rotational equilibrium, the sum of the clockwise moments about any point must be equal to the sum of the anticlockwise moments about the same point.

..... [2]

(b) A 15000 N raft is supported by two ropes as shown in Fig. 15.1. Point A indicates the center of gravity of the raft. The two ropes are 2.0 m apart.

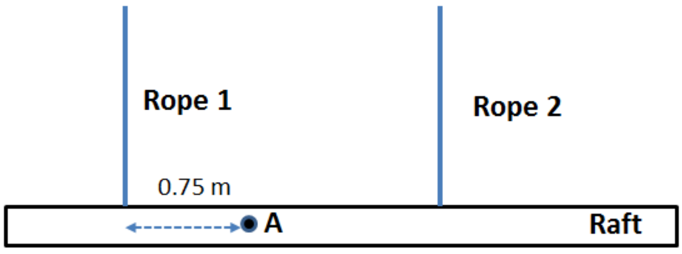


Fig. 15.1

(i) The position of the center of gravity is not at its midpoint. Suggest what this implies about the distribution of mass in the raft.

This suggests that the mass is distributed more heavily towards the left of the boat (near Rope 1)

..... [1]

(ii) Use the principle of moments to determine the tensions in the two ropes.

Taking moments about Rope 1, we have:

$0.75 \times 15000 = 2\text{m} \times T_2$

$T_2 = 5625 = 5.6 \times 10^3$ N

Taking moments about Rope 2, we have:

$(1.25 \times 15000) = 2\text{m} \times T_1$, $T_1 = 9375 = 9.4 \times 10^3$ N

tension in Rope 1 = ... N

tension in Rope 2 = ... N [4]

(c) Rope 2 breaks and the raft falls into the water as shown in Fig. 15.2. The volume of the raft submerged in the water is 0.300 m^3. The density of the boat is and water are 500 kgm^{-3} and 1000 kgm^{-3} respectively.

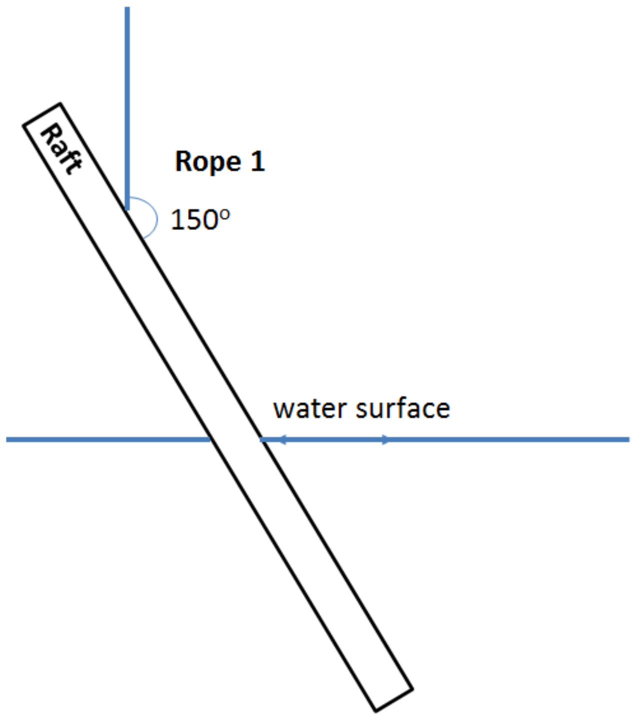


Fig. 15.2

(i) Draw a free body diagram in Fig. 15.2 showing all the forces acting on the raft. [1]

(ii) Calculate the new value of the tension in Rope 1.

Using upward forces = downwards forces, we have:

$W = \rho V g + T$

$15000 = 0.300(1000)(9.81) + T$

$T = 1.21 \times 10^4$ N

tension in Rope 1 = ... N [2]

[NJC 2013 P2 Q2]

16 A simplified drawing of a suspension bridge is shown in Fig. 16.1. The uniform bridge AB itself has a weight W and is supported at its edges A and B as well as by cables at C and D. The length of the bridge is $3d$ where AC = CD = DB = d. A tie joining the top of the pillar to the ground at E and another at F hold the pillars in a vertical position.

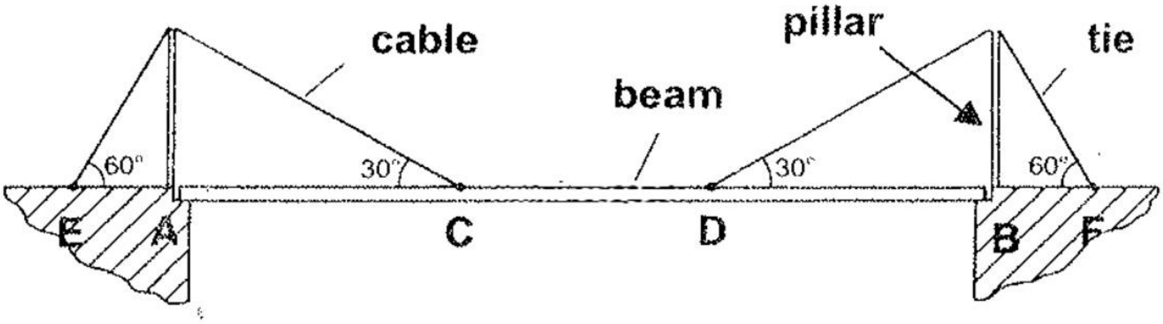


Fig. 16.1

(a) State the conditions necessary for a body to be in equilibrium.

Resultant force acting on the body in any direction is zero.

Resultant moment on the body about any axis is zero.

[2]

(b) **(i)** If the reaction on the bridge at each of the supports A and B is $\frac{W}{4}$ vertically upwards, find the tension, in terms of W, in the *tie*.

The net force acting vertically on the bridge is zero.

$2T\sin 30^\circ + 2\left(\frac{W}{4}\right) = W$ where T is the tension in the cable

$T = \frac{W}{2}$

The net force acting horizontally on the pillar is zero.

$T\sin 60^\circ = T_T \sin 30^\circ$ where T_T is the tension in the tie

$T_T = 0.87\,W$

[4]

(ii) State and explain the advantage of using a cable and a tie to hold the pillar.

The vertical component of the tensions in the cable and tie "pin" the pillar to the ground. Hence it will not be easily pushed to either side by any sideway force.

The tensions in the tie and cable provide moments about the base of the pillar which tend to cancel out, keeping the pillar upright.

[2]

[TJC 2013 P2 Q3]

17 **(a)** Define the terms moment of a force and torque of a couple.

The moment of a force *F* about an axis is the product of that force and the perpendicular distance from the line of action of the force to the axis.

The torque of a couple is the product of the force and the perpendicular distance between the forces.

[2]

(b) A uniform T-shaped structure, of mass per unit length 1.0 kg m^{-1}, is resting against a smooth wall and a rough floor, as shown in Fig. 17.1. Portion PQ, positioned perpendicularly to RS at its midpoint, is 1.5 times the length of RS. RS is given to be 2.0 m long.

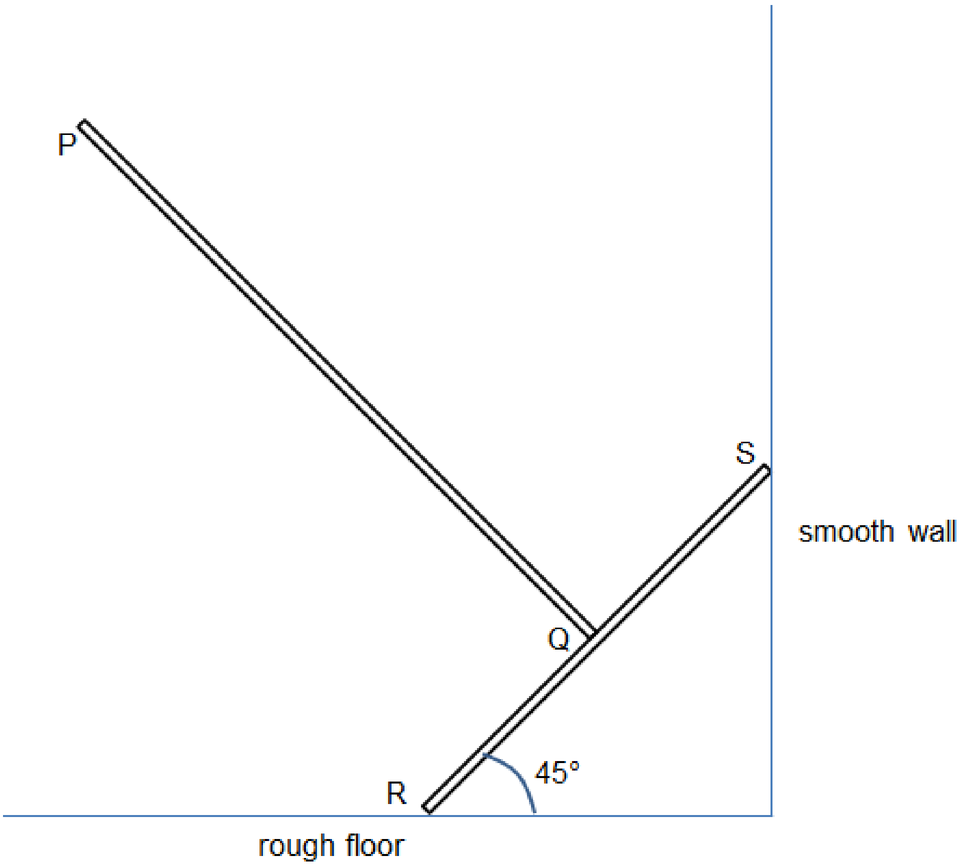


Fig. 17.1

Determine the magnitude of the resultant force the floor acts on the T-structure.

Taking moment about R,

$(3 \times 9.81)(0.5 \cos45°) + N_w(2 \sin45°) = (2 \times 9.81)(1)(\cos45°)$

N_w = 2.4525 N friction of rough floor, Normal reaction of floor = weight of PQ + RS

$N_{resultant}$ = 49.1 N

resultant force = ... N [3]

(c) Now the T-structure is placed on an elastic cable of spring constant 1 MN m^{-1} with the structure still pressing against the smooth wall, as shown in Fig. 17.2. Point R is positioned at the middle of the cable. The T-structure causes the cable to sag, causing a total extension of 0.010 m, making an angle θ to the horizontal. The entire system is suspended above the floor.

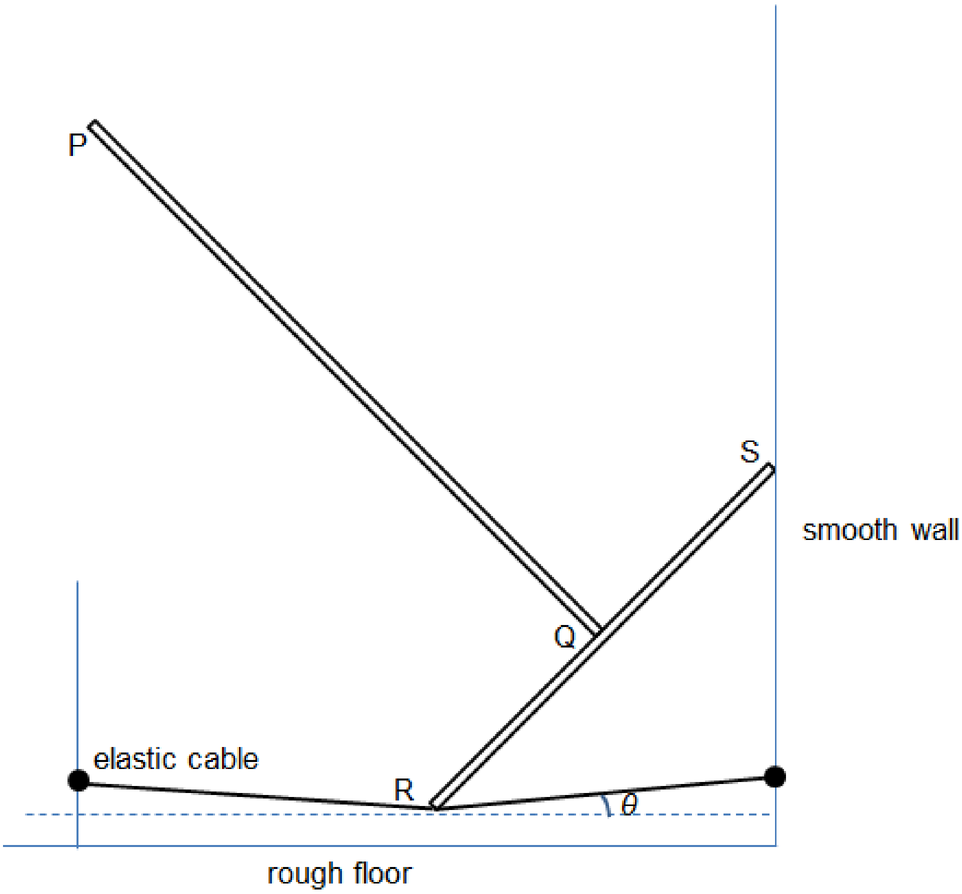


Fig. 17.2

Determine the angle *θ* if the system is in equilibrium.

2Tsinθ = normal reaction of floor = 49.05 N

Using Hooke's Law, T = kx, 2(1 × 10^6)(0.005)sinθ = 49.05

θ = 0.28°

angle = ... ° [3]

(d) Explain why the value of *θ* is unlikely to be zero.

From the equation, 2Tsinθ = weight of PQRS, for θ → 0°, T → ∞, which is impossible.

[1]

[RVHS 2013 P2 Q2]

18 **(a)** State the two conditions necessary for a body to be in equilibrium.

1

The two conditions necessary for a body to be in equilibrium are:

The resultant force on the object is zero, i.e., $\sum F = 0$.

2

The resultant torque on the object about any axis is zero, i.e., $\sum \tau = 0$.

(b) Fig. 18.1 shows a uniform beam AB of weight 2700 N and length 6.0 m which has been hoisted into the air by a crane. The lengths of the ropes AC and BC are both 6.0 m. The tension in AC is T_1 and that in BC is T_2.

A worker, of weight 900 N, was sitting on the beam when it was hoisted and now finds himself hanging onto the beam in mid-air. The worker is at point W where AW = 4.0 m and BW = 2.0 m.

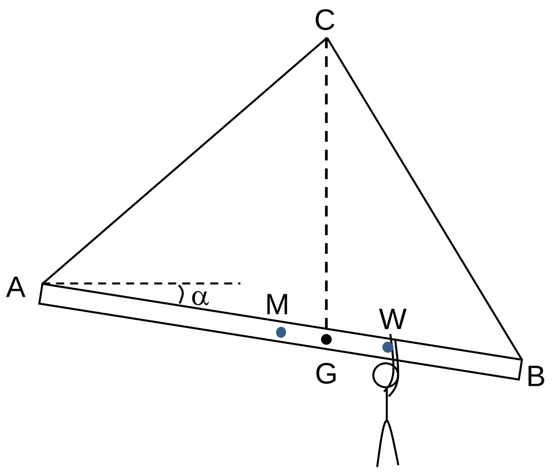


Fig. 18.1

When the beam and the worker are stationary, the beam makes an angle α to the horizontal. The point M is the mid-point of the beam and the point G on the beam is the position of the centre of gravity of the beam and the worker.

(i) Explain why the point G must lie directly below C.

The combined weight of the beam and the worker can be considered to be acting at G. Considering moments about C, moments due to T_1 and T_2 about C is zero as their lines of action pass through C. Hence for the resultant moments about C to be zero, the line of action of the combined weight of the beam and worker must pass through C as well so that the moment of the combined weight about C is zero. Hence the vertical line through G must pass through C. [2]

(ii) Calculate the distances MG and WG.

Considering moments due to the weight of beam, W_b, and reation force on beam by worker, R (=W_w) , about C,

$W_b \times$ (MG cos α) = $R \times$ (WG cos α)

(2700)(MG) = (900)(WG)

WG = 3 MG

MW = AW – AM = 4.0 – 3.0 = 1.0 m

MW = MG + GW = MG + 3 MG = 1.0 m

MG = 0.25 m

WG = 0.75 m

.... m

WG = ... m [3]

(iii) If the angle α is 2.8°, determine the magnitudes of the tensions T_1 and T_2.

<u>Method 1:</u>

For horizontal equilibrium,

T_1 cos (60.0° – 2.8°) = T_2 cos (60.0° + 2.8°)

$$T_1 = \left(\frac{\cos 62.8^\circ}{\cos 57.2^\circ}\right) T_2$$

$$T_1 = 0.8438\, T_2 \quad \cdots(1)$$

For vertical equilibrium,

T_1 sin 57.2° + T_2 sin 62.8° = 2700 + 900 ···(2)

Substituting equation (1) into equation (2) and solving for T_2 ,

T_2 = 2250 N (3 s.f.)

T_1 = 0.8438 × 2250 = 1900 N (3 s.f.)

N

3]

1]

<u>Method 2:</u>

Taking moments about point B,

(component of T_1 perpendicular to beam)(AB)

= (combined weight of beam and worker)(horizontal distance of G from B)

(T_1 sin 60.0°)(6.0) = (3600)(2.75 cos 2.8°)

$$T_1 = \frac{3600 \times 2.75 \cos 2.8^\circ}{6.0 \sin 60^\circ} = 1900 \text{ N (3 s.f.)}$$

Taking moments about point A,

(component of T_2 perpendicular to beam)(AB)

= (combined weight of beam and worker)(horizontal distance of G from A)

(T_2 sin 60.0°)(6.0) = (3600)(3.25 cos 2.8°)

$$T_2 = \frac{3600 \times 3.25 \cos 2.8^\circ}{6.0 \sin 60^\circ} = 2250 \text{ N (3 s.f.)}$$

Long structured questions

1 **(a)** State a scenario in which an object

(i) has an acceleration at right angles to its velocity

A ball moving in a horizontal circle/ object in circular motion

[1]

(ii) has an acceleration in the opposite direction to its velocity and

A ball thrown vertically upwards (not at the highest point of motion)/ object in simple harmonic motion oscillating from equilibrium position to maximum displacement position/ car is slowing down

[1]

(iii) has an acceleration but not moving.

the highest point of the motion when the ball is thrown vertically upwards/ object in simple harmonic motion at amplitude position

[1]

(b) The graph of Fig. 1.1 shows the variation with time *t* of the velocity of a raindrop of mass 5.0 μg from the moment it is dropped from a cloud. The raindrop falls vertically down and hits the ground.

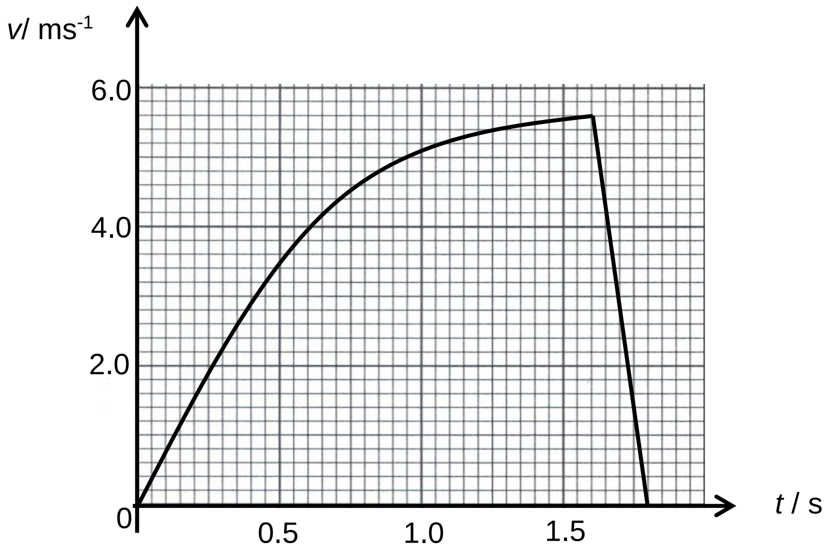


Fig. 1.1

(i) Determine the resistive force acting on raindrop at t = 1.0 s.

By Newton's 2nd Law,

net force acting on the raindrop = weight of raindrop – resistive force

$$ma = mg - F_{air}$$

$$F_{air} = 5.0\text{x}10^{-9}\,(9.81 - \frac{5.9-4.4}{1.5-0.5}) = 4.16 \times 10^{-8}\text{ N}$$

force = ... N [2]

(b) **(ii)** Explain how the average force exerted on the ground by the raindrop when it hit the ground may be determined from Fig. 1.1.

- Net force on raindrop, F_{net} = mass of raindrop(m) x acceleration of raindrop(a)
Acceleration of raindrop can be calculated from the gradient of v-t graph from t = 1.6s to t = 1.8s OR

Force to stop the motion = rate of change of momentum from t = 1.6s to t = 1.8s = $5\times10^{-9}(\frac{0-(-5.6)}{0.2})$ =1.4×10^{-7}N

- The net force acting on the raindrop is due to normal contact force by ground(F_{ground}) minus the weight of raindrop, i.e. i.e. $F_{net} = F_{ground} - mg$, acting upwards.

- By Newtons's 3rd law, force acting on the raindrop by ground is equal in magnitude but in opposite direction of force acting on ground by raindrop.

- Therefore, force exerted on ground by raindrop =1.89×10^{-7} N acting downwards.

[4]

(iii) State and explain what would happen to your answer in **b(ii)** if the raindrop rebounds with the same speed after it hits the ground.

In (b)(ii), the final speed of the water is zero. If raindrop rebound, the rate of change of momentum of the raindrop is higher and hence the force exerted on ground by raindrop will be larger.

[2]

(c) Fig. 1.2 below shows part of an experiment that is being used to estimate the speed of an air gun pellet.

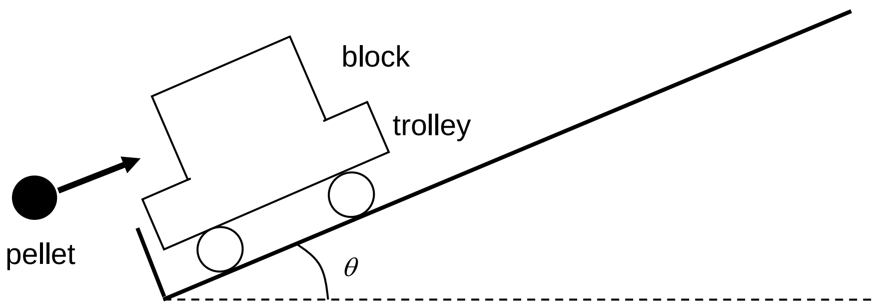


Fig. 1.2

The pellet which is moving parallel to the track, strikes the block with a speed of *u* before it is embedded to the block. The trolley and the block then move along the smooth track, rising a vertical height, *h*.

(i) Explain how the speed of the pellet *u* can be determined from measurements of *h*.

Speed of trolley after collision, V_f can be obtained from conservation of energy. Gain in gravitational of potential energy of trolley with block and pellet equal to loss in Kinetic Energy of trolley with block and pellet after collision. i.e. $Mgh = \frac{1}{2} M v_f^2$, $v_f = \sqrt{2gh}$

By conservation of linear momentum, initial momentum of pellet = final momentum of trolly with block and pellet, i.e. $mu = Mv_f$

Speed of pellet, $u = \frac{M}{m} v_f$

[2]

(ii) Explain whether the speed of pellet *u,* is an underestimated or overestimated if frictional forces are not negligible.

u is calculated assuming all kinetic energy of trolley with block and pellet after collision is converted to gravitational potential energy. With presence of friction, kinetic energy of trolley after collision are converted to gravitational potential energy of trolley and workdone against frictional force. Since h is a fixed measurement, u calculated earlier on is an underestimated.

[1]

(d) The following data is collected from the experiment:

Mass of trolley and block	0.50 kg
Mass of pellet	0.0020 kg
Speed of trolley and block immediately after impact	0.40 m s^{-1}

(i) State the principle of conservation of linear momentum.

If there is no resultant external force acting on a system of bodies, the total linear momentum of the system always remain constant.

[2]

(ii) Calculate the speed of the pellet just before impact.

By conservation of linear momentum,

$P_i = P_f$

$0.002(u) = (0.5002)(0.4)$

$u = 100$ m s^{-1}

speed = ... m s^{-1} [2]

(e) Use the answer from part (d) to show that the collision between the pellet and block is inelastic.

Kinetic Energy of pellet being hitting trolley with block = ½ (0.002)(100^2) = 10.0 J

Kinetic Energy of trolley with block and pellet after collision

= ½ (0.502)(0.4^2) = 0.0402 J

Since Kinetic Energy before and after collision are not the same value. The collision between pellet and block is not inelastic.

[2]

[NJC 2013 P3 Q7]

2 **(a)** Fig. 2.1 shows a table from a car driver's handbook.

On a dry road, a car in good condition driven by an alert driver will stop in the distances shown in the table.

Speed / km h^{-1}	Reaction distance / m	Braking distance / m	Overall stopping distance / m
15	2.5	1.45	3.95
30	5.0	5.80	10.8
45	7.5	13.0	20.5
60	10.0	23.1	33.1
75	12.5	36.1	48.6
90	15.0	52.1	67.1

The reaction distance is the distance travelled by the car during the driver's reaction time. The braking distance is the distance in which the car stops after the brakes have been applied.

Fig. 2.1

(i) Show that the value of constant deceleration assumed during the braking is approximately 6.0 m s^{-2}.

Consider when speed = 15 km h^{-1}

Using $v^2 = u^2 + 2as$,

$$0 = \left(\frac{15 \times 1000}{3600}\right)^2 + 2a(1.45)$$

$a = 6.0$ m s^{-2}

[2]

(ii) Calculate the overall stopping distance for a car travelling at 140 km h^{-1}.

$$\text{reaction time} = \frac{2.5}{15 \times \frac{1000}{3600}} = 0.60 \text{ s}$$

$$\text{braking distance} = \frac{u^2}{2a} = \frac{\left(\frac{140 \times 1000}{3600}\right)^2}{2 \times 6.0} = 126.03 \text{ m}$$

$$\text{overall stopping distance} = \left(\frac{140 \times 1000}{3600}\right)0.60 + 126.03 = 149 \text{ m}$$

overall stopping distance = ... m [3]

(iii) Calculate the overall stopping distance for a car travelling at 60 km h^{-1} down a hill at an angle of 10° to the horizontal.

$$\text{braking distance} = \frac{u^2}{2a} = \frac{\left(\frac{60 \times 1000}{3600}\right)^2}{2(6.0 - 9.81\sin 10°)} = 32.33 \text{ m}$$

$$\text{overall stopping distance} = 10.0 + 32.33 = 42.3 \text{ m}$$

overall stopping distance = ... m [2]

(iv) Explain why reaction distance is directly proportional to speed while braking distance is not.

Reaction time for a particular driver is almost constant and occurs at constant speed before brakes are applied.
Therefore reaction distance is proportional to speed.
For braking distance, at constant deceleration,
Braking distance is proportional to square of speed.

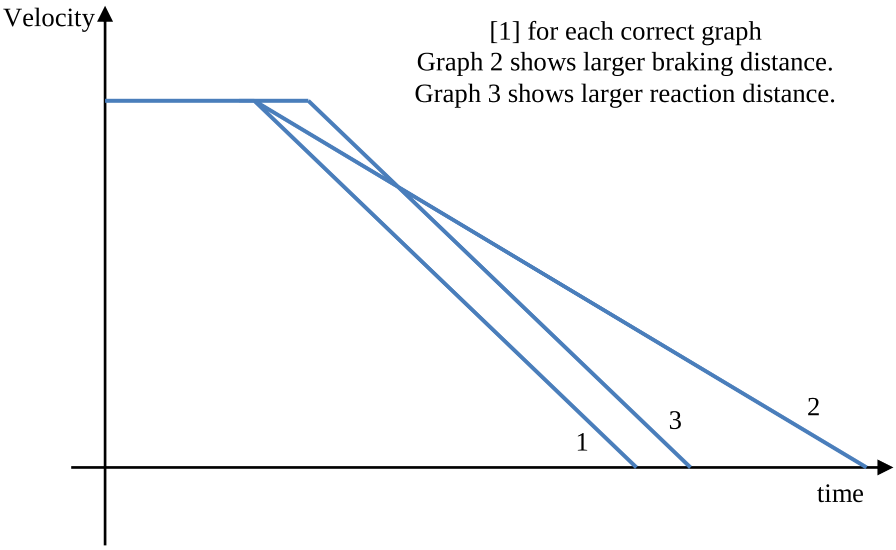
[3]

(v) On the axes provided in Fig. 2.2, sketch the graphs to show the variation of velocity with time under the following conditions:

1. The road is dry and the driver is fully alert.
2. The road is wet and the driver is fully alert.
3. The road is dry and the driver is not fully alert.

Label the graphs as 1, 2 and 3 respectively. [3]

Fig. 2.2



(b) After witnessing a head-on collision between a truck and a small passenger car travelling at the same speed, Frankie, an eager H2 Physics student makes the following comments:

(i) "The truck driver will have a higher chance of surviving the accident since the small passenger car will exert a smaller force on the truck during the collision."

(ii) "Principle of conservation of momentum is violated since the 2 moving vehicles came to a stop eventually after the collision."

Discuss the validity of Frankie's comments.

(i) By Newton's 3rd law, the forces the truck and car exert on each other are equal and opposite, the force by car is not smaller. Rate of change of momentum of truck and car is equal in magnitude.
By Newton's 2nd law, the deceleration experienced by the truck is smaller since the mass of the truck is bigger, given the same force. Hence the truck driver has higher chance of surviving as the force acting on him is smaller (his rate of change of momentum is smaller as it is calculated by his mass and the lower deceleration). [2]

(ii) For the system of car and truck, there is net external force due to resistive forces, friction and drag force.
Therefore momentum is not conserved and becomes zero. [2]

(iii) It was later found that regions at the front and rear of the passenger car are designed to collapse on impact, but the shell of the passenger compartment is of rigid construction.

Explain how these design features may help to protect passengers from serious injury during a collision.

On impact, the collapse of the regions at the front and rear of the car slows down the momentum change.
As force is proportional to rate of change of momentum, force on passenger is smaller and less damaging.
Rigid construction of passenger shell protects passengers from being crushed

.... [3]

[SAJC 2012 P3 Q5]

3 **(a)** Fig. 3.1 shows the displacement-time graph of a moving object from a point P.

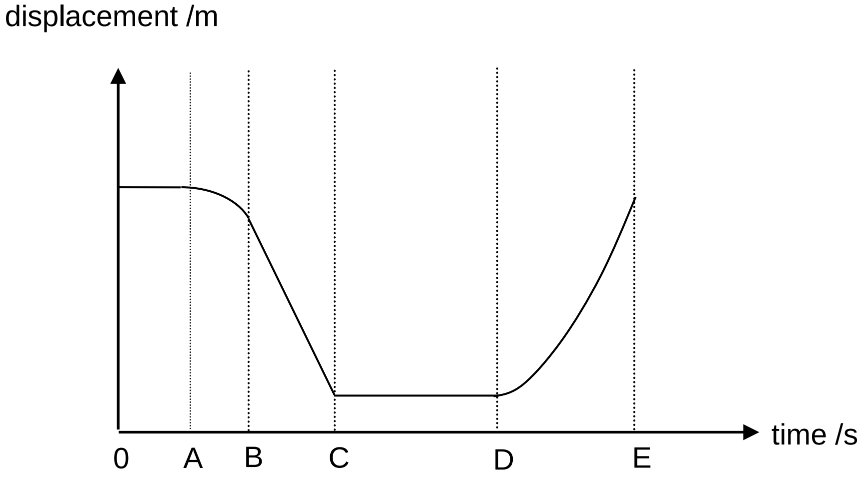


Fig. 3.1

A student describes the state of motion of the object for time interval A to B as follows:

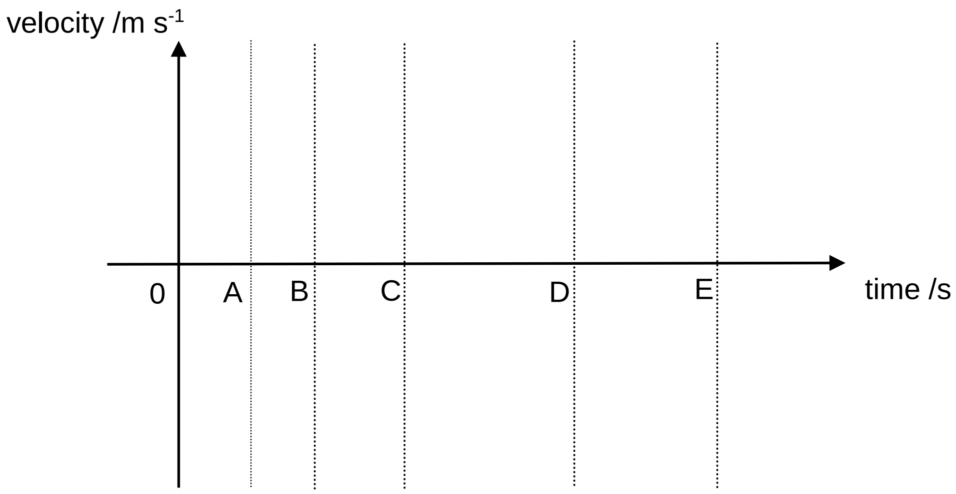
"The object is moving away from point P. It is slowing down and is decelerating towards P."

(i) Comment and explain the validity of the student's description.

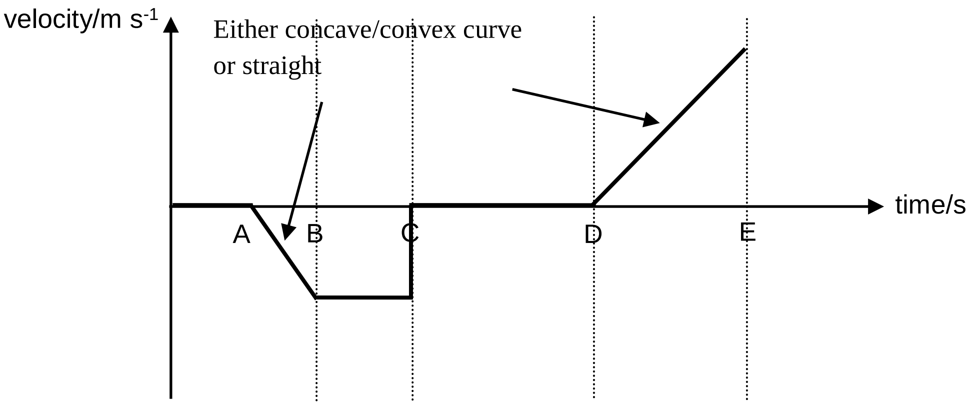
The statement is invalid because
Object is moving towards P as displacement from P is decreasing.
It is speeding up as (magnitude of) the gradient which denotes speed, is increasing with time
It is accelerating towards P as its speed is increasing towards P.

.... [3]

(ii) Sketch the corresponding velocity-time graph of the object for the whole journey.



[2]



(b) A tennis ball, released from rest, falls vertically to the floor and bounces up. Explain how Newton's Third Law and the Principle of Conservation of Momentum apply in this situation.

By Newton's law of gravitation, the tennis ball exerts a gravitational force upward on the floor & by Newton's 3rd law the floor exerts an equal and opposite force acting downward on the ball.

At the point of impact, the ball exerts a repulsive force on the floor downward, and the floor exerts an equal and opposite force upward on the ball.

When the ball is released from rest, the total momentum of the system of ball and the earth is zero. When the ball is falling down, it gains downward momentum. The floor is gaining upward momentum; and so the total momentum of the system is still zero and conserved. After the collision, the ball moves upwards with decreasing momentum in the upward direction. The floor will move downwards also with decreasing momentum so that the total momentum of the system is zero and conserved.
At a certain instant during impact where both have zero velocity, the total momentum of the system is zero and conserved.

[4]

(c) A 20-kg projectile is fired at an angle of 60° above the horizontal with a velocity of 400 m s^{-1}, from the ground. At the highest point of its trajectory, the projectile explodes into two fragments of equal mass. Assume air resistance is negligible.

(i) Explain why the velocity of the projectile at the highest point of the trajectory, just before the explosion, is 200 m s^{-1} in the horizontal direction.

Net force is provided by the gravitational force which acts on the projectile, vertically downwards.
Thus the horizontal component of the velocity of the projectile remains a constant.
Thus $v_x = u_x = u \cos\theta = 400 \cos 60^0 = 200$ m s^{-1}

[1]

(ii) After the explosion one of the fragments falls vertically with zero initial speed. Calculate the speed of the other fragment just after the explosion.

By principle of conservation of momentum in x-direction,

$M\,u_x = \frac{M}{2} v_{1,x} + \frac{M}{2} v_{2,x}$

Given that $v_{1,x} = 0$,

$\rightarrow M\,(200) = \frac{M}{2}(0) + \frac{M}{2} v_{2,x}$

$v_{2,x} = 400$ m s^{-1}

speed = ... m s^{-1} [2]

(iii) Both fragments eventually land on the ground, at the same level as the firing position. If the horizontal distance travelled by the projectile *just* before the explosion is 7100 m, determine the positions of both fragments when they land on the ground *relative to the firing position*.

Since the first fragment falls vertically downward, its displacement from the starting position is 7100 m also.

The second fragment takes the same time to fall to the ground as it takes to rise.

Time taken to rise is found from $S_x = u_x t$

ie $7\,100 = 200\,t$

$\rightarrow t = 35.5$ s

Horizontal distance travelled by second fragment as it falls $x_2 = v_{2,x} t = 400\,(35.5) = 14\,200$ m

Hence position of 2nd fragment relative to firing position

$= 7100 + x_2 = 7100 + 14200 = 21000$ m

position of 1st fragment = ... m;

position of 2nd fragment = ... m [4]

(d) A light spring is attached between two masses m_1 and m_2 resting on a frictionless floor. A force of 50 N is applied on m_1 as shown in Fig. 3.2.

Given: $m_1 = 2.0$ kg, $m_2 = 4.0$ kg, and the spring constant is 12 N cm^{-1}.

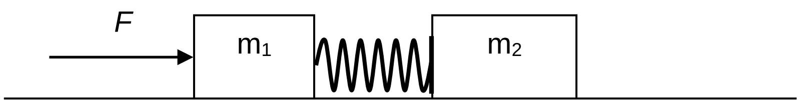


Fig. 3.2

(i) Calculate the acceleration of the masses.

$F = (m_1 + m_2)\, a$
ie $50 = (2.0 + 4.0)\, a$
$\rightarrow a = 8.3 \text{ m s}^{-2}$

acceleration = ... m s^{-2} [1]

(ii) Calculate the compression of the spring.

Consider m_1: $F - F_S = m_1 a$
$F_S = 50 - 2.0\,(8.33) = 33.33$ N

$F_S = kx$
$\rightarrow 33.33 = 12\, x$
$x = 2.8$ cm
Or,
Consider m_2: $F_s = m_2 a = 4 \times 8.33 = 33.3$ N
$F_S = kx$

$\rightarrow 33.33 = 12\, x$
$x = 2.8$ cm

ression = ... cm [3]

[SAJC 2013 P3 Q6]

4 **(a)** **(i)** Define acceleration.

Acceleration is the rate of change of velocity with respect to time. [1]

(ii) State Newton's second law of motion.

Newton's second law of motion states that the rate of change of momentum of a body is proportional to the resultant force acting on it and occurs in the direction of the force. [2]

(b) The variation with time t of vertical speed v of a parachutist falling from an aircraft is shown in Fig. 4.1. The mass of the parachutist is 95 kg.

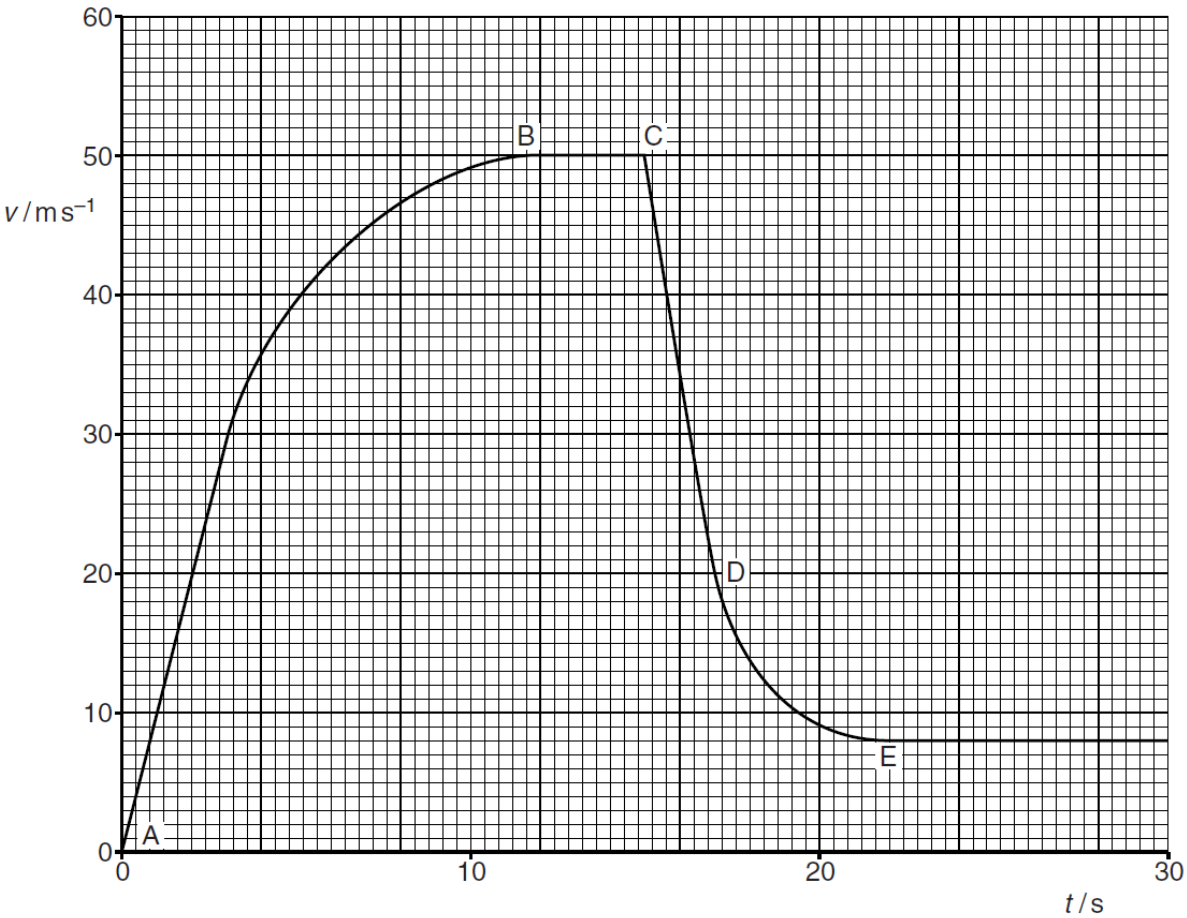


Fig. 4.1

(i) Calculate the distance travelled by the parachutist in the first 3.0 s of the motion.

distance is represented by the area under graph

distance = (½) (29.5) (3.0) = 44.3 m

distance = ... m [2]

(ii) Explain the variation of the resultant force acting on the parachutist from t = 0 (point A) to t = 15 s (point C).

resultant force = (weight – frictional force)

Since frictional force increases with speed (or speed^2 for high speeds), resultant force decreases.

At the start, the resultant force is constant as frictional force is negligible.

At the end, the resultant force is zero as frictional force = weight.

[3]

(iii) Describe the changes to the frictional force on the parachutist

1. at t = 15 s (point C),

The frictional force increases (drastically).

[1]

2. between t = 15 s (point C) and t = 22 s (point E).

Frictional force is constant between points C and D and then decreases between point D and E.

[2]

(iv) Calculate, for the parachutist between t = 15 s (point C) and t = 17 s (point D),

1. the average acceleration,

$$\langle a \rangle = \frac{(v_2 - v_1)}{(t_2 - t_1)} = \frac{(20.0 - 50.0)}{(17.0 - 15.0)} = -15 \text{ m s}^{-2}$$

acceleration = ... m s^{-2} [2]

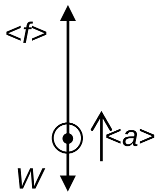
2. the average frictional force.

downward direction as positive

W – <f> = m<a>

W = (95)(9.81) N

<f> = (95)(9.81) – 95 (–15) = 2400 N (2360 N) (2357 N)



frictional force = ... N [3]

(v) The frictional force on an object falling through air at high speeds v is given by Dv^2, where D is a constant. The value of D depends on the characteristics of the object and on the density of the air.

1. Calculate the change in the value of D when the parachutist is at t = 14 s and at t = 24 s.

$$mg = Dv^2 \Rightarrow D = \frac{mg}{v^2}$$

$v = 50.0$ m s^{-1} at t = 14 s

$v = 8.0$ m s^{-1} at t = 24 s

$$\text{change in } D = mg\left(\frac{1}{8.0^2} - \frac{1}{50.0^2}\right) = (95)(9.81)\left(\frac{1}{8.0^2} - \frac{1}{50.0^2}\right) = 14 \text{ kg m}^{-1}$$

change in D = ... kg m^{-1} [3]

2. Explain why there is a change in the value of D.

An increase in the cross-sectional area measured in a plane perpendicular to its velocity, after the parachute opens or a change in shape after the parachute opens.

.... [1]

[RI 2013 P3 Q6]